Emergent 4D gravity

T. Gherghetta*
School of Physics and Astronomy, University of Minnesota
Minneapolis, MN 55455, USA

Abstract

It is shown that a massless tensor zero mode arises when the bulk and boundary masses in 5D warped gravity are tuned to special values. The tensor zero mode is a smooth deformation of the Randall-Sundrum graviton and can be localized anywhere in the bulk. When the tensor zero mode is localized near the IR brane, the dual interpretation is a composite graviton describing an emergent (induced) 4D theory of gravity at the IR scale. In this case Newton's law of gravity changes to a new power law below the millimeter scale, with an exponent that can even be irrational.

1 Introduction

One of the surprising facts about gravity in extra dimensions is that four-dimensional (4D) gravity can be localized in AdS_5 by tuning the bulk and brane cosmological constants [1]. It is even more surprising that this five-dimensional (5D) model has a dual 4D interpretation via the AdS/CFT correspondence [2]. This localization can be extended to fields of different spin such as scalars and fermions [3, 4], where the fermion and scalar zero modes can be localized anywhere in the 5D bulk. This unrestricted localization is achieved by introducing bulk and boundary masses with the degree of localization directly depending on the bulk mass parameter [4]. A natural question to ask is whether a similar construction can work for gravity whereby the graviton zero mode can be localized anywhere. The AdS/CFT correspondence provides the primary motivation for studying the delocalization of the graviton in the Randall-Sundrum (RS) scenario. If the graviton zero mode is localized on the IR brane, this would suggest that in the dual theory the graviton is a composite CFT state whereby dynamical gravity only emerges in the infrared. At the linear level one can indeed show that by tuning bulk and boundary mass terms for gravity, a graviton zero mode can be localized anywhere in the bulk [5].

Consider adding a bulk mass term for the tensor perturbation in the background Randall-Sundrum metric. The 5D bulk action becomes:

$$S = \int d^5x\sqrt{-g}\left[M^3 R - 2\Lambda - M^3\, k^2\, g^{(0)\,MN}\, g^{(0)\,AB}\,(a\,h_{MA}\,h_{NB} + b\,h_{MN}\,h_{AB})\right]\ , \qquad (1)$$

where a and b are real parameters and $h_{AB} \equiv g_{AB} - g^{(0)}_{AB}$, are perturbations of the Randall-Sundrum metric $g^{(0)}_{AB}$ defined as

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 = A^2(z)(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2) \equiv g^{(0)}_{AB}dx^A dx^B\ . \qquad (2)$$

We assume that the fifth dimension is compactified on an orbifold S^1/Z_2 of radius R, where $0 \leq y \leq \pi R$ is the "fundamental domain", k is the AdS curvature scale, and the Minkowski metric $\eta_{\mu\nu}$ has signature $(-+++)$. The Latin indices $(A, B, \ldots)$ label all the 5D coordinates,

* **e-mail**: tgher@physics.umn.edu

while Greek indices $(\mu, \nu, \dots)$ are restricted to the 4D coordinates. We will work with conformal coordinates defined by $z = (e^{ky} - 1)/k$ and $A(z) = (1+kz)^{-1}$ is the warp factor. At the orbifold fixed points $z_0 = 0$ and $z_1 = (e^{\pi kR} - 1)/k$ there are two three-branes, the UV and the IR brane, respectively.

The metric perturbations, h_{AB} around the background Randall-Sundrum metric $g^{(0)}_{AB}$, correspond to fifteen components, and a useful way to parametrize them is:

$$
\begin{aligned}
ds^2 &= A^2(z)\left[(1+2\phi)dz^2 + 2\,(B_{,\mu} + B_\mu)dz\,dx^\mu \right. \\
&\qquad \left. + \left((1+2\psi)\eta_{\mu\nu} + 2E_{,\mu\nu} + E_{(\mu,\nu)} + \widehat{h}_{\mu\nu}\right) dx^\mu dx^\nu \right] , \qquad (3) \\
&\equiv (g^{(0)}_{AB} + h_{AB})dx^A\,dx^B , \qquad (4)
\end{aligned}
$$

where $E_{(\mu,\nu)} \equiv \partial_\mu E_\nu + \partial_\nu E_\mu$. We see that the metric perturbations are divided into three sectors: scalar, vector, and tensor (with respect to the Poincaré symmetry of the 4D Minkowski background).

Specifically, the tensor mode $\widehat{h}_{\mu\nu}$, is taken to satisfy the transverse ($\partial^\mu \widehat{h}_{\mu\nu} = 0$) and traceless ($\widehat{h}^\mu_\mu = 0$) conditions. It is gauge invariant under infinitesimal coordinate transformations, and being symmetric, it contains $10-5=5$ components. A similar analysis for the remaining vector (B_μ, E_μ) and scalar (ϕ, ψ, B, E) modes can also be done [5].

The bulk Einstein equations following from the action (1) with mass terms leads to

$$
\delta G^A_B + 2M^3\,k^2\left[a\,h^A_B + b\,h\,\delta^A_B\right] = 0 , \qquad (5)
$$

where δG is the linear perturbation of the Einstein tensor, $h^A_B = g^{(0)AC}\,h_{CB}$ and $h = h^A_A$. The equation of motion for the tensor modes $\widehat{h}_{\mu\nu}$ is the transverse–traceless part of the $\mu\nu$ component of (5), and is given by:

$$
\Box\widehat{h}_{\mu\nu} + \widehat{h}''_{\mu\nu} + 3\,\frac{A'}{A}\,\widehat{h}'_{\mu\nu} - 4\,a\,k^2A^2\,\widehat{h}_{\mu\nu} = 0 , \qquad (6)
$$

where $\Box \equiv -\partial_t^2 + \partial_{\mathbf{x}}^2$. Notice that this equation does not depend on the b part of the mass term (1) since the tensor mode $\widehat{h}_{\mu\nu}$ is traceless. The solution of the equation of motion is obtained by a separation of variables $\widehat{h}_{\mu\nu}(x,z) = f(z)H_{\mu\nu}(x)$, where $\Box H_{\mu\nu}(x) = m^2 H_{\mu\nu}(x)$, with m representing the mass of the four-dimensional Kaluza-Klein modes. The massless mode solution is [5]

$$
\widehat{h}^{(0)}_{\mu\nu}(x,z) = \left[C_1\,A(z)^{-2\left(1-\sqrt{1+a}\right)} + C_2\,A(z)^{-2\left(1+\sqrt{1+a}\right)}\right] H^{(0)}_{\mu\nu}(x) , \qquad (7)
$$

while the massive modes are:

$$
\widehat{h}^{(n)}_{\mu\nu}(x,z) = A^{-2}(z)\left[C_1\,J_{2\sqrt{1+a}}\left(\frac{m_n}{kA(z)}\right) + C_2\,Y_{2\sqrt{1+a}}\left(\frac{m_n}{kA(z)}\right)\right] H^{(n)}_{\mu\nu}(x) , \qquad (8)
$$

where C_1, C_2 are arbitrary constants. We will consider only values $a \geq -1$, which include the Randall-Sundrum case ($a=0$). In the limiting case ($a=-1$), the massless solutions are degenerate. Also note that in the limit $a \to 0$ these modes become:

$$
\begin{aligned}
\widehat{h}^{RS,(0)}_{\mu\nu}(x,z) &= \left[C_1 + C_2\,A(z)^{-4}\right] H^{(0)}_{\mu\nu}(x) , \qquad (9) \\
\widehat{h}^{RS,(n)}_{\mu\nu}(x,z) &= A^{-2}(z)\left[C_1\,J_2\left(\frac{m_n}{kA(z)}\right) + C_2\,Y_2\left(\frac{m_n}{kA(z)}\right)\right] H^{(n)}_{\mu\nu}(x) , \qquad (10)
\end{aligned}
$$

which, together with appropriate boundary conditions (see below) smoothly reproduce the Randall-Sundrum solution [1]. The solutions to the equations of motion for the vector and scalar modes can also be obtained. It is found that at the massless level there is a vector mode, but no scalar modes [5]. The vector mode decouples from conserved sources at tree level and therefore does not play a phenomenological role.

2 Delocalizing gravity

The mass spectrum is obtained from the general solutions by imposing boundary conditions satisfied by the bulk modes on the branes. We will also add boundary mass terms on the branes since this will be crucial for obtaining a deformation of the RS solution. Hence, consider the following brane action at the location z_i:

$$\Delta S_i = -k\, M^3 \int d^4x \, \sqrt{-\gamma_0}\, h_{\mu\nu}\, h_{\alpha\beta} \left(\alpha_i\, \gamma_0^{\mu\alpha}\, \gamma_0^{\nu\beta} + \beta_i\, \gamma_0^{\mu\nu}\, \gamma_0^{\alpha\beta}\right) \,, \tag{11}$$

where $\gamma_{0,\mu\nu} = A^2\, \eta_{\mu\nu}$ is the background induced metric on the boundary, with A evaluated at the (unperturbed) location of the brane, and $h_{\mu\nu}$ are the perturbations of the induced metric, $\gamma_{\mu\nu} = \gamma_{0,\mu\nu} + h_{\mu\nu}$. The corresponding boundary condition for the tensor mode is given by:

$$\widehat{h}'_{\mu\nu} = \pm 4\, \alpha_i\, k\, A\, \widehat{h}_{\mu\nu} \,, \tag{12}$$

where this equation is evaluated at z_0 (upper sign) or z_1 (lower sign). Applying this boundary condition (12) to the tensor mode general solution (7) gives:

$$(1 - \sqrt{1+a} \mp 2\alpha_i) C_1\, A(z_i)^{2\sqrt{1+a}} + (1 + \sqrt{1+a} \mp 2\alpha_i) C_2\, A(z_i)^{-2\sqrt{1+a}} = 0 \,. \tag{13}$$

For generic mass parameters a, α_i the only solution is $C_1 = C_2 = 0$, and there is no massless graviton. However, when

$$\alpha_0 = -\alpha_1 \equiv \alpha \;\;, \;\; \alpha = \frac{1}{2}(1 \mp \sqrt{1+a}) \equiv \alpha_\mp \,, \tag{14}$$

the coefficient C_1 (C_2) drops from the boundary condition, and (13) simply gives $C_2 = 0$ ($C_1 = 0$). Hence, the massless tensor mode (7) becomes:

$$\widehat{h}_{\mu\nu}(x, z) = N_T A(z)^{-4\alpha}\, H^{(0)}_{\mu\nu}(x) \,, \tag{15}$$

where N_T is the overall normalization constant which is determined from the quadratic action in the perturbations and cannot be determined by the boundary conditions.

This form of the solution is only meaningful when the bulk mass parameter $a \geq -1$. This corresponds to $\alpha = \alpha_+ (\alpha_-)$ for $\alpha \geq 1/2$ ($\alpha \leq 1/2$), so that the full range of the boundary mass parameter α is covered by the two branches $\alpha_\pm$. When $a, \alpha_- \to 0$, the bulk and boundary mass terms become zero, and the α_- mode reduces to the usual RS tensor mode that is constant in the z coordinate. Hence the α_- mode is a smooth deformation of the RS tensor mode from $\alpha = 0$ to $-\infty < \alpha \leq 1/2$. On the other hand the α_+ mode can only exist when the boundary mass is nonzero and corresponds to the deformation of the RS tensor mode to values $1/2 \leq \alpha < \infty$.

The existence of zero modes follows from the relation (14) between the bulk and boundary mass parameters. However, this relation only fixes one of the parameters, say a, so that there is still freedom to choose α. The wavefunction dependence on α is given by [5]

$$f_T^{(0)}(y) \propto e^{-(1-4\alpha)ky} \,. \tag{16}$$

The RS tensor mode localized on the UV brane corresponds to $\alpha = 0$. However we now see that by varying α we can smoothly deform the tensor mode to be localized anywhere. For $\alpha < 0$ the mode becomes even more localized on the UV brane, compared to the original RS scenario. On the other hand, for $\alpha > 0$ the tensor mode is delocalized away from the UV brane towards the IR brane. The transition occurs for $\alpha = 1/4$ where the tensor mode is completely flat. As α becomes larger than $1/4$ the tensor mode becomes more and more localized on the IR brane. Hence, we have a continuous deformation of the original RS tensor mode from being completely localized on the UV brane to being completely localized on the IR brane!

For the massive solutions (8) the boundary condition (12) for $\alpha = \alpha_{\pm}$, leads to the Kaluza-Klein mass spectrum

$$m_n \simeq \left(n + \sqrt{1+a} - \frac{3}{4}\right) \pi\, k e^{-\pi k R} \ , \qquad n = 1, 2, 3, \dots \ , \tag{17}$$

where $k\, e^{-\pi k R} \ll m_n \ll k$. This approximation for the mass spectrum becomes increasingly better as n grows. When $a = 0$ we recover the RS Kaluza-Klein mass spectrum [1].

2.1 4D Planck mass

Assuming that there is matter on the branes with a stress-energy tensor, the graviton zero mode gives rise to the 4D (reduced) Planck mass (M_P) [5]

$$M_P^2 = \frac{M^3}{k} \frac{A^{8\alpha}(z_i)}{(4\alpha - 1)} \left[e^{2(4\alpha-1)\pi k R} - 1\right] \ . \tag{18}$$

When $\alpha = 0$ this expression reduces to the RS result $M_P^2 \simeq M^3/k$ [1], assuming $\pi k R \gg 1$.

The properties of the zero mode can be used to construct an interesting phenomenological scenario where the graviton is localized on the IR brane and all the standard model matter is located on the UV brane. If we associate the IR brane with the millimeter scale 10^{-3} eV, then for $M \sim k \sim$ TeV, we obtain the usual Planck mass M_P for $\alpha = 1/2$. The scale of the UV brane is the TeV scale. In the bulk the tensor mode has a profile e^{ky}, and it is therefore localized away from the UV brane. This explains the weakness of gravity with respect to the gauge interactions. One can also verify that the same exponential suppression holds for the coupling of the massive tensor modes.

2.2 Short range modifications of gravity

The gravitational interaction between nonrelativistic matter sources on the UV brane due to the zero mode and the massive Kaluza-Klein modes leads to the modification of Newtonian gravity. As we have seen in (17), the lowest Kaluza-Klein mass is at the IR scale $m_1 \sim k\, A\,(z_1)$. This scale sets the distance above which gravity is standard, so that gravity will be modified at distances r satisfying:

$$\frac{1}{k} \ll r \ll \frac{1}{k\, A_1} \ , \tag{19}$$

where we have denoted $A_1 \equiv A\,(z_1)$. The contribution from the Kaluza-Klein modes presents two distinct behaviors depending on whether α is smaller or greater than $1/4$. For either region, we define a positive parameter $\xi \equiv |4\alpha - 1|$. The gravitational potential is found to be [5]

$$V(r) \ \simeq \ -\frac{\mu}{M_P^2\, r} \left[1 + \frac{2\,\Gamma\,(2\xi)}{\xi\,\Gamma^2\,(\xi)} \frac{1}{(2\,k\,r)^{2\xi}}\right] \quad , \quad \xi = 1 - 4\,\alpha \ , \ \alpha < 1/4 \ , \tag{20}$$

$$V(r) \ \simeq \ -\frac{\mu}{M_P^2\, r} \times \begin{cases} 1 + \frac{2\,\Gamma(2\xi)}{\xi\,\Gamma^2(\xi)} \frac{1}{(2\,k\,A_1\,r)^{2\xi}} \quad , & r \gtrsim \frac{1}{kA_1} \\ \frac{2\,\Gamma(2\xi)}{\xi\,\Gamma^2(\xi)} \frac{1}{(2\,k\,A_1\,r)^{2\xi}} \quad , & r \lesssim \frac{1}{kA_1} \end{cases} ;\xi = 4\alpha - 1, \alpha > 1/4 \ . \tag{21}$$

The $1/r$ term in these two expressions is the contribution from the zero mode, which reproduces the standard Newtonian gravity at large distances. Instead, the second term represents the interaction mediated by the Kaluza-Klein massive modes in each case.

Note that there is a striking phenomenological difference between the two cases, shown in (20) and (21), respectively. Indeed, in the first case $\alpha < 1/4$, the corrections to the Newtonian potential are relevant only at UV distances $r \lesssim k^{-1}$. In particular the $1/r^3$ behavior of the

RS model is recovered for $\alpha = 0$. However, in the second case of $\alpha > 1/4$, the gravitational potential is strongly modified already at the much larger (and, possibly, phenomenologically relevant) IR scale, $r \simeq (A_1\, k)^{-1}$. Moreover, below this distance the $1/r$ term disappears and the potential is given solely by a power law $1/r^{2\xi+1}$. The cancellation of the $1/r$ term means that the corresponding force is stronger than the usual gravity. So, for $\alpha > 1/4$ standard gravity only emerges at infrared distance scales $r \gtrsim (A_1\, k)^{-1}$.

Actually, this behavior is due to the different localization of the zero mode in the two regimes. For $\alpha < 1/4$, the zero mode is localized towards the UV brane, and the relative contribution from the Kaluza-Klein modes can be neglected. On the contrary, for $\alpha > 1/4$ the zero mode is localized towards the IR brane and the relative contribution of the massive modes significantly increases becoming the dominant contribution for $r \lesssim (A_1\, k)^{-1}$—the regime indicated in (19). We will see that for $\alpha > 1/4$ this behavior is consistent with the graviton being a composite at the IR scale. Assuming that the IR (or compositeness) scale is related to the cosmological constant then the IR scale is $\sim 10^{-3}$ eV. The striking experimental signal of this model would then be that Newton's law of gravity changes to a new power law $r^{-1} \to r^{-2\xi-1}$ below ~ 0.1 mm, which could even be irrational!

3 The holographic interpretation

Motivated by the string theory AdS/CFT correspondence, bulk theories in a slice of AdS_5 can be given a holographic interpretation as dual to a CFT (at large N and large 't Hooft coupling) with conformal invariance broken in the IR, coupled to gravity and possibly other fields [6, 7, 8]. As opposed to the string-theory AdS/CFT, the UV boundary value of bulk fields are not only sources of operators in the dual CFT but acquire their own dynamics due to the presence of a UV cutoff.

The correlator can be calculated from the bulk theory and is given by [5]

$$\Sigma(p) = \left(\frac{M}{k}\right)^3 \frac{k^4}{2} \frac{q_0\,(I_\nu(q_0)K_\nu(q_1) - I_\nu(q_1)K_\nu(q_0))}{I_{\nu\mp1}(q_0)K_\nu(q_1) + I_\nu(q_1)K_{\nu\mp1}(q_0)}\,, \tag{22}$$

where $\nu_\pm = \pm(4\alpha_\pm - 1)$, and $q = p/(kA(z))$. The behavior of $\Sigma(p)$ can be studied for momenta p such that $kA_0 \gg p \gg kA_1$, or equivalently, $q_0 \ll 1, q_1 \gg 1$. In this energy regime, the effects of the conformal symmetry breaking (i.e., the IR brane) are completely negligible. The leading nonanalytic piece in $\Sigma(p)$ is then interpreted, by "matching" to the string AdS/CFT correspondence in the $A_0 \to \infty$ limit, as due to the strong dynamics of the dual CFT above the scale of conformal symmetry breaking. On the other hand, the analytic pieces in the correlator, which, in string AdS/CFT, are subtracted away by adding appropriate counterterms, are now interpreted as kinetic (and higher-derivative terms) of the dynamical source field in the holographic dual.

3.1 α_- branch holography

Consider first the α_- branch of the zero mode solution, which is the one continuously connected to the $\alpha = 0$ RS value. The scaling dimension $\Delta_{\mathcal{O}}$ of the operator $\mathcal{O}$—the energy momentum tensor of the dual theory—sourced by the metric perturbation h is:

$$\Delta_{\mathcal{O}} = 3 + \nu = 4 - 4\alpha_-\ . \tag{23}$$

Furthermore the leading analytic piece in the momentum expansion of the correlator (22) indicates that there is a kinetic term for the metric perturbation in the dual theory. Thus, the holographic description of this branch is that of a metric fluctuation $\widehat{h}_{\mu\nu}$ coupled to $T^{\mu\nu}_{CFT}$ of scaling dimension $4 - 4\alpha_-$. Note the unusual fact that the energy momentum tensor of the

CFT has an anomalous dimension. This, however, is required if the 4D dual is to evade the Weinberg-Witten theorem [9]—which assumes Poincare invariance, broken here by the presence of a nontrivial background metric, as in theories of induced gravity. The Lagrangian of the dual theory is, then, at a UV scale $\sim k$, with a canonically normalized metric perturbation:

$$\mathcal{L}_{UV} = \epsilon_\nu \frac{1}{4} h_{\mu\rho}\Box h^{\mu\rho} + \frac{\lambda_{UV}}{k} h_{\mu\rho}T^{\mu\rho}_{CFT} + \frac{\lambda_{UV}}{k} h_{\mu\rho}T^{\mu\rho}_{matter} + \mathcal{L}_{CFT} , \tag{24}$$

where $\epsilon_\nu = \text{sign } \nu$ and $\lambda_{UV} = |\nu|^{1/2}(M/k)^{-3/2}$. We have included the coupling to observable matter fields (UV-brane localized in the gravity dual). From eqn. (24), taking into account the anomalous scaling dimension of T_{CFT} from (23), one can derive the correct Planck mass consistent with the bulk calculation [5].

Similarly the leading correction to Newton's law at intermediate distances, $r < 1/(kA_1)$, can be calculated from the point of view of the dual interpretation. In the irrelevant case ($\nu_- > 0$), the coupling of matter to the CFT can be treated perturbatively and the leading correction arises from a single insertion of the CFT correlator and two insertions of the source field, as in the RS case. The leading and first subleading contribution to the Newton potential is [5]

$$\begin{aligned} V(r) &= -\mu\frac{\lambda_{UV}^2}{k^2}\int\frac{d^3p}{2\pi^2}\, e^{ipx}\left(\frac{1}{p^2} - \frac{\lambda_{UV}^2}{k^2}\frac{\langle\mathcal{O}\mathcal{O}\rangle(p)}{p^4}\right) , \\ &= -\mu\frac{\lambda_{UV}^2}{k^2}\int\frac{d^3p}{2\pi^2}\, e^{ip\cdot x}\left(p^{-2} - p^{2\nu-2}k^{-2\nu}\frac{\Gamma(-\nu)}{2^{2\nu}\Gamma(\nu)}\right) , \end{aligned} \tag{25}$$

where:

$$\langle\mathcal{O}\mathcal{O}\rangle(p) = -\left(\frac{M}{k}\right)^3 k^4 \left(\frac{p}{2k}\right)^{2\nu+2}\frac{4\Gamma(-\nu)}{\nu\Gamma(\nu)} , \tag{26}$$

is the nonanalytic piece in the momentum expansion of $\Sigma(p)$, interpreted as the CFT correlator at the relevant energy scale. Performing a Fourier transform of (25) and using various gamma-function identities (see [5]) we find *precisely* our result from the gravity calculation (20).

Consider next the correction, at $r < 1/(kA_1)$, for the case when the interaction with the CFT is relevant ($\nu_- < 0$). Then, we have to sum the chain of bubble graphs as indicated below (recall $\epsilon_\nu = -1$ now):

$$\begin{aligned} V(r) &= -\mu\frac{\lambda_{UV}^2}{k^2}\int\frac{d^3p}{2\pi^2}\frac{e^{ip\cdot x}}{p^2}\left[\epsilon_\nu - \frac{\lambda_{UV}^2}{k^2}\frac{\langle\mathcal{O}\mathcal{O}\rangle(p)}{p^2} + \epsilon_\nu\frac{\lambda_{UV}^4}{k^4}\left(\frac{\langle\mathcal{O}\mathcal{O}\rangle(p)}{p^2}\right)^2 - \ldots\right] , \\ &= \mu\frac{\lambda_{UV}^2}{k^2}\int\frac{d^3p}{2\pi^2}\, e^{ip\cdot x}\frac{1}{p^2 - \frac{\lambda_{UV}^2}{k^2}\langle\mathcal{O}\mathcal{O}\rangle(p)} \simeq -\mu\int\frac{d^3p}{2\pi^2}\, e^{ip\cdot x}\frac{1}{\langle\mathcal{O}\mathcal{O}\rangle(p)} , \end{aligned} \tag{27}$$

where we notice that for the distance scales of interest the CFT correlator dominates over p^2 in the denominator, as appropriate for a relevant coupling. Finally, computing the Fourier transform as before, we again recover precisely the leading term of the potential from Eq. (21) on the gravity side.

3.2 α_+ branch holography

Now consider $\nu = \nu_+ = 4\alpha_+ - 1 > 1$. In this case the graviton is always localized on the IR brane. The scaling dimension of the operator $\mathcal{O}$ is given by

$$\Delta_{\mathcal{O}} = \nu + 1 = 4\alpha_+ . \tag{28}$$

Thus when $\nu_+ > 2$ the source coupling to the CFT is irrelevant, marginal for $\nu_+ = 2$, while for $1 \le \nu_+ < 2$ the coupling is relevant.

The analytic terms of the momentum expansion of the correlator can be used to obtain the long-distance Lagrangian:

$$\mathcal{L}_{IR} = \frac{1}{4} h_{\mu\rho}(\Box - m_h^2) h^{\mu\rho} + \frac{\chi}{k} h_{\mu\rho} T^{\mu\rho}_{CFT} + \frac{\chi}{k} h_{\mu\rho} T^{\mu\rho}_{matter} + \mathcal{L}_{CFT} \, , \tag{29}$$

where $\chi = (\nu_+ - 2)^{1/2}(M/k)^{-3/2}$ and $m_h^2 = 4(\nu-1)(\nu-2)k^2$. If we now write the small-momentum expansion of the correlator as:

$$\langle \mathcal{O}\mathcal{O} \rangle \simeq (Mk)^3 \, 16\nu_+(\nu_+ - 1)^2 A_1^{2\nu_+} \frac{1}{p^2} \, , \tag{30}$$

where $A_0 = 1$, then the leading contribution to the gravitational potential at large distances is given by:

$$\begin{aligned} V(r) &\simeq -\mu \frac{\chi^2}{k^2} \int \frac{d^3p}{2\pi^2} \, e^{ip\cdot x} \, \frac{\chi^2}{k^2} \frac{\langle \mathcal{O}\mathcal{O}\rangle(p)}{m_h^4} \, , \\ &= -\mu \frac{\chi^4}{m_h^4} \frac{M^3}{k} 16\nu_+(\nu_+ - 1)^2 A_1^{2\nu_+} \int \frac{d^3p}{2\pi^2} \, e^{ip\cdot x} \frac{1}{p^2} \, , \\ &= -\frac{\mu}{M_P^2 r} \, , \end{aligned} \tag{31}$$

where the source propagator has been approximated by $1/(p^2 + m_h^2) \simeq 1/m_h^2$ and the Planck mass is given by:

$$M_P^2 = \left(\frac{M}{k}\right)^3 \frac{k^2}{\nu_+} A_1^{-2\nu_+} = \frac{M^3}{k(4\alpha_+ - 1)} e^{(8\alpha_+ - 2)\pi k R} \, . \tag{32}$$

This agrees with the Planck mass formula derived from the bulk for $\alpha_+ > 1/2$. Note also that further insertions of $\langle \mathcal{O}\mathcal{O}\rangle$ in (31) are negligible at large distances.

When $\nu_+ > 2$ the source coupling to the CFT is irrelevant and the gravitational potential follows from the coupling to the CFT. The UV Lagrangian is the same as (29). The nonanalytic part of the correlator expansion is given by:

$$\langle \mathcal{O}\mathcal{O} \rangle = -\left(\frac{M}{k}\right)^3 k^4 \left(\frac{p}{2k}\right)^{2\nu-2} \frac{4\,\Gamma(2-\nu)}{\Gamma(\nu-1)} \, . \tag{33}$$

Since there is no longer any massless pole, the leading contribution to the potential is given by:

$$\begin{aligned} V(r) &\simeq -\mu \frac{\chi^2}{k^2} \int \frac{d^3p}{2\pi^2} \, e^{ip\cdot x} \, \frac{\chi^2}{k^2} \frac{\langle \mathcal{O}\mathcal{O}\rangle(p)}{m_h^4} \, , \\ &= \mu \frac{\chi^4}{m_h^4} \left(\frac{M}{k}\right)^3 \int \frac{d^3p}{2\pi^2} \, e^{ip\cdot x} \left(\frac{p}{2k}\right)^{2\nu-2} \frac{4\,\Gamma(2-\nu)}{\Gamma(\nu-1)} \, . \end{aligned} \tag{34}$$

Performing the Fourier transform leads precisely to the result derived purely on the gravity side (21).

When $1 < \nu_+ < 2$ the source coupling to the CFT is relevant but the nonanalytic term in the expansion of the correlator is still subdominant compared to the leading mass term. In this case no summation is needed beyond the leading CFT correction and so the contribution to the potential is identical to that obtained in (34). The corresponding Fourier transform then leads to the same expression (21).

4 Discussion

We have seen that the graviton zero mode can be smoothly deformed away from the Planck brane. This deformation requires modifying the bulk covariant theory at the linear level by introducing bulk and boundary mass terms. A massless mode then occurs only for a special choice of the bulk and boundary masses. This is an additional tuning beyond the usual tuning of bulk and brane cosmological constants in the RS model.

However, general relativity is an inherently nonlinear theory, and it is apparent that higher order nonlinear interactions will spoil this symmetry. This requires modifying the scenario at the nonlinear level by introducing nonlinear terms in the bulk and on the branes, in order to at least preserve the 4D general covariance. After an extra fine-tuning of the nonlinear terms the zero mode is expected to remain massless in the nonlinear theory, although this analysis remains to be done.

As shown in Ref [5] the scalar sector is trivially zero because this is the only solution consistent with the bulk and boundary equations. Clearly this is due to the fact that we are working at the linear level, and the scalar modes can possibly appear at the nonlinear level. Nonetheless even though a smooth deformation exists at the linear level without the presence of ghosts, the appearance of scalar modes at the nonlinear level could lead to a strong-coupling problem. This issue remains to be investigated. Scalar modes may also arise when matter is added on the brane. On the phenomenological side, they are certainly needed to reproduce the correct gravitational law if the stress–energy tensor of the matter fields is not traceless. A similar situation takes place in the usual RS case. In the compact version with a stabilized radion there are no massless scalar excitations. However, a massless scalar mode (most easily interpreted as the brane bending mode) arises when matter is present on the brane, and allows for the recovery of standard 4D gravity at large scales [10]. A similar analysis should also be carried out in the set-up we have discussed here.

By the AdS/CFT correpondence there is an interesting 4D dual interpretation of our model, especially in the case when the graviton zero mode is localized on the IR brane. This is because zero modes localized on the IR brane correspond to CFT bound states and therefore the dual CFT interpretation would correspond to gravity emerging from the strongly coupled gauge theory. In this model of emergent gravity the UV theory is a gauge (string) theory at the TeV scale, and the graviton is a composite particle which can be associated with the millimeter scale. Thus gravity emerges as a low energy phenomenon in the IR. This is different from the conventional viewpoint that gravity is a fundamental degree of freedom in the UV theory. The setup looks intriguing at the linear level and if the remaining issues at the nonlinear level can be addressed then this would represent a novel possibility for gravity.

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