# Gravity in a Higgs phase

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**Abstract**

In this presentation I review basic properties of the simplest Higgs phase of gravity called ghost condensation, and discuss possible applications and observational bounds.

## 1 Introduction

Acceleration of the cosmic expansion today is one of the greatest mysteries in both cosmology and fundamental physics. Assuming that Einstein's general relativity is the genuine description of gravity all the way up to cosmological distance and time scales, the so called concordance cosmological model requires that about 70% of our universe should be some sort of energy with negative pressure, called dark energy. However, since the nature of gravity at cosmological scales has never been probed directly, we do not know whether the general relativity is really correct at such infrared (IR) scales. Therefore, it seems natural to consider modification of general relativity in IR as an alternative to dark energy. Dark energy, IR modification of gravity and their combination should be tested and distinguished by future observations and experiments.

From the theoretical point of view, however, IR modification of general relativity is not an easy subject. Most of the previous proposals are one way or another scalar-tensor theories of gravity, and are strongly constrained by e.g. solar system experiments [1] and the theoretical requirement that ghosts be absent [2, 3, 4]. The massive gravity theory [5] and the Dvali-Gabadadze-Porrati (DGP) brane model [6] are much more interesting IR modification of gravity, but they are known to have macroscopic UV scales [7, 8]. A UV scale of a theory is the scale at which the theory breaks down and loses its predictability. For example, the UV scale of the $4D$ general relativity is the Planck scale, at which quantum gravity effects are believed to become important. Since the Planck scale is microscopic, the general relativity maintains its predictability at essentially all scales we can directly probe. On the other hand, in the massive gravity theory and the DGP brane model, the UV scale is macroscopic. For example, if the scale of IR modification is the Hubble scale today or longer then the UV scale would be $\sim 1,000km$ or longer. At the UV scale an extra degree of freedom, which is coupled to matter, becomes strongly coupled and its quantum effects cannot be ignored. This itself does not immediately exclude those theories, but means that we need UV completion in order to predict what we think we know about gravity within $\sim 1,000km$. Since this issue is originated from the IR modification and the extra degree of freedom cannot be decoupled from matter, it is not clear whether the physics in IR is insensitive to unknown properties of the UV completion. In particular, there is no guarantee that properties of the IR modification of gravity will persist even qualitatively when the theories are UV completed in a way that they give correct predictions about gravity at scales between $\sim 1,000km$ and $\sim 0.1mm$.

Ghost condensation is an analogue of the Higgs mechanism in general relativity and modifies gravity in IR in a way that avoids the macroscopic UV scale [9] [1]. In ghost condensation the

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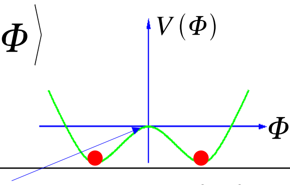
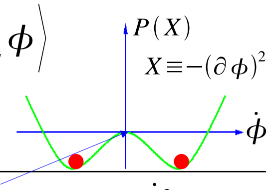
[1] See e.g.[10, 11, 12, 13, 14, 15, 16] for other related proposals.

theory is expanded around a background without ghost and the low energy effective theory has a universal structure determined solely by the symmetry breaking pattern. While the Higgs mechanism in a gauge theory spontaneously breaks gauge symmetry, the ghost condensation spontaneously breaks a part of Lorentz symmetry since this is the symmetry relevant to gravity. In a gauge theory the Higgs mechanism makes it possible to give a mass term to the gauge boson and to modify the force law in a theoretically controllable way. Similarly, the ghost condensation gives a "mass term" to the scalar sector of gravity and modifies gravitational force in the linearized level even in Minkowski and de Sitter spacetimes. The Higgs phase of gravity provided by the ghost condensation is simplest in the sense that the number of Nambu-Goldstone bosons associated with spontaneous Lorentz breaking is just one and that only the scalar sector is essentially modified.

## 2 Ghost Condensation

The ghost condensation can be pedagogically explained by comparison with the usual Higgs mechanism as in the table shown below. First, the order parameter for ghost condensation is the vacuum expectation value (vev) of the derivative $\partial_\mu\phi$ of a scalar field $\phi$, while the order parameter for Higgs mechanism is the vev of a scalar field $\Phi$ itself. Second, both have instabilities in their symmetric phases: a tachyonic instability around $\Phi = 0$ for Higgs mechanism and a ghost instability around $\partial_\mu\phi = 0$ for ghost condensation. In both cases, because of the instabilities, the system should deviate from the symmetric phase and the order parameter should obtain a non-vanishing vev. Third, there are stable point where small fluctuations do not contain tachyons nor ghosts. For Higgs mechanism, such a point is characterized by the vev of the order parameter satisfying $V' = 0$ and $V'' > 0$. On the other hand, for ghost condensation a stable point is characterized by $P' = 0$ and $P'' > 0$. Fourth, while the usual Higgs mechanism breaks usual gauge symmetry and changes gauge force law, the ghost condensation spontaneously breaks a part of Lorentz symmetry (the time translation symmetry) and changes linearized gravity force law even in Minkowski background. Finally, generated corrections to the standard Gauss-law potential is Yukawa-type for Higgs mechanism but oscillating for ghost condensation.

| | ***Higgs Mechanism*** | ***Ghost Condensation*** |
|---|---|---|
| ***Order Parameter*** | $\langle \Phi \rangle$  | $\langle \partial_\mu \phi \rangle$  |
| ***Instability*** | Tachyon $-m^2\Phi^2$ | Ghost $-\dot{\phi}^2$ |
| ***Condensate*** | V'=0, V''>0 | P'=0, P''>0 |
| ***Spontaneous breaking*** | Gauge symmetry | Lorents symmetry (Time translation) |
| ***Modifying*** | Gauge force | Gravitational force (in flat background) |
| ***New potential*** | Yukawa-type | Oscillating |

At this point one might wonder if the system really reach a configuration where $P' = 0$ and $P'' > 0$. Actually, it is easy to show that this is the case. For simplicity let us consider a Lagrangian $L_\phi = P(-(\partial\phi)^2)$ in the expanding FRW background with $P$ of the form shown in the upper right part of the table. We assume the shift symmetry, the symmetry under the constant shift $\phi \to \phi + c$ of the scalar field. This symmetry prevents potential terms of $\phi$ from being generated. The equation of motion for $\phi$ is simply $\partial_t[a^3 P'\dot{\phi}] = 0$, where $a$ is the scale factor of the universe. This means that $a^3 P'\dot{\phi}$ is constant and that

$$P'\dot{\phi} \propto a^{-3} \to 0 \quad (a \to \infty) \tag{1}$$

as the universe expands. We have two choices: $P' = 0$ or $\dot{\phi} = 0$, namely one of the two bottoms of the function $P$ or the top of the hill between them. Obviously, we cannot take the latter choice since it is a ghosty background and anyway unstable. Thus, we are automatically driven to $P' = 0$ by the expansion of the universe. In this sense the background with $P' = 0$ is an attractor.

Having shown that the ghost condensate is an attractor, let us construct a low energy effective field theory around this background. For this purpose let us consider a small fluctuation around the background with $P' = 0$. For $\phi = M^2 t + \pi$, the quadratic action for $\pi$ coming from the Lagrangian $P$ is $\int d^4x[(P'(M^4) + M^4 P''(M^4))\dot{\pi}^2 - P'(M^4)(\nabla\pi)^2]$. By setting $P'(M^4) = 0$ we obtain the time kinetic term $M^4 P''(M^4)\dot{\pi}^2$ with the correct sign. Unless the function $P$ is fine-tuned, $P''$ is non-zero at $P' = 0$. This means that the coefficient of the time kinetic term is non-vanishing and, thus, we do not have the strong coupling issue which the massive gravity and the DGP brane model are facing with. On the other hand, the coefficient of $(\nabla\pi)^2$ vanishes at $P' = 0$ and the simple Lagrangian $P$ does not give us a spatial kinetic term for $\pi$. However, this does not mean that there is no spatial kinetic term in the low energy EFT for $\pi$. This just says that the leading spatial kinetic term is not contained in $P$ and that we should look for the leading term in different parts. Indeed, other terms like $\tilde{P}((\partial\phi)^2)Q(\Box\phi)$ do contain spatial kinetic terms for $\pi$ but the spatial-derivative expansion starts with the fourth derivative: $(\nabla^2\pi)^2 + \cdots$. If there is a non-vanishing second-order spatial kinetic term $(\nabla\pi)^2$ then it can be included in $P$ by redefinition and the redefined $P'$ goes to zero by the expansion of the universe as shown above. Namely, the expansion of the universe ensures that the spatial-derivative expansion starts from $(\nabla^2\pi)^2 + \cdots$. Combining this spatial kinetic term with the previously obtained time kinetic term and properly normalizing $\pi$, we obtain the low energy effective action of the form

$$M^4 \int d^4x \left[ \frac{1}{2}\dot{\pi}^2 - \frac{\alpha}{M^2}(\nabla^2\pi)^2 + \cdots \right], \tag{2}$$

where $\alpha$ is a dimensionless parameter of order unity [2]. One might worry that other (nonlinear) terms in effective theory such as $\dot{\pi}(\nabla\pi)^2$ might mess up the effective action. In fact, it turns out that all such terms are irrelevant at low energy [9]. An important fact to show this is that the scaling dimension of $\pi$ is not the same as its mass dimension 1 but is 1/4, reflecting the situation that the Lorentz symmetry is broken spontaneously. Moreover, it is also straightforward to show that all spurious modes associates with higher time derivative terms such as $(\ddot{\phi})^2$ have frequency above the cutoff $M$ and, thus, should be ignored. In this sense, we are assuming the existence of a UV completion but not assuming any properties of it. Finally, it must be noted that the effective action of the form (2) is stable against radiative corrections. Indeed, the only would-be more relevant term in the effective theory is the usual spatial kinetic term $(\nabla\pi)^2$, but its coefficient $P'$ is driven to an extremely small value by the expansion of the universe even if it is radiatively generated.

The effective action (2) would imply the low energy dispersion relation for $\pi$ is $\omega^2 \simeq \alpha k^4/M^2$. However, since the background spontaneously breaks Lorentz invariance, $\pi$ couples to gravity

[2] With this normalization, $\pi$ has the dimension of length.

in the linearized level even in Minkowski or de Sitter background. Hence, mixing with gravity introduces an order $M^2/M_{pl}^2$ correction to the dispersion relation. As a result the dispersion relation in the presence of gravity is $\omega^2 \simeq \alpha k^4/M^2 - \alpha M^2 k^2/2M_{pl}^2$. This dispersion relation leads to IR modification of gravity due to Jean's instability. Note that there is no ghost around the stable background $P' = 0$ and the Jeans's instability is nothing to do with a ghost.

In the above we have expanded a general Lagrangian consistent with the shift symmetry around the stable background in order to construct the low energy EFT. This is the most straightforward approach. An alternative, more powerful way is to use the symmetry breaking pattern. In this approach, we actually do not need to specify a concrete way of the spontaneous symmetry breaking. In this sense, the ghost around $\dot{\phi} = 0$ has nothing to do with the construction of the EFT around $P' = 0$. Indeed, it is suffice to assume the symmetry breaking pattern, namely from the full 4-dimensional Lorentz symmetry to the 3-dimensional spatial diffeomorphism [9].

Here, let us briefly review this approach based on the symmetry breaking pattern. This leads to the exactly same conclusion as above, but is more universal and can be applied to any situations as far as the symmetry breaking pattern is the same. We assume that (i) the 4-dimensional Lorentz symmetry is spontaneously broken down to a 3-dimensional spatial diffeomorphism and that (ii) the background spacetime metric is maximally symmetric, either Minkowski or de Sitter. With the assumption (i), we are left with the 3-dimensional spatial diffeomorphism $\vec{x} \to \vec{x}'(t, \vec{x})$. Our strategy here is to write down the most general action invariant under this residual symmetry. After that, the action for the Nambu-Goldstone (NG) boson $\pi$ is obtained by undoing the unitary gauge.

For simplicity let us consider the Minkowski background plus perturbation: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. The infinitesimal gauge transformation is $\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$, where $\xi^\mu$ is a 4-vector representing the gauge freedom. Under the residual gauge transformation $\xi^i$ $(i = 1, 2, 3)$, the metric perturbation transforms as

$$\delta h_{00} = 0, \quad \delta h_{0i} = \partial_0 \xi_i, \quad \delta h_{ij} = \partial_i \xi_j + \partial_j \xi_i. \tag{3}$$

Now let us seek terms invariant under the residual gauge transformation. Those terms must begin at quadratic order since we assumed that the flat spacetime is a solution to the equation of motion. The leading term (without derivatives acted on the metric perturbations) is $\int dx^4 M^4 h_{00}^2$. This is indeed invariant under the residual gauge transformation (3). From this term, we can obtain the corresponding term in the effective action for the NG boson $\pi$. Since $h_{00} \to h_{00} + 2\partial_0 \xi_0$, by promoting the broken symmetry $\xi^0$ to a physical degree of freedom $\pi$, we obtain the term $\int dx^4 M^4 (h_{00} - 2\dot{\pi})^2$. This includes a time kinetic term for $\pi$ as well as a mixing term. At this point we wonder if we can get the usual space kinetic term $(\vec{\nabla}\pi)^2$ or not. The only possibility would be from $(h_{0i})^2$ since $h_{0i} \to h_{0i} - \partial_i \pi$ under the broken symmetry transformation $\xi^0 = \pi$. However, this term is not invariant under the residual spatial diffeomorphism $\xi^i$ and, thus, cannot enter the effective action. Actually, there are combinations invariant under the spatial diffeomorphism. They are made of the geometrical quantity called extrinsic curvature. The extrinsic curvature $K_{ij}$ in the linear order is $K_{ij} = \partial_j h_{0j} + \partial_i h_{0j} - \partial_0 h_{ij}$ and transforms as a tensor under the spatial diffeomorphism. Thus, $\int dx^4 \tilde{M}^2 (K_i^i)^2$ and $\int dx^4 \bar{M}^2 K^{ij} K_{ij}$ are invariant under spatial diffeomorphism and can be used in the action. Since $K_{ij} \to K_{ij} - \partial_i \partial_j \pi$ under the broken symmetry $\xi^0 = \pi$, we obtain $\int dx^4 (\tilde{M}^2 + \bar{M}^2)(\vec{\nabla}^2 \pi)^2$. Combining these terms with the above time kinetic term and properly normalizing the definition of $\pi$ and $M$, we obtain

$$L_{eff} = M^4 \left\{ \frac{1}{2} \left( \dot{\pi} - \frac{1}{2} h_{00} \right)^2 - \frac{\alpha}{M^2} (\vec{\nabla}^2 \pi)^2 + \cdots \right\}, \tag{4}$$

where $\alpha$ is a dimensionless constant of order unity. By setting $h_{00} = 0$, this completely agrees with (2), which was obtained by expanding the scalar field action explicitly around the stable background. Here, in deriving the effective action all we needed was the symmetry breaking pattern. Thus, the low energy EFT of the ghost condensation is universal and should hold as far as the symmetry breaking pattern is the same.

In ghost condensation the linearized gravitational potential is modified at the length scale $r_c$ in the time scale $t_c$, where $r_c$ and $t_c$ are related to the scale of spontaneous Lorentz breaking $M$ as

$$r_c \simeq \frac{M_{\rm Pl}}{M^2}, \quad t_c \simeq \frac{M_{\rm Pl}^2}{M^3}. \tag{5}$$

Note that $r_c$ and $t_c$ are much longer than $1/M$. The way gravity is modified is peculiar. At the time when a gravitational source is turned on, the potential is exactly the same as that in general relativity. After that, however, the standard form of the potential is modulated with oscillation in space and with exponential growth in time. This is an analogue of Jeans instability, but unlike the usual Jeans instability, it persists in the linearized level even in Minkowski background. The length scale $r_c$ and the time scale $t_c$ above are for the oscillation and the exponential growth, respectively. At the time $\sim t_c$, the modification part of the linear potential will have an appreciable peak only at the distance $\sim r_c$. At larger distances, it will take more time for excitations of the Nambu-Goldstone boson to propagate from the source and to modify the gravitational potential. At shorter distances, the modification is smaller than at the peak position because of the spatial oscillation with the boundary condition at the origin. The behavior explained here applies to Minkowski background, but in ref. [9] the modification of gravity in de Sitter spacetime was also analyzed. It was shown that the growing mode of the linear gravitational potential disappears when the Hubble expansion rate exceeds a critical value $H_c \sim 1/t_c$. Thus, the onset of the IR modification starts at the time when the Hubble expansion rate becomes as low as $H_c$.

If we take the $M/M_{\rm Pl} \to 0$ limit then the Higgs sector is completely decoupled from the gravity and the matter sectors and, thus, the general relativity is safely recovered. Therefore, cosmological and astrophysical considerations in general do not set a lower bound on the scale $M$ of spontaneous Lorentz breaking, but provide upper bounds on $M$. If we trusted the linear approximation for all gravitational sources for all times then the requirement $H_c \lesssim H_0$ would give the bound $M \lesssim (M_{\rm Pl}^2 H_0)^{1/3} \simeq 10MeV$, where $H_0$ is the Hubble parameter today [9]. However, for virtually all interesting gravitational sources the nonlinear dynamics dominates in time scales shorter than the age of the universe. As a result the nonlinear dynamics cuts off the Jeans instability of the linear theory, and allows $M \lesssim 100GeV$ [17].

Note that the ghost condensate provides the second most symmetric class of backgrounds for the system of field theory plus gravity. The most symmetric class is of course maximally symmetric solutions: Minkowski, de Sitter and anti-de Sitter. The ghost condensate minimally breaks the maximal symmetry and introduces only one Nambu-Goldstone boson.

Because of the universality of the low energy EFT, it is worthwhile investigating properties of the Higgs phase of gravity, whether or not it leads to interesting physical phenomena. Actually, it turns out that the physics in the Higgs phase of gravity is extremely rich and intriguing. They include IR modification of gravity [9], a new spin-dependent force [18], a qualitatively different picture of inflationary de Sitter phase [19, 20], effects of moving sources [21, 22], nonlinear dynamics [23, 17], properties of black holes [24, 25, 26], implications to galaxy rotation curves [27, 28, 29], dark energy models [30, 31, 32], other classical dynamics [33, 34], attempts towards UV completion [35, 36], and so on.

# 3 Possible Applications

**Dark energy:** In the usual Higgs mechanism, the cosmological constant (cc) would be negative in the broken phase if it is zero in the symmetric phase. Therefore, it seems difficult to imagine how the Higgs mechanism provides a source of dark energy. On the other hand, the situation is opposite with the ghost condensation: the cc would be positive in the broken phase if it is zero in the symmetric phase. Hence, while this by itself does not solve the cc problem, this can be a source of dark energy.

**Dark matter:** If we consider a small, positive deviation of $P'$ from zero then the homogeneous part of the energy density is proportional to $a^{-3}$ and behaves like dark matter. Inhomogeneous linear perturbations around the homogeneous deviation also behaves like dark matter. However, at this moment it is not clear whether we can replace dark matter with ghost condensate. We need to see if it clumps properly. Ref. [17] can be thought to be a step towards this direction.

**Inflation:** We can also consider inflation within the regime of the validity of the EFT with ghost condensation. In the very early universe where $H$ is higher than the cutoff $M$, we do not have a good EFT describing the sector of ghost condensation. However, the contribution of this sector to the total energy density $\rho_{tot}$ is naturally expected to be negligible: $\rho_{ghost} \sim M^4 \ll M_p^2 H^2 \simeq \rho_{tot}$. As the Hubble expansion rate decreases, the sector of ghost condensation enters the regime of validity of the EFT and the Hubble friction drives $P'$ to zero. If we take into account quantum fluctuations then $P'$ is not quite zero but is $\sim (H/M)^{5/2} \sim (\delta\rho/\rho)^2 \sim 10^{-10}$ in the end of ghost inflation. In this way, we have a consistent story, starting from the outside the regime of validity of the EFT and dynamically entering the regime of validity. All predictions of the ghost inflation are derived within the validity of the EFT, including the relatively low-$H$ de Sitter phase, the scale invariant spectrum and the large non-Gaussianity [19].

**Black hole:** In ref. [25] we consider the question "what happens near a black hole?" A ghost condensate defines a hypersurface-orthogonal congruence of timelike curves, each of which has the tangent vector $u^\mu = -g^{\mu\nu}\partial_\nu\phi$. It is argued that the ghost condensate in this picture approximately corresponds to a congruence of geodesics and the accretion rate of the ghost condensate into a black hole should be negligible for a sufficiently large black hole. This argument is confirmed by a detailed calculation based on the perturbative expansion w.r.t. the higher spatial kinetic term. The essential reason for the smallness of the accretion rate is the same as that for the smallness of the tidal force acted on an extended object freely falling into a large black hole.

# 4 Bounds

In this section we consider the bounds on the symmetry breaking scale $M$. We argue that the nonlinear dynamics cuts off the Jeans instability of the linear theory, and allows $M \lesssim 100$ MeV [17].

## 4.1 Jeans Instability

For $M \gtrsim 10$ MeV, the Jeans instability time is shorter than the lifetime of the universe, and we must consider the effects of this instability. We have seen that the nonlinear effects dominate near interesting gravitational sources, but the linear dynamics still controls the behavior of the system for sufficiently weak ghostone amplitudes. In the linear regime, fluctuations with wavelength $\lambda \gtrsim L_{\rm J}$ grow on a time scale

$$\tau \sim T_{\rm J}\frac{\lambda}{L_{\rm J}}, \tag{6}$$

where

$$L_{\rm J} \sim \frac{M_{\rm Pl}}{M^2}, \qquad T_{\rm J} \sim \frac{M_{\rm Pl}^2}{M^3} \tag{7}$$

are the Jeans length and time scales. Wavelengths of order $L_{\rm J}$ become unstable first, and longer wavelengths take longer to grow. Since fluctuations on wavelength shorter than $L_{\rm J}$ are stable, we expect the minimum size of a positive or negative energy region to be $L_{\rm J}$. On the other hand, the maximum size is determined by requiring that the time scale $\tau$ above be shorter than the Hubble time. Hence, a positive or negative region can grow within the age of the universe if its size $L$ is in the range

$$L_{\rm J} \lesssim L \lesssim L_{\max}, \tag{8}$$

where

$$L_{\max} \sim \frac{M}{M_{\rm Pl} H_0} \sim R_{\odot} \left( \frac{M}{100\ {\rm GeV}} \right). \tag{9}$$

The unstable modes grow at least until nonlinear effects become important. This happens for $\pi \gtrsim \pi_{\rm c}$, where

$$\pi_{\rm c} \sim \frac{\lambda^2}{\tau}. \tag{10}$$

or equivalently $\Sigma \gtrsim \Sigma_{\rm c}$ with

$$\Sigma_{\rm c} \sim \frac{\pi_{\rm c}}{\tau} \sim \frac{\lambda^2}{\tau^2} \sim \frac{M^2}{M_{\rm Pl}^2}. \tag{11}$$

It is reasonable to assume that the nonlinear effects cut off the Jeans instability at this critical amplitude. This mechanism will fill the universe with regions of positive and negative ghostone field with amplitude of order $\pm\Sigma_{\rm c}$ and the size in the range (8). Since $\Sigma$ is a conserved charge, there will be equal amounts of positive and negative $\Sigma$.

The sun's Newtonian potential triggers the Jeans instability of the ghost condensate and, thus, it is expected that there be a positive or negative region around the sun. This is justified if the 'aether' is efficiently dragged by the sun and we now argue that this is indeed the case. To do this, it is useful to work in the rest frame of the sun. Far from the sun, the aether is moving with constant velocity $v \sim 10^{-3}$, but near the sun the velocity field will be distorted by the presence of the sun. By using the fluid picture of the ghostone field, we estimate the effect on a fluid particle with speed $v$ and impact parameter $r$. The fluid particle will be a distance of order $r$ away for a time $\Delta t \sim r/v$, so the change in the particle velocity in the impulse approximation is

$$\Delta v \sim \frac{R_{\rm S}}{r^2} \cdot \frac{r}{v} \sim \frac{R_{\rm S}}{vr}, \tag{12}$$

where $R_{\rm S}$ is the Schwarzschild radius of the source. Thus, the change in the velocity of a fluid particle becomes comparable to or greater than the initial velocity if $r < r_{\rm drag}$, where

$$r_{\rm drag} \sim \frac{R_{\rm S}}{v^2}, \tag{13}$$

For our sun, $r_{\rm drag} \sim 10 R_{\odot}$, so the dragged region extends *outside* the solar radius.[3]

We require that the absolute value of the mass of the lump with the critical density $\rho_c$ and the size $L_{\max}$ be at worst less than the solar mass:

$$\rho_c L_{\max}^3 \lesssim M_{\odot}. \tag{14}$$

This requirement gives the bound

$$M \lesssim 10^3\ {\rm GeV}. \tag{15}$$

Since the high power of $M$ (the l.h.s. $\propto M^9$) is involved in (14), a more stringent requirement on the mass of the lump will not substantially improve the bound.

[3]This radius is still much less than the orbital radius of Mercury.

### 4.2 Twinkling from Lensing

We have argued that if $M \gtrsim 10$ MeV, then the Jeans instability fills the universe with regions of positive and negative energy of size $L \gtrsim L_{\rm J} \sim M_{\rm Pl}/M^2$ with energy density $\rho_{\rm c} \sim M^6/M_{\rm Pl}^2$. This will happen everywhere, in particular in the voids between galaxies. Any light that travels to us from far away will therefore be lensed by these positive and negative regions. These positive and negative energy regions move, because the local rest frame of the lensing regions is different from that of our galaxy, so the result is that the observed luminosity of any point source will change with time. This is similar to the twinkling of the stars in the night sky caused by time dependent temperature differences in the atmosphere. In this subsection, we work out the bounds on the ghost condensate from this effect.

Suppose that the universe is filled with regions of positive and negative energy with size $L$ and density $\rho_{\rm c}$. A light ray traveling through such a region will lens by an angle

$$\Delta\theta \sim \Phi \sim \frac{\rho_{\rm c} L^2}{M_{\rm Pl}^2} \sim \frac{M^6 L^2}{M_{\rm Pl}^4}. \tag{16}$$

If a light ray travels a distance $d \gg L$, then it will undergo $N \sim d/L$ uncorrelated lensing events, so the total angular deviation will be enhanced by a $N^{1/2}$ random walk factor:

$$\Delta\theta_{\rm tot} \sim \left(\frac{d}{L}\right)^{1/2} \frac{M^6 L^2}{M_{\rm Pl}^4}. \tag{17}$$

We see that the largest angular deviation comes from the largest $L$ and largest $d$.

The size of $L$ is limited by the time for the Jeans instability to form as in (8). If the source is the cosmic microwave background, then $d \sim H_0^{-1}$ and we obtain

$$\Delta\theta_{\rm CMB} \sim \frac{M^{15/2}}{M_{\rm Pl}^{11/2} H_0^2} \sim \left(\frac{M}{100 \text{ GeV}}\right)^{15/2}, \tag{18}$$

for the largest regions with the size $L \sim L_{max}$. The high power of $M$ makes the precise experimental limit on $\Delta\theta_{\rm CMB}$ irrelevant, and we obtain the bound

$$M \lesssim 100 \text{ GeV}. \tag{19}$$

For $M \sim 100$ GeV, the size of the largest critical region is $L \sim 10^{12}$ cm, approximately the radius of the sun. The local velocity of these regions relative to our galaxy is of order $10^{-3}$, so the time scale for one of these regions to cross the line of sight is of order a day, which is therefore the time scale of the variation.

If there is a distant astrophysical source that is observed to shine with very little time variation, it may give a competitive bound. But given the high power of $M$ involved, it seems difficult to improve on this bound significantly.

### 4.3 Supernova time-delay

Gravitational lensing considered in the previous subsection induces a time-delay for light-rays coming from far distances. With this time-delay effect, observed supernovae should be older than they appear. Thus, this effect would change the estimate of dark energy by observation of Type Ia supernovae. Since the determination of the dark energy by supernovae observation is known to be consistent with the WMAP data, we require that the time-delay is sufficiently shorter than the total time:

$$\frac{\Delta t}{t} \sim (\Delta\theta)^2 \sim \frac{M^6 L_{\rm max}^2}{M_{\rm Pl}^4} \leq 1. \tag{20}$$

Note that the precise experimental limit on the $\Delta t/t$ is irrelevant because of the higher power of $M$ involved in the l.h.s. From this we obtain the bound

$$M \lesssim 10^3 \text{ GeV}. \tag{21}$$

# 5 Summary

The usual Higgs mechanism gives a mass to a gauge boson in a theoretically controllable way by spontaneously breaking the gauge symmetry. Similarly, the ghost condensation gives a "mass" to the scalar-sector of gravity by spontaneously breaking a part of Lorentz symmetry, the invariance under time re-parameterization. It has been shown that the structure of low energy effective field theory of ghost condensation is determined by the symmetry breaking pattern and does not depend at all on the way the symmetry is broken. In this sense the low energy effective field theory of ghost condensation has nothing to do with ghost.

The theory of ghost condensation opens up a number of new avenues for attacking cosmological problems, including inflation, dark matter, dark energy and black holes. Finally, it has been argued that the theory is compatible with all current experimental observations if the scale of spontaneous Lorentz breaking is lower than $\sim 100$ MeV. Our current understanding of the dynamics of gravity in Higgs phase is very immature. Most of its properties still remain unexplored.

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