A stabilized brane world in five-dimensional Brans-Dicke theory

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Abstract

We discuss brane world models, which can be constructed in the five-dimensional Brans-Dicke theory. For different choices of potentials we get an unstabilized model with the Randall-Sundrum solution for the metric and constant solution for the scalar field and a stabilized brane world model, which reproduces the Randall-Sundrum solution for the metric and gives an exponential solution for the scalar field.

1 Introduction

Brane world models and their phenomenology have been widely discussed in the last years [1, 2]. One of the most interesting brane world models is the Randall-Sundrum model with two branes, – the RS1 model [3]. This model solves the hierarchy problem due to the warp factor in the metric and predicts an interesting new physics in the TeV range of energies. A flaw of the RS1 model is the presence of a massless scalar mode, – the radion, which describes fluctuations of the branes with respect to each other. Its interactions contradict the existing experimental data, and in order the model be phenomenologically acceptable the radion must acquire a mass, which is equivalent to the stabilization of the brane separation distance. The latter can be achieved, for example, by introducing a five-dimensional scalar field with bulk and brane potentials, see, for esample, [4]. However the background metric of this solution is rather complicated and differs significantly from the Randall-Sundrum solution. An interesting problem is, whether it is possible to find a stabilized model, where the Randall-Sundrum form of the metric is retained despite the interaction with the scalar field.

A solution to this problem based on a non-minimal coupling of the scalar field to gravity was put forward in paper [5], in which the form of the metric was found to be the Randall-Sundrum one, whereas the solution for the scalar field again turned out to be rather complicated.

One of the standard forms of the non-minimal coupling of the scalar field to gravity is the linear coupling to scalar curvature used in the Brans-Dicke theory of gravity. Brans-Dicke theory in the brane world context was already discussed in the literature, see, for example, [6].

In the present paper we use this type of coupling to construct a stabilized model with the Randall-Sundrum solution for the metric, the solution for the scalar field being also a simple exponential function. Our paper is organized as follows. First, we present a method for constructing different backgrounds in five-dimensional Brans-Dicke theory by considering bulk and brane scalar field potentials of a special form. Then we discuss models which can be obtained with different choices of the potentials and parameters. And finally, we discuss the obtained results.

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2 Setup

Let us denote the coordinates in five-dimensional space-time $E = M_4 \times S^1/Z_2$ by $\{x^M\} \equiv \{x^\mu, y\}$, $M = 0,1,2,3,4$, $\mu = 0,1,2,3$, the coordinate $x^4 \equiv y$, $-L \le y \le L$ parameterizing the fifth dimension. It forms the orbifold, which is realized as the circle of the circumference $2L$ with the points y and $-y$ identified. Correspondingly, the metric g_{MN} and the scalar field ϕ satisfy the orbifold symmetry conditions

$$\begin{aligned} g_{\mu\nu}(x,-y) = g_{\mu\nu}(x,y), \quad g_{\mu 4}(x,-y) = -g_{\mu 4}(x,y), \\ g_{44}(x,-y) = g_{44}(x,y), \quad \phi(x,-y) = \phi(x,y). \end{aligned} \tag{1}$$

The branes are located at the fixed points of the orbifold, $y = 0$ and $y = L$.

The action of the brane world models can be written as

$$\begin{aligned} S &= \int d^4x \int_{-L}^{L} dy \sqrt{-g}\left[\phi R - \frac{\omega}{\phi} g^{MN}\partial_M\phi\partial_N\phi - V(\phi)\right] - \\ &- \int_{y=0} \sqrt{-\tilde{g}}\lambda_1(\phi) d^4x - \int_{y=L} \sqrt{-\tilde{g}}\lambda_2(\phi) d^4x. \end{aligned} \tag{2}$$

Here $V(\phi)$ is a bulk scalar field potential and $\lambda_{1,2}(\phi)$ are brane scalar field potentials, $\tilde{g} = det\tilde{g}_{\mu\nu}$, $\tilde{g}_{\mu\nu}$ denotes the metric induced on the branes and ω is the five-dimensional Brans-Dicke parameter. The formal restriction on this parameter in the five-dimensional Brans-Dicke theory is $\omega > -\frac{4}{3}$ (which can be easily seen, for example, from the formulas of paper [7]), and further restrictions on its value can be obtained only after studying an effective four-dimensional theory for a certain background solution. The only difference from the classical Brans-Dicke theory is the presence of the bulk scalar field potential $V(\phi)$ and the branes. The signature of the metric g_{MN} is chosen to be $(-,+,+,+,+)$.

The standard ansatz for the metric and the scalar field, which preserves the Poincaré invariance in any four-dimensional subspace $y = const$, looks like

$$ds^2 = e^{2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \quad \phi(x,y) = \phi(y), \tag{3}$$

$\eta_{\mu\nu}$ denoting the flat Minkowski metric. If one substitutes this ansatz into the equations corresponding to action (2), one gets a rather complicated system of nonlinear differential equations for functions $\sigma(y), \phi(y)$:

$$3\sigma''\phi + \frac{\omega}{\phi}(\phi')^2 + \phi'' - \sigma'\phi' + \frac{\lambda_1}{2}\delta(y) + \frac{\lambda_2}{2}\delta(y-L) = 0, \tag{4}$$

$$6(\sigma')^2\phi - \frac{1}{2}\left(\frac{\omega}{\phi}(\phi')^2 - V\right) + 4\sigma'\phi' = 0, \tag{5}$$

$$\frac{\omega}{\phi}\left(\phi'' + 4\sigma'\phi'\right) - 4\sigma'' - 10(\sigma')^2 - \frac{\omega}{2\phi^2}(\phi')^2 = \frac{1}{2}\frac{dV}{d\phi} + \frac{1}{2}\frac{d\lambda_1}{d\phi}\delta(y) + \frac{1}{2}\frac{d\lambda_2}{d\phi}\delta(y-L), \tag{6}$$

where prime denotes the derivative with respect to extra dimension coordinate y.

We will consider a special class of bulk potentials, namely

$$V(\phi) = -(3\omega+4)\left[(4\omega+5)\,\phi F^2 + 2\phi^2 F\frac{dF}{d\phi} - 3\phi^3\left(\frac{dF}{d\phi}\right)^2\right], \tag{7}$$

where $F \equiv F(\phi)$ is a function. One can check that in this case any solution of the equations

$$\phi' = \phi F - 3\phi^2\frac{dF}{d\phi}, \tag{8}$$

$$\sigma' = (\omega + 1) F + \phi \frac{dF}{d\phi} \tag{9}$$

also satisfies equations (4), (5) and (6) in the interval $[0, L]$, provided that the following boundary conditions on the branes are satisfied

$$(3\omega + 4) F|_{y=+0} = -\frac{\lambda_1}{4\phi}, \tag{10}$$

$$(3\omega + 4) F|_{y=L-0} = \frac{\lambda_2}{4\phi}, \tag{11}$$

$$(3\omega + 4) \frac{dF}{d\phi}|_{y=+0} = -\frac{1}{4}\frac{d(\lambda_1/\phi)}{d\phi}, \tag{12}$$

$$(3\omega + 4) \frac{dF}{d\phi}|_{y=L-0} = \frac{1}{4}\frac{d(\lambda_2/\phi)}{d\phi}. \tag{13}$$

It is necessary to note that the symmetry conditions (1) were used to obtain these relations.

Thus we get first order differential equations instead of the initial second order differential equations. One can recall papers [4, 8], where an analogous situation arises, if the bulk and the brane potentials for the scalar field minimally coupled to five-dimensional gravity are chosen in an appropriate way.

3 The Randall-Sundrum solution

Let us choose the function $F(\phi)$ in the following form

$$F = B\phi^{\frac{1}{3}}, \tag{14}$$

where B is a constant. It follows from (8) that $\phi = const$ in this case. Bulk and brane potentials are

$$V(\phi) = -\frac{4}{3}B^2\phi^{\frac{5}{3}} (3\omega + 4)^2, \tag{15}$$

$$\lambda_{1,2} = \mp 4B (3\omega + 4) \phi^{\frac{4}{3}}, \tag{16}$$

and solution for the warp factor is

$$\sigma' = \frac{B}{3} (3\omega + 4) \phi^{\frac{1}{3}} \tag{17}$$

in the interval $[0, L]$. Making redefinition

$$\frac{B}{3} (3\omega + 4) \phi^{\frac{1}{3}} = -k, \tag{18}$$

$$\phi = M^3, \tag{19}$$

we get

$$V(\phi) = -12k^2M^3, \tag{20}$$

$$\lambda_{1,2} = \pm 12kM^3, \tag{21}$$

which formally coincide with the original RS1 solution [3]. Nevertheless the linearized theory in this background differs from that in the RS1 model, because the fluctuations of the scalar field add an extra degree of freedom. In this case the model cannot be stabilized (i.e. the size of the extra dimension cannot be fixed) by adding positively defined potentials on the branes, since the solution for the scalar field does not depend on y.

4 Stabilized brane world with the Randall-Sundrum solution for the metric

Now let us consider the case $F = const$, i.e. it does not depend on the field ϕ. Bulk and brane potentials, corresponding to such a choice of $F(\phi)$, can be chosen to be (see (7) and (10)–(13))

$$V(\phi) = \Lambda\phi, \tag{22}$$

$$\lambda_{1,2} = \pm\lambda\phi. \tag{23}$$

Let us suppose that $\lambda > 0$. It is not difficult to check that from equations (8)–(13) follows

$$\sigma = -k|y|, \tag{24}$$

$$\phi = Ce^{-u|y|} \tag{25}$$

with

$$u = \sqrt{\frac{-\Lambda}{(3\omega+4)\,(4\omega+5)}}, \tag{26}$$

$$k = (\omega+1)u, \tag{27}$$

and the fine-tuning relation

$$\lambda = 4\sqrt{-\Lambda}\sqrt{\frac{3\omega+4}{4\omega+5}}. \tag{28}$$

Constant C is not defined by the equations. One can see that in the limit $\omega \to \infty$ we arrive at the standard Randall-Sundrum solution.

Now let us discuss stabilization mechanism which can be utilized in the case under consideration. We will follow the way proposed in [4] and add stabilizing quadratic potentials on the branes, namely

$$\Delta\lambda_{1,2} = \gamma_{1,2}\,(\phi - v_{1,2})^2\,, \quad \gamma_{1,2} > 0. \tag{29}$$

Such an addition will not affect equations of motion provided

$$\phi|_{y=0} = v_1, \quad \phi|_{y=L} = v_2. \tag{30}$$

Thus, now the constant C appears to be defined and is equal $C = v_1$, whereas the size of extra dimension is now defined by the relation

$$L = \frac{1}{u}\ln\left(\frac{v_1}{v_2}\right), \tag{31}$$

which is the same as in the model of paper [4]. Obviously, this stabilization mechanism works only for solutions with the scalar field depending on the extra coordinate, and does not work for the solution in section 3.

In fact, the power law potentials in this approach, as well as in the approach of paper [4], can be unbounded from below. The question, whether such systems with the unbounded from below bulk potentials are stable against arbitrary fluctuations of gravitational and scalar fields, demands a thorough investigation. The final answer can be obtained, for example, with the help of the second variation Lagrangian formalism, as it was done in paper [9] for the model of paper [4], which turned out to be stable. There is a good reason to believe that the stabilized models found in the present paper are stable as well.

Now let us find the relationship between the four-dimensional Planck mass and the parameters of the theory. We assume that the brane at $y = L$ is "our" brane. To this end one should choose $\sigma = -k|y| + kL$ to make four-dimensional coordinates $\{x^\mu\}$ Galilean on this brane (this problem was discussed in detail in [10]). Naive considerations suggest that the wave function of massless four-dimensional tensor graviton has the same form as that in the unstabilized Randall-Sundrum model, namely $h^0_{\mu\nu}(x,y) = e^{2\sigma}\alpha_{\mu\nu}(x)$. Moreover, the form of the residual gauge transformation, which are left after imposing the gauge on the fluctuations of metric (see [9, 10]), also suggests the same form of the wave function. Thus, substituting the following ansatz

$$g_{\mu\nu}(x,y) = e^{2\sigma}g^{(4)}_{\mu\nu}(x), \quad \phi(x,y) = \phi(y), \quad g_{44}(x,y) = 1, \quad g_{\mu 4}(x,y) = 0 \tag{32}$$

into action (2), we get

$$S = \int_{-L}^{L} \phi e^{2\sigma} dy \int R_{(4)}\sqrt{-g_{(4)}}\, d^4x \tag{33}$$

and

$$2M_{Pl}^2 = \int_{-L}^{L} \phi e^{2\sigma} dy = \frac{2v_1}{2k+u}\left(e^{2kL} - e^{-uL}\right). \tag{34}$$

If $uL < 1$ we get a formula analogous to that in the unstabilized Randall-Sundrum model (see [2, 10]). To solve the hierarchy problem one needs kL to be of the order $kL \sim 30$. If one chooses relatively large ω (for example, $\omega \geq 30$), then k would go to the value corresponding to the unstabilized Randall-Sundrum model, namely

$$k \approx \sqrt{\frac{-\Lambda}{12}}, \tag{35}$$

(compare with (19) and (20)) whereas uL could be made less than unity ($uL < 1$), since $u = k/(\omega+1)$. Under this assumption the parameter v_1 can be chosen to be of the same order as v_2. In this case the parameters of the model, made dimensionless with the help of a fundamental scale in the TeV range, do not contain a hierarchical difference. The situation turns out to be completely analogous to that in the model proposed in [4]. At the same time, solution for the warp factor in stabilized brane world model found in [4] is quite complicated, which impedes the analysis of the equations of motion for linearized gravity (approximate solutions can be found only under certain assumptions and simplifications, see [9]). As for our case, one can think that though the general structure of action (2) is more complicated than that of the action used in [4], the simplicity of solutions (24) and (25) could result in simpler equations of motion for linearized gravity.

Another advantage of the solution presented above is that one can use all the results, obtained for the case of the universal extra dimensions in the Randall-Sundrum model (i.e. if one allows additional fields to propagate in the bulk, see, for example, [11] and references therein), in our case too. This happens because of the equivalence of the solutions for the warp factors in both models. Of course, it is true in the case of the standard coupling of five-dimensional gravity to matter in the bulk. But since the size of extra dimension in our model appears to be stabilized, one can think that this would allow us to avoid possible problems caused by the radion field, which are inherent in the unstabilized Randall-Sundrum model.

One can pose the question about the correspondence of the solution found above with the one, which can be obtained in the Einstein frame. Indeed, if we make conformal rescaling in action (2) with the potential and parameters discussed in this section, allowing one to turn from the Jordan frame to the Einstein frame, we would get a model describing scalar field with the Liouville potential, minimally coupled to five-dimensional gravity. Such "dilatonic" brane worlds were widely discussed in the literature, see, for example [12, 13]. In particular, our solutions can be brought to the form of the general solutions of paper [12] by a conformal rescaling of the metric and an appropriate transformation of the extra dimension coordinate, the latter being necessary for retaining the same ansatz for the metric (of the form (3)). Nevertheless, it is not evident that the general solutions of paper [12] for certain values of the parameters can be transformed to our solution, discussed in this section. Moreover, our solution gives a simple solution to the problem of finding a stabilized brane world model with the Randall-Sundrum form of the metric, which was discussed in [5]. In this paper a similar solution in a theory with the scalar field non-minimally coupled to gravity, which preserves the Randall-Sundrum form of the metric, was found. But this solution and ours cannot be transformed to each other by a redefinition of fields, rescaling of metric and coordinate transformations.

One more important point is that the theories obtained by a rescaling of the metric are not equivalent, if one considers the standard coupling of gravity to matter on the branes (the so-called conformal ambiguity [14]). Since in stabilized brane world models the fluctuations of the scalar field describe also the radion (see [9]), this ambiguity modifies, in particular, the radion coupling to matter on the branes. Because we do not know, which frame is the "real" one, there are no strong objections against choosing the Jordan one. In this connection, an interesting problem is to compare the physical consequences of the models in different frames, which can be transformed one into another in the absence of matter on the branes.

A reasonable question arises – what happens to the mass of the radion? Naive calculations give

$$m_{rad}^2 = \frac{\gamma_2 u^2 v_1 e^{-2uL}}{4k}, \tag{36}$$

which is similar in some sense to the result obtained in [9] for the radion mass in the stabilized brane world model proposed in [4]. It is evident, that with an appropriate choice of parameters the radion mass (36) can be made to be in the TeV range, which can be interesting from the experimental point of view and does not contradict the known data. Nevertheless, an answer to the question posed above can be obtained only after a thorough examination of linearized gravity in the models. This problem calls for further investigation.

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