Induced gravity and Universe generation on domain walls in five-dimensional space-time

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Abstract

We present a non-compact *4 + 1* dimensional model with a local strong four-fermion interaction supplementing it with gravity. In the strong coupling regime it reveals the spontaneous translational symmetry breaking which eventually leads to the formation of thick 3-brane, embedded in the AdS_5 manifold. To describe this phenomenon we construct the appropriate low-energy effective action and investigate kink-type vacuum solutions in a quasiflat Riemannian metric. We discuss the physics of light particles in *3 + 1* dimensions and establish the relation among the bulk five-dimensional gravitational constant, the brane Newton's constant and the curvature of AdS_5 space-time, the compositeness scale of the scalar matter and the symmetry breaking scale. The induced cosmological constant on the brane does vanish due to exact cancelation of matter and gravity contributions.

Dedicated to 100 birthday of Matvey P. Bronstein

1 Introduction

In the present talk we study and explore a non-compact *4 + 1* dimensional fermion model [1] with a local strong four-fermion interaction supplementing it with induced gravity [2]. In strong coupling regime there arises the spontaneous breaking of translational symmetry which leads to the localization of light particles on *3 + 1* flat domain wall in *4 + 1*- dimensional AdS_5 space-time.

The possibility about that our *3 + 1* dimensional world might be allocated on a brane in a multi-dimensional space-time has recently attracted much interest [3]-[8], giving new tools to solve the long standing mass and scale hierarchy problems in particle theory. New extra dimensional physics could manifest itself in accessible experiments and observations, when the size of extra dimensions is relatively large or even infinite (see review articles [9]–[15]).

The thick (or fat) brane (or domain wall) formation and the trapping of light particles in its layer might be obtained [16]–[21] by a number of particular background scalar and/or gravitational fields living in the multi-dimensional bulk, when their vacuum configuration has a non-trivial topology, thereby generating zero-energy states localized on the brane.

Respectively, the mechanism of how such background fields might emerge and further induce the spontaneous breaking of translational symmetry is worthy to be elaborated[1, 2].

Let's elucidate the domain wall phenomenology in more details and start from the model of one five-dimensional fermion bi-spinor $\psi_j(X)$ coupled to a scalar field $\Phi(X)$.

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$(X_A) = (x_\mu, z)$, $x_\mu = (x_0, x_1, x_2, x_3)$, $(\eta_{AA}) = (+, -, -, -, -)$ and the subspace of coordinates x_μ eventually corresponds to the four dimensional Minkowski space. The extra-dimension size is supposed to be infinite (or large enough).

The fermion wave function is then described by the Dirac equation

$$[\, i\gamma^A \partial_A \mp \Phi(X)\,]\psi_j(X) = 0 \ , \quad \gamma^A = (\gamma^\mu, i\gamma_5) \ , \quad \{\gamma^A, \gamma^B\} = 2\eta^{AB} \ , \tag{1}$$

γ^μ, γ_5 being a standard set of four dimensional Dirac matrices in the chiral (or Weyl) representation.

The trapping of light fermions on a four dimensional hyper-plane – the domain wall $\equiv$ 3-brane – localized in the fifth dimension at $z = z_0$ can be promoted by a certain *topological*, z-dependent configuration of the v. e. v. of scalar field $\langle \Phi(X) \rangle_0 = \varphi(z)$ – for instance $\varphi(z) = M \tanh(Mz)$ – due to the appearance of zero-modes with a certain chiralities in the spectrum of the four dimensional Dirac operator [3, 9]. It turns out that real fermions - quarks and leptons of the Standard Model are mainly massive. Therefore, for each light fermion in the Brane World one needs two five-dimensional promo-fermions $\psi_1(X), \psi_2(X)$ [20, 1] to generate left- and right-handed parts of a four dimensional Dirac bi-spinor as zero modes. Those fermions clearly to have couple with opposite charges to the scalar field $\Phi(X)$ in order to produce the required zero modes with different chiralities,

$$[\, i \not{\partial} - \tau_3 \Phi(X)\,]\Psi(X) = 0 \ , \quad \not{\partial} \equiv \widehat{\gamma}^A \partial_A \ , \quad \Psi(X) = \begin{pmatrix} \psi_1(X) \\ \psi_2(X) \end{pmatrix} \ , \tag{2}$$

where $\widehat{\gamma}^A \equiv \gamma^A \otimes \mathbf{1}_2$ – the Dirac matrices, and $\tau_a \equiv \mathbf{1}_4 \otimes \sigma_a$, $a = 1, 2, 3$ are the generalizations of "Pauli matrices" for bi-spinor components field $\psi_i(X)$. In addition to the trapping scalar field, a further one is required to supply light domain wall fermions with a mass. Its coupling must mix left and right chiralities as the mass term breaks the chiral invariance. Thus we introduce two types of four-fermion self-interactions to reveal two composite scalar fields with a proper coupling to fermions. These two scalar fields acquire mass spectra similar to fermions with light counterparts located on the domain wall. The dynamical scheme of creation of domain wall particles turns out to be quite economical and few predictions on masses and decay constants of fermion and boson particles have been derived [1]. However the allocation of matter on the domain wall certainly leads to strong gravitational effects.

2 Model of fermions in five-dimensional space-time with self-interaction

Thus based on the above reasonings, we come to the following classical Lagrange density of a fermionic model in five-dimensional Euclidean space,

$$\mathcal{L}^{(5)}(\overline{\Psi}_j, \Psi_j) = \sum_{j=1}^{N_f} \overline{\Psi}_j \, i \not{\partial} \Psi_j + \frac{g_1}{4N\Lambda^3} \left(\sum_{j=1}^{N_f} \overline{\Psi}_j \tau_3 \Psi_j \right)^2 + \frac{1}{4N\Lambda^3} \sum_{j,k=1}^{N_f} G_{2,jklm} \, \overline{\Psi}_j \tau_1 \Psi_k \overline{\Psi}_l \tau_1 \Psi_m, \tag{3}$$

where $\Psi(X)$ is the eight-component five-dimensional fermionic field with the flavor j, which can also be a "color" multiplet with N_c degrees of freedom (d.o.f.). The total number N of color and flavor d.o.f. of massive fermions is approximately equal to twenty in the Standard Model.If we take the massive Dirac neutrinos in the SM, then $N = 24$. Being nonrenormalizable, this model can be considered an effective model, arising at the compositeness scale Λ as a result of reduction from a more fundamental theory valid at the Planck scale. It contains two dimensional coupling constants expressed in units of the compositeness scale Λ. The ultraviolet cutoff scale Λ plays the role of a cutoff for virtual fermion energies and momenta. We take the structure of the matrix of coupling constants $G_{2,jklm}$ in minimal form, which suffices for the dynamical fermionic mass generation $G_{2,jklm} = g_{2,j}\, g_{2,l}\, \delta_{jk}\, \delta_{lm}$, and neglect transitions with flavor changing. Then Lagrangian density (3) can be bosonized with the help of two scalar fields $\Phi(X)$ and $H(X)$,

$$\mathcal{L}^{(5)}(\overline{\Psi}_j, \Psi_j, \Phi, H) = \sum_{j=1}^{N_f} \overline{\Psi}_j (i\, \partial\!\!\!/ - \tau_3 \Phi - \tau_1 \bar{g}_j\, H) \Psi_j - \frac{N\Lambda^3}{g_1}\, \Phi^2 - \frac{N\Lambda^3}{g_2}\, H^2 \, . \tag{4}$$

Here we introduce an average coupling constant $g_2 = \sum g_{2,j}^2 / N_c$ and the relative interaction constants $\bar{g}_j = g_{2,j}/\sqrt{g_2}$. The sum in the definition of the constant g_2 ranges both flavor and color d.o.f. These constants eventually parameterize masses of fermions of the SM. As the Yukawa constant for top quark considerably exceeds the corresponding constants for all other fermions, we can assume with good accuracy that $g_2 \simeq g_t^2$, and the relative constant in the top channel is then $\bar{g}_t \simeq 1$ in this approximation.

All interactions lead coherently, first, to the discrete symmetry breaking and, further on, to the breaking of translational invariance. Namely, for sufficiently large values of the coupling constants, this system undergoes a phase transition to the state in which the condensation of fermion-antifermion pairs does spontaneously break – partially or completely – the so-called τ-symmetry: $\Psi_j \longrightarrow \tau_1 \Psi_j$; $\Phi \longrightarrow -\,\Phi$; and $\Psi_j \longrightarrow \tau_3 \Psi_j$; $H \longrightarrow -\,H$. To investigate this phenomenon we must calculate the low-energy effective action with kinetic terms for composite scalar fields generated by high-energy fermions (the one-loop approximation with using the invariant separation of high-and low-energy scales [24]). In large N- approximation (in Euclidean space) and after integrating out the high-energy part of the fermion spectrum we come to the following one-loop effective low-energy Lagrange density,

$$\begin{aligned}
&\mathcal{L}^{(5)}_{\text{low}}(\overline{\Psi}^{(l)}_j, \Psi^{(l)}_j, \Phi, H) = \sum_{j=1}^{N_f} \overline{\Psi}^{(l)}_j i(\partial\!\!\!/ + \tau_3 \Phi(X) + \tau_1 \bar{g}_j\, H(X)) \Psi^{(l)}_j \;\; + \\
&\frac{\Lambda}{4\pi^3} \Big\{ [N \partial_\mu \Phi(X) \partial_\mu \Phi(X) + N_c \partial_\mu H(X) \partial_\mu H(X)] \\
&+ N\, [\,\partial_z \Phi(X)\,]^2 + N_c\, [\,\partial_z H(X)\,]^2 - 2N\Delta_1 \Phi^2(X) - 2N_c \Delta_2 H^2(X) \\
&+ N\Phi^4(X) + 2N_c \Phi^2(X) H^2(X) + N_c H^4(X) \Big\},
\end{aligned} \tag{5}$$

where two mass scale parameters Δ_i characterize the deviation from the critical point g_i^{cr} [1],

$$\Delta_1(g_1) = \frac{2\Lambda^2}{9 g_1} \left(g_1 - 9\pi^3\right); \quad \Delta_2(g_t) = \frac{2\Lambda^2}{9 g_t^2} \left(g_t^2 - \frac{9N\pi^3}{N_c} \right). \tag{6}$$

In turn, the pair of equations of motion for scalar fields is

$$\Phi'' = 2\Phi \left(\Phi^2 + \frac{N_c}{N} H^2 \right) - 2\Delta_1 \Phi; \quad H'' = 2H \left(\Phi^2 + H^2 \right) - 2\Delta_2 H. \tag{7}$$

As in [1] one can discover kink–like solutions for Eqs. (7): namely,

$$(J) \quad \Phi_J \equiv \langle \Phi(X) \rangle_0 = M \tanh(Mz) \;, \quad H_J \equiv \langle H(X) \rangle_0 = 0 \;; \tag{8}$$

$$\begin{aligned}
(K) \quad \Phi_K &\equiv \langle \Phi(X) \rangle_0 = M \left[\tanh(\beta z) + \mathrm{O}(\frac{\mathrm{N_c}\mu^2}{\mathrm{NM}^2}) \right], \\
H_K &\equiv \langle H(X) \rangle_0 = \mu \left[\mathrm{sech}(\beta z) + \mathrm{O}(\frac{\mu^2}{\mathrm{M}^2}) \right] \;,
\end{aligned} \tag{9}$$

where we introduce the notation

$$\beta^2 = M^2 - \tilde{\mu}^2; \quad \tilde{\mu}^2 = \frac{N_c}{N} \mu^2 = 2\Delta_2 - M^2 \, . \tag{10}$$

Solution (K) is presented approximately in (9) in the expansion in the parameter μ/M , which characterizes the ratio of the SM particle masses to the scale of translational symmetry breaking. It is easy to see that solution (K) exists only in the interval $\Delta_2 < M^2 < 2\Delta_2$ and coincides with the extremum of solution (J) in the limit $\Delta_2 \to M^2/2,\ \mu \to 0,\ \beta \to M$.

3 Ultra-low energy physics on the domain wall in the matter sector

As a result of creating the domain wall, we expect the localized the SM fermions to acquire masses m_j^f much less than the low-energy scale M, i.e., we wait for particle physics to appear naturally in the $(3+1)$- dimensional space-time. In our consideration we have three scales, $\mu \ll M \ll \Lambda$, among which the least one, μ, is the scale of the physics of ultra-low energies.

We describe the structure of the spectrum and of particle interaction on the brane in the absence of gravity. The kinetic operators (second variations of the effective action) of the two scalars $\Phi(X)$ and $H(X)$ and of the spinor field $\Psi(X)$ do exhibit normalizable zero-modes in the extra dimension, in the vicinity of the vacuum background (8) or (9), at the scaling point $M^2 = \Delta_1 = 2\Delta_2$ or $\mu = 0$. Those zero-modes $\phi_0(z)$, $h_0(z)$ and $\psi_{0,j}(z)$, respectively, are localized at the origin of the z-axis, with thickness width $\sim 1/M$ and, at ultra-low energies, the fluctuations of the matter fields can be parameterized as follows: namely,

$$\begin{aligned}
&\Phi(X) \simeq \langle\Phi(X)\rangle_0 + \phi(x)\phi_0(z)\ ; \quad \phi_0(z) \simeq \mathrm{sech}^2(Mz)\sqrt{\frac{3M\pi^3}{2\Lambda N}}\ ;\\
&H(X) \simeq \langle H(X)\rangle_0 + h(x)h_0(z)\ ; \quad h_0(z) \simeq (\mathrm{sech}(Mz))^{1-2\epsilon}\sqrt{\frac{M\pi^3}{\Lambda N_c}}\ ; \quad \epsilon \equiv \frac{\mu^2}{M^2};\\
&\Psi_j(X) \simeq \psi_j(x)\,\psi_{0,j}(z)\ ; \quad \psi_{0,j}(z) \simeq \mathrm{sech}(Mz)\sqrt{\frac{M}{2}}\ .
\end{aligned} \tag{11}$$

For these configurations, the effective Lagrangian density (in the Minkowski space) is

$$\begin{aligned}
\mathcal{L}^{(4)}\big|_{\mu=0} &= \sum_{j=1}^{N_f}\overline{\psi}_j(x)\left(i\,\partial\!\!\!/ - g_j^{(Y)}\,h(x)\right)\psi_j(x) + \frac{1}{2}(\,\partial_\mu\phi(x)\,)^2 + \frac{1}{2}(\,\partial_\mu h(x)\,)^2\\
&- \lambda_1\phi^4(x) - \lambda_2\phi^2(x)h^2(x) - \lambda_3 h^4(x)\ ,
\end{aligned} \tag{12}$$

with the effective constants at ultra-low energies being the Yukawa coupling $g_j^{(Y)}$ for fermions,the self-interaction constants λ_k for the Higgs-type scalar fields $h(x)$, and for branons $\phi(x)$

$$g_j^{(Y)} = \frac{\pi}{4}\bar{g}_j\sqrt{\frac{N\zeta}{N_c}}\ , \quad \lambda_1 = \frac{18}{35}\zeta\ , \quad \lambda_2 = \frac{4}{5}\zeta\ , \quad \lambda_3 = \frac{N}{3N_c}\zeta\ , \quad \zeta \equiv \frac{M\pi^3}{\Lambda N} = \frac{\pi^3}{3N}\sqrt{\kappa}. \tag{13}$$

As we to see in the next Sec., in the presence of gravity [2], the zero modes remain localized and the vacuum solution that corresponds to the AdS_5 geometry in the asymptotic domain does not play essential role in determining the constants in expression (12).

For vacuum configuration (K), the deviation from the scaling point at $\mu \ll M$ generates masses for the Higgs particles and fermions. Furthermore, one obtains trilinear scalar vertices

$$\begin{aligned}
&\Delta\mathcal{L}_\mu^{(4)} = -\frac{1}{2}m_h^2\,h^2(x) - \sum_{j=1}^{N_f} m_j^{(f)}\,\overline{\psi}_j(x)\psi_j(x) - \lambda_4\,h^3(x) - \lambda_5\,\phi^2(x)h(x)\ ;\\
&m_h^2 = \mu^2\left(4 - \frac{2N_c}{N}\right)\ ; \quad m_j^{(f)} = \frac{\pi}{4}\bar{g}_j\mu\ ; \quad \lambda_4 = \frac{4}{3}\mu\sqrt{\zeta\frac{N}{N_c}}; \quad \lambda_5 = \frac{8}{5}\mu\sqrt{\zeta\frac{N_c}{N}}\ .
\end{aligned} \tag{14}$$

The tree-level coupling of light fermions to the massless scalar $\bar{\psi}(x)\psi(x)\phi(x)$ does not appear: it is suppressed by additional powers of μ^2/M^2(heavy fermion exchange). Thereby the low-energy Standard Model matter is essentially stable. Also, we see, that all the interaction vertices are controlled by the parameter $\zeta \sim M/\Lambda$, and if $\zeta \ll 1$, then the scalar matter practically interacts neither with fermions nor with itself (we do not consider the effects of gauge fields). But it is difficult to predict the value of the parameter ζ in the absence of gravity, and we can only bound it above by experimental data. On the other hand, the masses of the Higgs-type scalars and the fermions masses are controlled by the ultra-low scale μ independently of ζ. Following the Standard Top Model [27],we can set $\bar{g}_t = 1$ for the heaviest top quark, which therefore provides the leading contribution to the dynamical symmetry breaking, and the scale $\mu \sim m_{\text{top}} \sim 200GeV$ is therefore of the order of the electroweak symmetry breaking scale.

4 Induced gravity and Brane generation in AdS_5

Here we study the dynamical mechanism of the fermionic self-interaction under which the Hilbert-Einstein gravitational action is completely induced by the high-energy spinor field and are mainly interested in the creation of a "thick" brane, which reconstructs the flat $(3+1)$-dimensional Minkowski space in the limit of zero thickness. Gravity becomes nontrivial in the direction of the fifth dimension orthogonal to the Minkowski space if vacuum configurations for composite scalar fields break the translational invariance.

We introduce gravity on a five-dimensional Riemannian manifold $\mathcal{M}_5$ using the metric field $g_{AB}(X)$. The invariant measure in the integral over the manifold in the classical action S is normalized to the determinant of this metric $\mathbf{g} \equiv \det(g_{AB})$,

$$S(\Phi(X), H(X), \overline{\Psi}(X)_j^{(l)}, \Psi(X)_j^{(l)}, \mathbf{g}) = \int_{\mathcal{M}_5} d^5X \sqrt{\mathbf{g}} \left[\mathcal{L}^{(5)}_{\text{fermion}} + \mathcal{L}^{(5)}_{\text{boson}}\right] , \tag{15}$$

while the fermionic (spinor) and bosonic (scalar and gravitational) parts of the Lagrangian density are defined below. In Eq.(15), the Lagrangian density bilinear in fermionic fields and invariant w.r.t. diffeomorphisms can be defined in terms of pentad fields $e^i_A(X)$, which locally relate the curvilinear manifold $\mathcal{M}_5$ to the flat space with the Euclidean signature. The fermion part is defined in the Euclidean five-dimensional space as,

$$\begin{aligned}
\mathcal{L}^{(5)}_{\text{fermion}}(\overline{\Psi}_j^{(l)}, \Psi_j^{(l)}, \Phi, H, \mathbf{g}) &= i\sum_{j=1}^{N_f} \overline{\Psi}_j^{(l)} \left[\,\widehat{\gamma}_k e^A_k \left(\partial_A + \omega_A\right) + \tau_3\Phi + \tau_1 \bar{g}_j \, H\,\right] \Psi_j^{(l)} \\
&= i\sum_{j=1}^{N_f} \overline{\Psi}_j^{(l)} \left(\not{\nabla} + \tau_3\Phi + \tau_1 \bar{g}_j \, H\right) \Psi_j^{(l)} \equiv \sum_{j=1}^{N_f} \overline{\Psi}_j^{(l)} \not{D}_j \Psi_j^{(l)} \ ,
\end{aligned} \tag{16}$$

where ω_A is a spin connection(see [22, 23] for definitions and technical details). To compensate large one-loop contributions induced by fermionic matter, we must add the classical bosonic Lagrangian density with large (primary) coefficients of proper values and signs,

$$\mathcal{L}^{(5)}_{\text{boson}} = N\Lambda^3 \left(\frac{\Phi^2}{g_1} + \frac{H^2}{g_2}\right) + \Lambda^3 \lambda_0 \ . \tag{17}$$

In this equation, the quantity $\lambda_0 \sim \Lambda^2$ is the primary cosmological constant of the five-dimensional Universe. In our approach, the gravitational Hilbert-Einstein Lagrangian density is completely induced by the fermionic matter and does not appear in the primary action. Gravity therefore becomes very strong on the compositeness scale, and its description, strictly speaking, requires a consistent quantization. But in the low-energy region, the gravitational

Lagrangian density acquires the large coefficient resulting from vacuum contributions of high-energy fermions, and the five-dimensional gravitational physics that provides the standard Newton gravity in our Universe situated on the brane. After integration over high-energy fermions one obtains the low-energy effective action in the curved five-dimensional space of gravity [2] and the total Euclidean low-energy Lagrangian density can write in the form,

$$\begin{aligned}
\mathcal{L}^{(5)}_{\text{low}} &\equiv \mathcal{L}^{(5)}_{\text{fermion}}(\overline{\Psi}^{(l)}, \Psi^{(l)}, \Phi, H, \mathbf{g}) + \mathcal{L}^{(5)}_{\text{boson}}(\Phi, H, \mathbf{g}) + \Delta\mathcal{L}^{(5)}(\Phi, H, \mathbf{g}) \\
&= i\sum_{j=1}^{N_f} \overline{\Psi}^{(l)}_j(X)\,[\not\nabla + \tau_3\Phi(X) + \tau_1\bar{g}_j H(X)\,]\,\Psi^{(l)}_j(X) \\
&+ \frac{\Lambda}{4\pi^3}\left\{N\partial_A\Phi(X)\partial^A\Phi(X) + N_c\partial_A H(X)\partial^A H(X) - 2N\Delta_1\Phi^2(X) - 2N_c\Delta_2 H^2(X)\right\} \\
&- \frac{N\Lambda^3}{108\pi^3}\{\,R(X) - 2\lambda\,\} \\
&+ \frac{\Lambda}{4\pi^3}\left[N\Phi^4(X) + 2N_c\Phi^2(X)H^2(X) + N_c H^4(X) + \frac{R(X)}{6}(\,N\Phi^2(X) + N_c H^2(X))\right] \\
&+ \frac{N\Lambda}{2880\pi^3}\left\{5R^2(X) - 8R_{AB}(X)R^{AB}(X) - 7R_{ABCD}(X)R^{ABCD}(X)\right\}, \qquad (18)
\end{aligned}$$

where R_{ABCD}, R_{AB}, R are the Riemann curvature tensor, the Ricci tensor and the scalar curvature respectively. The renormalized cosmological constant

$$\lambda = \lambda_0 + \frac{18\Lambda^2}{25}\;, \qquad (19)$$

results from the balance between the classical and induced contributions, while the gravitational five-dimensional Planck scale

$$M_*^3 \equiv \frac{N\Lambda^3}{54\pi^3}, \qquad (20)$$

is completely induced by high-energy fermions. Because the primary gravitational action is absent, we have the scenario of induced gravity. Let us assume that the effective matter coupling constants result in the low-energy scale $M \ll \Lambda$, which is determined by the value of the energy gap $\Delta_1 = M^2$. The search for classical vacuum configurations of gravity and scalar fields is performed by analyzing the low-energy effective action (18), restricting ourselves to the class of conformal-like metrics (warped geometries) with the flat Minkowski hyperplanes at each point along the fifth coordinate,

$$ds^2 = g_{AB}(X)\,dX^A\,dX^B = \exp\{-2\rho(z)\}\,dx_\mu dx_\mu + dz^2 \qquad (21)$$

with the Euclidean signature. The low-energy Lagrangian density (18) for this metric becomes,

$$\begin{aligned}
&\mathcal{L}^{(5)}_{\text{boson}}(\Phi, H, \mathbf{g}) + \Delta\mathcal{L}^{(5)}(\Phi, H, \mathbf{g}) \;= \\
&\frac{\Lambda}{4\pi^3}\Big\{\exp\{2\rho(z)\}\;[\,N\partial_\mu\Phi(X)\partial_\mu\Phi(X) + N_c\partial_\mu H(X)\partial_\mu H(X)] \\
&+N\,[\,\partial_z\Phi(X)\,]^2 + N_c\,[\,\partial_z H(X)\,]^2 - 2N\Delta_1\Phi^2(X) - 2N_c\Delta_2 H^2(X) \\
&+N\Phi^4(X) + 2N_c\Phi^2(X)H^2(X) + N_c H^4(X)\Big\} \\
&+ \frac{\Lambda}{3\pi^3}\,[\,N\Phi^2(X) + N_c H^2(X)\,]\left\{\rho''(z) - \frac{5}{2}\,[\,\rho'(z)\,]^2\right\} \\
&+ \frac{N\Lambda^3}{54\pi^3}\,\{-4\rho''(z) +\; 10\,[\,\rho'(z)\,]^2 \;+\; \lambda\} \\
&+ \frac{N\Lambda}{120\pi^3}\,\{2[\,\rho''(z)\,]^2 - 36\rho''(z)[\,\rho'(z)\,]^2 + 45[\,\rho'(z)\,]^4\}\,, \qquad (22)
\end{aligned}$$

where $\rho'(z) \equiv d\rho/dz$, and $\rho''(z) \equiv d^2\rho/dz^2$. We find classical solutions for metric (21) in the regime of weak coupling to the gravitational field. The latter means that $|\rho'(z)|/M = \o(1)$, $|\rho''(z)|/M^2 = \o(1)$ along the additional dimension. This condition together with that $M \ll \Lambda$ results in the terms quadratic in the curvature and in the curvature tensor in Lagrangian density (18) being negligible in the last term in (22).

In the weak gravitational interaction approximation, the dynamics are basically determined by the Hilbert-Einstein action on the five-dimensional manifold $\mathcal{M}_5$ and by its relation to the scalar matter fields. The field equations can therefore be written as,

$$R_{AB} - \frac{1}{2} g_{AB} (R - 2\lambda) \equiv G_{AB} + \lambda g_{AB} = \frac{27}{\Lambda^2} t_{AB}, \tag{23}$$

where the normalized energy-momentum tensor for the scalar matter fields is

$$\begin{aligned} t_{AB} \equiv & (4\pi^3/N\Lambda) T_{AB} \equiv \partial_A \Phi \, \partial_B \Phi + \frac{N_c}{N} \partial_A H \, \partial_B H - \frac{1}{2} g_{AB} \Big(\partial_C \Phi \, \partial^C \Phi + \frac{N_c}{N} \partial_C H \, \partial^C H \\ - & 2\Delta_1 \Phi^2 - \frac{2N_c}{N} \Delta_2 H^2 + \Phi^4 + \frac{2N_c}{N} \Phi^2 H^2 + \frac{N_c}{N} H^4 \Big) \\ + & \frac{1}{6} \Big(R_{AB} - \frac{1}{2} g_{AB} R + g_{AB} D^C \partial_C - D_B \, \partial_A \Big) \Big(\Phi^2 + \frac{N_c}{N} H^2 \Big) \; . \end{aligned} \tag{24}$$

In quasiflat metric case (21) and for classical configurations of kink-type scalar fields $\langle \Phi(X) \rangle_0 = \Phi(z)$, and $\langle H(X) \rangle_0 = H(z)$, the equations for the scalar matter fields become

$$\Phi'' = 2\Phi \left(\Phi^2 + \frac{N_c}{N} H^2 \right) - 2\Delta_1 \Phi + 4\rho' \, \Phi' + \frac{2}{3} \Phi \left(2\rho'' - 5{\rho'}^2 \right) \; , \tag{25}$$

$$H'' = 2H \left(\Phi^2 + H^2 \right) - 2\Delta_2 H + 4\rho' \, H' + \frac{2}{3} H \left(2\rho'' - 5{\rho'}^2 \right) \; . \tag{26}$$

The equations for the sum of the diagonal components '$\mu\mu$' + '55'

$$\rho'' = \frac{\kappa}{M^2} \left\{ {\Phi'}^2 + \frac{N_c}{N} {H'}^2 + \frac{1}{2} \left(\rho'' - \frac{1}{3} \frac{d^2}{dz^2} - \frac{1}{3} \rho' \frac{d}{dz} \right) \left(\Phi^2 + \frac{N_c}{N} H^2 \right) \right\} \; , \tag{27}$$

and for the component '55',

$$\begin{aligned} \frac{2M^2}{3\kappa} \lambda = & {\Phi'}^2 + \frac{N_c}{N} {H'}^2 + 2\Delta_1 \Phi^2 + \frac{2N_c}{N} \Delta_2 H^2 \\ - & \Phi^4 - \frac{2N_c}{N} \Phi^2 H^2 - \frac{N_c}{N} H^4 \\ + & \left(2{\rho'}^2 - \frac{4}{3} \rho' \frac{d}{dz} \right) \left(\Phi^2 + \frac{N_c}{N} H^2 \right) - \frac{4M^2}{\kappa} {\rho'}^2. \end{aligned} \tag{28}$$

Here we introduce the dimensionless low-energy gravitational parameter

$$\kappa \equiv \frac{9M^2}{\Lambda^2} \ll 1. \tag{29}$$

The last equation actually represents the integral of motion (after Eqs. of motion for scalar fields are taken into account), *i.e.* the rescaled five dimensional cosmological constant plays the role of an integration constant. In these equations we have neglected by terms originated from the last part of the Lagrangian density (18) quadratic in the curvature as they do not contribute into the leading order in the weak gravitational coupling expansion. It is easy to check that for $\kappa \ll 1$ kink-like solutions remain in the flat space. Both solutions consistently give

$$\lambda = \frac{27M^4}{2\Lambda^2} = \frac{3}{2} \kappa M^2 \; . \tag{30}$$

¿From Eq.(27) one can find the conformal factor in the form

$$\rho(z) \simeq \frac{2\kappa}{3} \ln\cosh(Mz). \tag{31}$$

This solution obviously represents the symmetric metric of the AdS_5 space for large z, namely,

$$\rho(z) \overset{|z|\to\infty}{\sim} k|z| \ ; \quad k \approx \frac{2}{3}\,\kappa M \ , \tag{32}$$

and k characterizes the curvature of the AdS_5 space, whose value determines the deviation from the Newton law at small distances [25].

5 Newton's constant and other scales of the model

We find the relation between the five-dimensional gravity in the AdS_5 world and the gravity on the brane using the factored Riemannian metric

$$ds^2 = \exp\{-2\rho(z)\}\, g_{\mu\nu}(x) dx^\mu dx^\nu + dz^2 \ . \tag{33}$$

For this metric, the effective four-dimensional Hilbert-Einstein action in the leading order in the parameter κ is

$$\begin{aligned} S[\,g\,] = & - \frac{N\Lambda^3}{108\pi^3} \int d^5X \sqrt{\mathbf{g}(X)}\, \{R(X) \ - \ 2\lambda\} \\ \simeq & - \frac{N\Lambda^3}{108\pi^3} \int d^4x \sqrt{\mathbf{g}(x)}\, R(x) \int_{-\infty}^{+\infty} dz \ \exp\{-2\rho(z)\} \\ & - \frac{N\Lambda^3}{54\pi^3} \int d^4x \sqrt{\mathbf{g}(x)} \int_{-\infty}^{+\infty} dz \ \exp\{-4\rho(z)\}\, \{6[\,\rho'(z)\,]^2 - \lambda\} \\ \equiv & - \frac{1}{16\pi G_N} \int d^4x \sqrt{\mathbf{g}(x)}\, \{R(x) \ - \ 2\Lambda_{\mathrm{grav}}\} \ , \end{aligned} \tag{34}$$

whence we eventually obtain the Planck mass scale $M_P \sim 1.22\times 10^{19}$ GeV/c^2 which corresponds to the Newton's gravitational constant

$$M_P^2 = G_N^{-1} \equiv \frac{4N\Lambda^3}{27\pi^2} \int_{-\infty}^{+\infty} dz \ \exp\{-2\rho(z)\}. \tag{35}$$

In turn, the gravitational part of the four-dimensional cosmological constant is

$$\Lambda_{\mathrm{grav}} \equiv \frac{4NG_N\Lambda^3}{27\pi^2} \int_{-\infty}^{+\infty} dz \ \exp\{-4\rho(z)\}\, \{\lambda - 6[\,\rho'(z)\,]^2\} \ . \tag{36}$$

Remarkably, the full value of the cosmological constant, including the gravitational as well as the matter vacuum energy densities, exactly vanishes to all orders in the perturbative expansion in powers of κ,

$$\begin{aligned} \Lambda_{\mathrm{cosmo}} \ \equiv \ & \frac{2\Lambda G_N}{\pi^2} \int_{-\infty}^{+\infty} dz \ \exp\{-4\rho(z)\} \Big\{ \frac{2NM^2}{3\kappa} \Big(\lambda - 6[\,\rho'(z)\,]^2 \Big) \\ + \ & N\Phi'^2(z) + N_c H'^2(z) - 2N\Delta_1\Phi^2(z) - 2N_c\Delta_2 H^2(z) \\ + \ & N\Phi^4(z) + N_c\Phi^2(z)H^2(z) + N_c H^4(z) \\ + \ & \frac{2}{3}\big[\,N\Phi^2(z) + N_c H^2(z)\,\big]\,\big[\,2\rho''(z) - 5\rho'^2(z)\,\big] \Big\} \ = \ 0 \ , \end{aligned} \tag{37}$$

where the vacuum expectation values (25) and (26) of the scalar fields together with the field equation (27) of the conformal factor have been suitably taken into account. It makes consistently endorsed the *ansatz* for the flat Minkowski's hyperplanes.

We now obtain the relations between the AdS_5 curvature of the AdS_5 space $k \simeq 2\kappa\, M/3$, the Planck mass M_P and the spontaneous translational symmetry breaking scale M

$$k^5 M_P^4 = \frac{128N^2}{27\pi^4}\, M^9 \ , \tag{38}$$

$$k M_P^2 = \frac{4N}{27\pi^3}\, \Lambda^3 \ . \tag{39}$$

We can also relate the five-dimensional Planck scale to the Planck scale of our Universe using the curvature of the AdS_5 space,

$$M_*^3 = \frac{k M_P^2}{8\pi} \ . \tag{40}$$

This results in the estimate of the lower bound for the five-dimensional Planck mass $M_* > 10^8$ GeV, which follows from the experimental estimate for the curvature $k > 10\text{mm}^{-1} = 2 \cdot 10^{-3}$ eV. The latter is based on the absence of observed deviations from the Newton low at distances larger than 0.1 mm [25]. ¿From the above experimental estimate and formulas follows the lower bounds for other scales in the model:

$$M \geq 100 \text{ GeV} \ , \qquad \Lambda \geq 10^9 \text{ GeV} \ , \tag{41}$$

and the parameters characterizing the interactions become rather small,

$$\zeta \sim M/\Lambda \sim 10^{-7} \ , \qquad \kappa \sim 10^{-13} \ , \tag{42}$$

the direct interaction of light particles is then strongly suppressed, and only their interaction with gravity remains. Obviously, the bound $M \sim 100$ GeV is unlawfully low, and it is excluded by experiments with modern colliders because it determines the energy barrier above which fermions can freely propagate in the fifth dimension.

For energies of the order $M \sim 1$ TeV, we can estimate the scale of curvature of the AdS_5 space as $k \sim 10^{-10}$GeV, which corresponds to distances of the order of μm. Such scales are inaccessible for seeking deviations from the Newton law in experiments in the nearest future [25]. In these limits, scalar particles are essentially separated from the fermion world and from each other because

$$M_* \sim 10^9\text{GeV}; \quad \zeta \sim M/\Lambda \sim 10^{-6.5}; \quad \kappa \sim 10^{-12}. \tag{43}$$

Although the Higgs-like particles may be involved into the gauge boson interaction and be observable by gauge boson mediation, it turns out that branons [26], *i.e.* the quanta of the field ϕ, decouple from any other kind of matter that makes them a perfect candidate for the dark matter/energy, depending on their mass. If the curvature of the AdS_5 space is related to the scale of electroweak interaction breaking $k \sim 200$GeV, then the boundary $M \sim 10^{10}$GeV is too high to observe any real physics in laboratory conditions on Earth. Because $\Lambda \sim 10^{14}$GeV and $\zeta \sim M/\Lambda \sim 10^{-4}$,the branons in this case interact rather weakly and still belong to the dark universe.

The parameter ζ for induced gravity can therefore become not negligibly small only at $k \sim M \sim \Lambda$ at the Planck mass scale where gravity indeed becomes strong ($\kappa \sim 1$) and a theory of quantum gravity must be involved.

6 Conclusion and outlook

We have performed the mechanism of how the universe can be generated dynamically by matter self-interaction from a spontaneous breaking of the translational invariance. This breaking is related to the τ-symmetry breaking in our model. It turned out that both gravitational and scalar fields are responsible for localizing light matter on the brane. The classical vacuum configurations of these brane-type fields were obtained in the mean-field approximation. The gravity plays a negligible role in forming the masses and interaction constants of light particles.

We concluded that the induced gravity results in splitting branons from the Standard Model matter in a wide range of admissible scales and coupling constants, which makes branons natural candidates for explaining the phenomenon of dark matter (or energy).

The dimensionless parameter characterizing the gravitational interaction force turned out to be very small, of the order $\kappa \leq 10^{-8}$. This justifies using the perturbation theory both to calculate vacuum configurations of matter fields and of the background gravitational field and to derive the mass spectrum of localized particles.

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