Violating general covariance

Yu. F. Pirogov*
Theory Division, Institute for High Energy Physics,
Protvino, 142281 Moscow Region, Russia

Abstract

The explicit violation of the general covariance on the whole and its minimal violation to the unimodular covariance specifically is considered. The proper extension of General Relativity is shown to describe consistently the massive scalar graviton together with the massless tensor one, as the parts of the metric. The bearing of the scalar graviton to the dark matter and dark energy is indicated.

1 Motivation

The General Relativity (GR) is the viable theory of gravity, very robust in the underlying principles. It is known to consistently describe the massless tensor graviton as a part of the metric field. This is insured by the general covariance (GC) which serves as the gauge symmetry to eliminate the degrees of freedom contained in the metric in excess of the massless tensor graviton. Nevertheless, phenomenologically, the application of GR to cosmology encounters a number of problems, superior of which are those of the dark energy (DE) and the dark matter (DM). In particular, to solve the latter problem one adjusts usually the conventional or hypothetical matter particles, remaining still in the realm of GR. The ultimate goal of DM being in essence to participate only in the gravitational interactions, one can try to attribute to the aforesaid purpose the additional degrees of freedom contained in the metric, going thus beyond GC. With this in mind, I discuss in the given report the self-consistent extension of GR, with the explicit violation of GC to the residual unimodular covariance (UC). In addition to the massless tensor graviton, such an extension describes the massive scalar graviton as a part of the metric field. The scalar graviton is proposed as a resource of the gravitational DM, as well as the scale dependent part of DE.[1]

2 GC and beyond

Poincare group Let us first discuss the problem of the GC violation from the point of view of the particle representation in the relativistic quantum mechanics. The free particles are described by the irreducible finite-dimensional unitary representations of the Poincare group $ISO(1,3)$ [2]. The proper representations (m,s) are characterized by the mass m and spin s. The massless particles, $m=0$, possess the isotropic momentum k_μ, $k\cdot k=0$. The invariance group of the momentum (the "little" group) proves to be $ISO(2)$, which is noncompact. The unitary representations of the noncompact groups are known to be infinite-dimensional, but for the scalar representations. Thus, for a unitary representation of the Poincare group to be finite-dimensional the noncompact generators of the little group (here the "translations"of $ISO(2)$) should act trivially on the representation. It follows thereof that the massless particles

*e-mail: pirogov@ihep.ru

[1]The report is partly based on ref. [1], where more details can be found.

of the spin $s \geq 1$ should be described not by the rays in a Hilbert space but by the respective equivalence classes. This means that the theory for the spin $s \geq 1$ should possess the invariance relative to transformations within the proper equivalence classes, in other words, be gauge invariant. Thus, the gauge invariance is not a mere accident but is in fact deeply rooted in the unitarity requirement for the relativistic quantum theory.

Remind that the spin-one massless particle, say, photon is described by the transverse vector $\hat{A}_\mu(k)$, $k \cdot \hat{A} = 0$. The gauge transformations required for the triviality of the noncompact generators, and thus for the unitarity, is $\hat{A}_\mu \to \hat{A}_\mu + \alpha k_\mu$, with $\alpha(k)$ being a scalar. The respective gauge group is $U(1)$. Due to this, one is left with the two-component photon possessing helicities $\lambda = \pm 1$. Likewise, the spin-two massless particle, the graviton, is described by the transverse-traceless symmetric tensor $\hat{h}_{\mu\nu}(k)$, with $k^\mu \hat{h}_{\mu\nu} = 0$ and $\hat{h}^\mu_\mu = 0$ [3]. The gauge transformations required for the triviality of the $ISO(2)$ translations prove to be

$$\hat{h}_{\mu\nu} \to \hat{h}_{\mu\nu} + \xi_\mu k_\nu + \xi_\nu k_\mu, \tag{1}$$

with $\xi_\mu(k)$ restricted by $k \cdot \xi = 0$. The respective three-parameter group corresponds precisely to UC. Altogether, one arrives at the two-component graviton with the helicities $\lambda = \pm 2$. Thus, UC is necessary and sufficient for the consistent description of the massless tensor graviton. In this, the massive scalar graviton can additionally be represented by the independent scalar $\hat{h}(k)$ for the time-like momentum k_μ, $k \cdot k = m^2 > 0$. The little group of the momentum being the compact $SO(3)$, the respective gauge transformations are trivial.

One can abandon the reducibility requirement for the representation of the massless tensor graviton, describing the latter at $k \cdot k = 0$ by the arbitrary transverse symmetric tensor $\hat{h}_{\mu\nu}(k)$, $\hat{h}^\mu_\mu \neq 0$. For consistency, this requires the whole gauge group, with arbitary ξ_μ corresponding to GC. Under these transformations, the trace changes as $\hat{h}^\mu_\mu \to \hat{h}^\mu_\mu + 2k \cdot \xi$ and thus can be removed, leaving no scalar graviton. It follows thereof that GC, with ξ_μ unrestricted, though being commonly used and sufficient to consistently describe the massless tensor graviton, is in fact redundant.

Field theory Let x^μ, $\mu = 0, \ldots, 3$, be the arbitrary observer's coordinates. Let us now consider the same problem of the GC violation in the framework of the Lorentz-invariant local field theory of the symmetric tensor $h_{\mu\nu}(x)$. The latter is treated as a part of the dynamical metric field $g_{\mu\nu}(x)$. The effective field theory of the metric is to be built of the metric itself and its first derivatives $\partial_\lambda g_{\mu\nu}$ (as well as, generally, the higher ones). Otherwise, one can use the Christoffel connection ${\Gamma^\lambda}_{\mu\nu}(g_{\rho\sigma})$ which is in the one-to-one correspondence with the first derivatives of the metric. Now, ${\Gamma^\lambda}_{\mu\nu}$ is not a tensor and as such can not generally be used as the Lagrangian field variable. To remedy this introduce the new field variable

$${\Omega^\lambda}_{\mu\nu} = {\Gamma^\lambda}_{\mu\nu} - {\tilde{\Gamma}^\lambda}_{\mu\nu}, \tag{2}$$

with the compensating term ${\tilde{\Gamma}^\lambda}_{\mu\nu}$ being an external nondynamical affine connection. As the difference of the two connections, ${\Omega^\lambda}_{\mu\nu}$ is the tensor and can thus serve as the Lagrangian field variable. Generally, ${\tilde{\Gamma}^\lambda}_{\mu\nu}$ contains forty components. Allowing for the four-parameter coordinate freedom to bring four components of ${\tilde{\Gamma}^\lambda}_{\mu\nu}$ to a canonical form, there are still left thirty six free components. Thus, GC is completely violated. But for the field theory of the metric to be consistent, at least the three-parameter residual covariance is obligatory. This can be shown as follows.

Consider the linearized approximation (LA) of the metric theory by putting $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, with $h_{\mu\nu}$ being the symmetric tensor field, $|h_{\mu\nu}| \ll 1$, and $\eta_{\mu\nu}$ being the Minkowski symbol. Specify some coordinates $x^\mu = (x^0, x^m)$, $m = 1, 2, 3$, and decompose the symmetric Lorentz-tensor $h_{\mu\nu}(x)$ in terms of the $SO(3)$ fields as $h_{\mu\nu} = (h_{00}, h_{m0}, h_{mn})$. The second, namely, the three-vector component in the decomposition possesses the wrong norm, violating thus

unitarity. The unitarity to be preserved, the "dangerous" component should be eliminated. This requires the three-parameter residual gauge symmetry, at the least. In GR, one invokes the four-parameter gauge transformations

$$h_{\mu\nu}(x) \to h_{\mu\nu}(x) + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu \tag{3}$$

with arbitrary $\xi_\mu(x)$ in accord with GC. Together with the three wrong-norm components h_{m0}, these transformations eliminate one more right-norm component. In the transverse gauge, $\partial^\mu h_{\mu\nu} = 0$, on the mass shell, $\partial \cdot \partial h_{\mu\nu} = 0$, accounting for the residual gauge freedom with the harmonic parameters, $\partial \cdot \partial \xi_\mu = 0$, one arrives explicitly at the two-component graviton. (Here one puts $\partial \cdot \partial = \partial_\mu \partial^\mu$ and similarly for any two vectors in what follows.) This procedure is quite reminiscent of the electrodynamics where the vector field $A_\mu(x) = (A_0, A_m)$ possesses one, namely, scalar component with the wrong norm. To eliminate this component the one-parameter gauge symmetry $U(1)$ is required: $A_\mu \to A_\mu + \partial_\mu \alpha$, with arbitrary $\alpha(x)$. In the transverse gauge, $\partial \cdot A = 0$, on the mass shell, $\partial \cdot \partial A_\mu = 0$, with account for the residual harmonic transformations, $\partial \cdot \partial \alpha = 0$, one is left explicitly with the two-component photon.

To allow for some residual covariance one should reduce the number of the free components in $\tilde{\Gamma}^\lambda{}_{\mu\nu}$. To this end, suppose that $\tilde{\Gamma}^\lambda{}_{\mu\nu}$ is the Christoffel connection for an external nondynamical metric $\tilde{g}_{\mu\nu}$. The latter contains generally ten free components. Allowing for the four-parameter coordinate freedom there are left six independent nondynamical fields. Thus, the reduction of the number of the fields is insufficient to leave some residual covariance. The possible caveat is to confine oneself to the contraction $\tilde{\Gamma}^\lambda{}_{\mu\lambda}$. Due to the relation $\tilde{\Gamma}^\lambda{}_{\mu\lambda} = \partial_\mu \sqrt{-\tilde{g}}$, with $\tilde{g}$ being the determinant of $\tilde{g}_{\mu\nu}$, the theory depends in this case just on one nondynamical field. The respective Lagrangian field variable becomes

$$\Omega_\mu = \Gamma^\lambda{}_{\mu\lambda} - \tilde{\Gamma}^\lambda{}_{\mu\lambda} = \partial_\mu \ln \sqrt{g/\tilde{g}} \tag{4}$$

In this marginal case, the nondynamical metric entering only through $\tilde{g}$, one can consider the latter just as a scalar density of the proper weight. One can always choose the coordinates so that $\tilde{g} = -1$. Under the variation of the coordinates $\delta x^\mu = -\xi^\mu$, the scalar density $\tilde{g}$ varies as $\delta\sqrt{-\tilde{g}} = \partial \cdot (\sqrt{-\tilde{g}}\xi)$. The residual covariance is that which leaves the canonical value $\tilde{g} = -1$ invariant, requiring $\partial \cdot \xi = 0$. This is the three-parameter UC. In this case, there is left one more independent component in the dynamical metric. Precisely this extra component corresponds to the scalar graviton which can be supplemented to the tensor graviton not violating the consistency of the theory. Note finally that the dependence on the external nondynamical field $\tilde{g}$ (more generally, on $\tilde{g}_{\mu\nu}$) would tacitly imply that the metric Universe, contrary to what is assumed in GR, should be not a self-contained system and could not entirely be described in the internal dynamical terms.

3 Scalar graviton

Lagrangian Let us study the theory of the dynamical metric field $g_{\mu\nu}$ and the generic matter field $\phi_{\rm m}$ with the generic action

$$I = \int \Big(L_{\rm g}(g_{\mu\nu}) + \Delta L_{\rm g}(g_{\mu\nu}, \chi) + L_{\rm m}(\phi_{\rm m}, g_{\mu\nu}) + \Delta L_{\rm m}(\phi_{\rm m}, g_{\mu\nu}, \chi) \Big) \sqrt{-g}\, d^4x, \tag{5}$$

where

$$\chi = \ln \sqrt{g/\tilde{g}}. \tag{6}$$

Here $g = \det g_{\mu\nu}$ and $\tilde{g}$ is a nondynamical scalar density of the same weight as g. Being the function of the ratio of the two similar scalar densities, χ itself is the scalar and thus can serve as the Lagrangian field variable. In the above, $L_{\rm g}$ and $\Delta L_{\rm g}$ are, respectively, the generally

covariant and the GC violating contributions of the gravity. Likewise, $L_{\rm m}$ and $\Delta L_{\rm m}$ are the matter Lagrangian, respectively, preserving and violating GC. All the Lagrangians above are assumed to be the scalars.

Conventionally, take as $L_{\rm g}$ the Λ-grafted Einstein-Hilbert Lagrangian:

$$L_{\rm g} = -\frac{1}{2}M_{\rm P}^2\Big(R(g_{\mu\nu}) - 2\Lambda\Big), \tag{7}$$

where $R = g^{\mu\nu}R_{\mu\nu}$ is the Ricci scalar, with $R_{\mu\nu}$ being the Ricci curvature, and Λ is the cosmological constant. Also, $M_{\rm P} = (8\pi G_{\rm N})^{-1/2}$ is the Planck mass, with $G_{\rm N}$ being the Newtonian constant. Present the scalar graviton Lagrangian $\Delta L_{\rm g}$ as

$$\Delta L_{\rm g} = \Delta K_{\rm g}(\partial_\mu\chi, \chi) - \Delta V_{\rm g}(\chi), \tag{8}$$

with $\Delta V_{\rm g}$ being the potential. In the lowest order, the kinetic term $\Delta K_{\rm g}$ looks like

$$\Delta K_{\rm g} = \frac{1}{2}\kappa_0^2\,\partial\chi\cdot\partial\chi, \tag{9}$$

with κ_0 being a constant with the dimension of mass.

The proposed extension of GR is more deeply rooted in the affine Goldstone approach to gravity [4]. This approach is based on two symmetries: the global affine symmetry (AS) and GC. AS terminates the theory in the local tangent space, whereas GC insures the matching among the various tangent spaces. Most generally, such a theory depends on an external nondynamical metric $\tilde{g}_{\mu\nu}$. This dependence violates GC and reveals the extra degrees of freedom contained in the dynamical metric $g_{\mu\nu}$. Call such an extended metric theory of gravity the "metagravity". Its minimal version, as considered in the report, depends just on $\tilde{g}$ and describes only the scalar graviton in addition to the tensor one. Call specifically the so reduced theory – the "scalar-tensor metagravity".[2] More generally, the metagravity can encompass also the vector graviton [7], though in this case the unitarity is to be violated as well.

In the Lagrangian $\Delta L_{\rm g}$ above, $\Delta K_{\rm g}$ violates only GC, with $\Delta V_{\rm g}(\chi)$ violating also AS. The GC violating part of the matter Lagrangian, $\Delta L_{\rm m}$, can be postulated in the simplest form as

$$\Delta L_{\rm m} = -f_0 J_{\rm m}(\phi_{\rm m}, g_{\mu\nu})\cdot\partial\chi, \tag{10}$$

where $J_{{\rm m}\mu}$ is the matter current and f_0 is a scalar. In the case when f_0 is a constant, $\Delta L_{\rm m}$ above violates only GC, still preserving AS. The possible dependence of f_0 on χ would reflect the violation of AS, though still preserving UC. Allowing for $f_0 \to 0$, independent of κ_0, the matter sector can be made as safe in confrontation between the theory and experiment as desired. For this reason, $\Delta L_{\rm m}$ will be disregarded in what follows.

Classical equations By varying the action (5) with respect to $g^{\mu\nu}$, $\tilde{g}$ being fixed, one arrives at the modified gravity equation:

$$G_{\mu\nu} = M_{\rm P}^{-2}\Big(T^{({\rm m})}_{\mu\nu} + \Delta T^{({\rm g})}_{\mu\nu}\Big). \tag{11}$$

Here

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}(R - 2\Lambda)g_{\mu\nu} \tag{12}$$

[2]This theory is not to be mixed with the "scalar-tensor gravity" [5]. The latter is the generally covariant extension of GR by means of a genuine scalar field, which can not completely be absorbed by the metric. Also, the theory proposed is to be distinguished from the "Unimodular Relativity" based on UC but with the dynamical metric scale completely changed for the nondynamical one [6].

is the usual gravity tensor and $T^{(\mathrm{m})}_{\mu\nu}$ is the matter energy-momentum tensor defined by L_{m}. The term $\Delta T^{(\mathrm{g})}_{\mu\nu}$ is the scalar graviton contribution looking as follows:

$$\begin{aligned}\Delta T^{(\mathrm{g})}_{\mu\nu} &= \kappa_0^2\Big(\partial_\mu\chi\partial_\nu\chi - \frac{1}{2}\partial\chi\cdot\partial\chi g_{\mu\nu}\Big) + \Delta V_{\mathrm{g}} g_{\mu\nu} \\ &+\Big(\kappa_0^2\nabla\cdot\nabla\chi + \frac{\partial\Delta V_{\mathrm{g}}}{\partial\chi}\Big)g_{\mu\nu}. \end{aligned} \tag{13}$$

Mutatis mutandis, the first line of the equation above is the ordinary energy-momentum tensor of the scalar field. The second line is the effective wave operator of the field, with ∇_μ being the covariant derivative, $\nabla_\mu\chi = \partial_\mu\chi$. This line appeared solely due to the dependence of χ on the metric and would be absent for the genuine scalar field. We interpret the above contributions, respectively, as those of the gravitational DM and the scale dependent part of DE, caused by the scalar graviton. The latter having no specific quantum numbers and undergoing only the gravitational interactions, such an association is quite a natural one.[3,4]

The r.h.s. of eq. (11) is thus proportional to the total energy momentum of the nontensor-graviton origin, produced by the nongravitational matter and the scalar graviton. Due to the Bianchi identity

$$\nabla_\mu G^{\mu\nu} = 0, \tag{14}$$

the total energy-momentum is conserved:

$$\nabla_\mu(T^{\mu\nu}_{\mathrm{m}} + \Delta T^{\mu\nu}_{\mathrm{g}}) = 0, \tag{15}$$

whereas the energy-momentum of the nongravitational matter alone, $T^{(\mathrm{m})}_{\mu\nu}$, ceases to conserve.

To really solve the gravity equations one should impose the four coordinate fixing conditions. E.g., one can choose the canonical coordinates where $\tilde{g} = -1$, supplemented by the three more independent conditions on the dynamical metric $g_{\mu\nu}$. As a result, $g_{\mu\nu}$ contains generally seven independent components. Having solved the equations in the distinguished coordinates one can recover the solution in the arbitrary observer's coordinates. Confronting the latter solution with experiment one could conceivably extract the sought $\tilde{g}$.

Linearized approximation To facilitate the problem of finding $\tilde{g}$ one could rely on LA. Not knowing $\tilde{g}$, guess from some physical considerations the background metric $\bar{g}_{\mu\nu}$. Decompose the dynamical metric in LA as follows

$$\begin{aligned} g_{\mu\nu} &= \bar{g}_{\mu\nu} + h_{\mu\nu}, \\ g^{\mu\nu} &= \bar{g}^{\mu\nu} - h^{\mu\nu} + \mathcal{O}((h_{\mu\nu})^2), \end{aligned} \tag{16}$$

with $\bar{g}^{\mu\nu}$ being the inverse background metric. For the consistency, it is to be supposed that $|h_{\mu\nu}| \ll 1$. The indices are raised and lowered with $\bar{g}^{\mu\nu}$ and $\bar{g}_{\mu\nu}$, respectively, so that $h^{\mu\nu} = \bar{g}^{\mu\lambda}\bar{g}^{\nu\rho}h_{\lambda\rho}$, etc. Then one gets

$$\chi = (h_0 + h)/2 + \mathcal{O}(h^2), \tag{17}$$

where $h \equiv \bar{g}^{\mu\nu}h_{\mu\nu}$ and $h_0 = \ln(\bar{g}/\tilde{g})$. The latter term is a scalar parameter-field, not bound in general to be small. Physically, it reflects the discrepancy between the background scale $\sqrt{-\bar{g}}$, which is at our disposal, and the nondynamical scale $\sqrt{-\tilde{g}}$, which is given a priori.

The GR Lagrangian in LA becomes as follows

$$L_{\mathrm{g}} = \frac{1}{8}M_{\mathrm{P}}^2\Big((\bar{\nabla}_\lambda h_{\mu\nu})^2 - 2(\bar{\nabla}^\lambda h_{\lambda\mu})^2 + 2\bar{\nabla}^\lambda h_{\lambda\mu}\bar{\nabla}^\mu h - (\bar{\nabla}_\lambda h)^2\Big) + \mathcal{O}((h_{\mu\nu})^3), \tag{18}$$

[3]The above division on DM and DE is rather conventional. In particular in the limit $\kappa_0 \to 0$, the whole contribution of the scalar graviton looks like DE.

[4]The other kinds of DM, if any, are to be included in the matter Lagrangian.

with $\bar{\nabla}_\mu$ being the background covariant derivative and $\bar{\nabla}_\mu h = \partial_\mu h$. The Λ-term is omitted here and in what follows. For the respective gravity tensor, one gets

$$G_{\mu\nu} = -\frac{1}{2}\Big(\bar{\nabla}\cdot\bar{\nabla}h_{\mu\nu} - \bar{\nabla}_\mu\bar{\nabla}^\lambda h_{\lambda\nu} - \bar{\nabla}_\nu\bar{\nabla}^\lambda h_{\lambda\mu} + \bar{\nabla}_\mu\bar{\nabla}_\nu h\Big) - \frac{1}{2}\Big(\bar{\nabla}^\lambda\bar{\nabla}^\rho h_{\lambda\rho} - \bar{\nabla}\cdot\bar{\nabla}h\Big)\bar{g}_{\mu\nu}, \quad (19)$$

independent of h_0. The Lagrangian above is invariant under the gauge transformations

$$h_{\mu\nu}(x) \to h_{\mu\nu}(x) + \bar{\nabla}_\mu\xi_\nu + \bar{\nabla}_\nu\xi_\mu, \quad (20)$$

with arbitrary ξ_μ corresponding to GC. In particular, one has $h(x) \to h(x) + 2\bar{\nabla}\cdot\xi$. By this token, h can be removed, and thus $L_{\rm g}$, taken alone, does not produce any physical manifestations for the scalar graviton.

The contribution of $\Delta L_{\rm g}$ to the gravity equations in terms of h_0 and h can be read off from eqs. (13), (16) and (17). This contribution is invariant only under the restricted gauge transformations with $\bar{\nabla}\cdot\xi = 0$ or, otherwise, $\partial\cdot(\sqrt{-\bar{g}}\xi) = 0$. In the curved background, this corresponds to the residual UC. To solve the gravity equations one should impose on $h_{\mu\nu}$ the three gauge fixing conditions, leaving thus seven independent components. Comparing the solution with observations one can conceivably extract thereof h_0 and, under the chosen $\bar{g}$, the looked for $\tilde{g}$.

Quantization Assuming to have found $\tilde{g}$, rescale the background metric to adjust it to the external nondynamical scale, so that $\bar{g} = \tilde{g}$. Under this choice, h_0 vanishes. The GC preserving part of the gravity Lagrangian stays as before. The GC violating part reads

$$\Delta L_{\rm g} = \frac{1}{8}\Big(\kappa_0^2(\bar{\nabla}_\lambda h)^2 - \mu_0^4 h^2\Big) + \mathcal{O}(h^4), \quad (21)$$

with the potential supposed to be as follows

$$\Delta V_{\rm g}(h) = \frac{1}{8}\mu_0^4 h^2 + \mathcal{O}(h^4) \quad (22)$$

and μ_0 being a constant with the dimension of mass. The Lagrangian $\Delta L_{\rm g}$ possesses only the residual UC, with $\bar{\nabla}\cdot\xi = 0$ insuring $h \to h$. Normalized properly, the true field for the scalar graviton is $\kappa_0 h/2$, with the constant κ_0 characterizing thus the scale of the wave function. At $\kappa_0 \to 0$, the wave function squeezes formally to dot. The other free constant, μ_0, characterizes the scalar graviton mass, $m_0 = \mu_0^2/\kappa_0$.

Finally, the gauge fixing Lagrangian in the case of UC can be chosen similar to ref. [8] as

$$L_{\rm gf} = -\lambda(\bar{\nabla}_\mu\bar{\nabla}^\lambda h_{\lambda\nu} - \bar{\nabla}_\nu\bar{\nabla}^\lambda h_{\lambda\mu})^2, \quad (23)$$

with λ being the indefinite Lagrange multiplier. This condition fixes three components in $h_{\mu\nu}$, the scalar h remaining untouched. The forth independent gauge condition which is to be imposed in GR is now abandoned. It is superseded by the GC violating term. The latter looks superficially as the gauge fixing term but with the definite coefficients. This is the principle difference between the two kinds of terms. In the GC limit, $\kappa_0 \to 0$ and $\mu_0 \to 0$, the given quantum theory becomes underdetermined and requires one more gauge condition. For this reason, the GC restoration is, generally, singular.

Altogether, one should study the present theory of the field $h_{\mu\nu}$ in the curved background. As usually, this requires the transition to the local inertial coordinates, what can in principle be done. To facilitate the quantization procedure suppose the Lorentzian background, $\bar{g}_{\mu\nu} = \eta_{\mu\nu}$, with the effect that $\bar{\nabla}_\mu = \partial_\mu$. The required ghost system is found in this case in ref. [8]. The respective propagator can be shown to become

$$D_{\mu\nu\rho\sigma}(x-x') = \frac{1}{4}\Big(P^{(2)}_{\mu\nu\rho\sigma}(\lambda)\frac{1}{\partial\cdot\partial} + \frac{1}{\epsilon_0^2}P^{(0)}_{\mu\nu\rho\sigma}\frac{1}{\partial\cdot\partial + m_0^2}\Big)i\delta^4(x-x'), \quad (24)$$

where $\epsilon_0 = \kappa_0/M_{\rm P}$. The first term in the propagator corresponds to the massless tensor graviton. The tensor projector $P^{(2)}_{\mu\nu\rho\sigma}$, unspecified here, corresponds to the six components of the tensor graviton off the mass shell, as in GR. The second term, with the scalar projector $P^{(0)}_{\mu\nu\rho\sigma} = \partial_\mu\partial_\nu\partial_\rho\partial_\sigma/(\partial\cdot\partial)^2$, describes additionally the scalar graviton. Altogether, the theory describes the seven propagating degrees of freedom reflecting ultimately the residual three-parameter UC.

In the limit $\kappa_0 \to 0$, μ_0 being fixed, one gets for the scalar part of the propagator

$$D^{(0)}_{\mu\nu\rho\sigma}(x-x') \simeq \frac{1}{4\omega_0^2} P^{(0)}_{\mu\nu\rho\sigma} i\delta^4(x-x'), \tag{25}$$

with $\omega_0 \equiv \epsilon_0 m_0 = \mu_0^2/M_{\rm P}$ being finite. In this limit, the theory describes the massless tensor graviton, as in GR, plus the contact scalar interactions. The GC restoration limit, $\kappa_0 \to 0$ and $\mu_0 \to 0$, is indefinite in accord with the necessity of adding one more gauge condition.[5]

4 Conclusion

In conclusion, the self-consistent extension of GR, with the explicit violation of GC to the residual UC, is developed. Being based on the gauge principle, though with the reduced covariance, the extension is as consistent theoretically as GR itself. In addition to the massless tensor graviton, the respective theory – the scalar-tensor metagravity – describes the massive scalar graviton as the part of the metric field. The scalar graviton is the natural challenger for the gravitational DM and/or the scale dependent part of DE. The restoration of GR being unattainable on the whole, the extension may be not quite safe vs. observations. Its experimental consistency needs investigation.

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[5] Conceivably, this is the particular manifestation of a more general singularity at $\mu_0 \to 0$ but κ_0 fixed, corresponding to the massless limit for the scalar graviton.