Conventional cosmology from multidimensional models

A. I. Zhuk*
Department of Theoretical Physics and Astronomical Observatory,
Odessa National University,
2 Dvoryanskaya St., Odessa 65026, Ukraine

Abstract

We investigate a possibility for construction of the conventional Friedmann cosmology for our observable Universe if underlying theory is multidimensional Kaluza-Klein model endowed with a perfect fluid. We show that effective Friedmann model obtained by dynamical compactification of the multidimensional one is faced with too strong variations of the fundamental "constants". From other hand, models with stable compactification of the internal space are free from this problem and also result in conventional 4D cosmological behavior for our Universe. We prove a no-go theorem which shows that stable compactification of the internal spaces is possible only if equations of state in the external and internal spaces are properly adjusted to each other. With a proper choice of parameters (fine tuning), effective cosmological constant in this model provides the late time acceleration of the Universe. The fine tuning problem is resolved in the case of the internal spaces in the form of orbifolds with branes in fixed points. However, in this case the effective potential is too flat (mass gravexcitons is very small) to provide necessary constancy of the effective fundamental "constants".

1 Introduction

Multidimensionality of our Universe is one of the most intriguing assumption in modern physics. It is a natural ingredient of theories which unify different fundamental interactions with gravity, such like string/M-theory. It is of great interest to investigate the cosmological consequences of this assumption. One of the most simple and natural generalization leads to the Kaluza-Klein (KK) models with warped product topology

$$M = M_0 \times M_{D'} \tag{1.1}$$

consisting of an external ("our") four-dimensional spacetime manifold M_0 and a D'-dimensional compact space component $M_{D'}$. Here, dynamics of factor spaces is described by their own scale factors. Obviously, dynamical picture of this model can be very complicated and significantly differ from the evolution of the Friedmann-Robertson-Walker (FRW) Universe. This deviation may have dramatic consequences. For example, the abundance of the light elements is very sensitive to the rate of the evolution. Thus, the deviations from evolution of the FRW Universe may contradict observable data. Hence, it is very important to show that proposed new cosmological models are in concordance with conventional FRW cosmology (at least during the radiation and matter dominated stages). It is also very desirable within these models to have stages of the initial inflation as well as late time acceleration. The main purpose of the present paper consists in investigation of a possibility of the conventional description of effective four-dimensional cosmological models obtained from Kaluza-Klein models.

*e-mail: zhuk@paco.net

We investigate KK models where spacetime is endowed with a multicomponent perfect fluid [1]. Within the standard KK models and according to the present level of the experimental data, the internal spaces are unobservable if their scales are of order or less than the Fermi length $L_{\rm F} \sim 10^{-17}$cm ~ 1TeV^{-1}. Such small scales can be achieved by two ways. First, the internal dimensions behave dynamically toward the decrease of their size below $L_{\rm F}$. Here, the internal spaces undergo dynamical evolution all the time. This behavior is called dynamical compactification. Second, the internal spaces can be stabilized near some fixed value, e.g. $L_{\rm F}$. This behavior is called stable compactification.

For the first class of models (with dynamical compactification), in Ref. [2] it was proposed an approach for the reduction of multidimensional models with perfect fluid to an effective four-dimensional ones which have the form of the conventional cosmology. In the present paper we elaborate this model and show that it provides very interesting gravitational "constant" tuning effect. In spite of dynamical behavior of an effective 4-D gravitational "constant" and a non-conventional dynamics of an effective 4-D energy density, their product behaves exactly as in FRW scenario. Precisely this product enters into Friedmann equations. Thus, the external space has dynamical evolution in accordance with the standard FRW cosmology. However, it is well known that in KK approach dynamical internal spaces lead to variation of the fundamental constants (see e.g. [3] - [7] and references therein). As result, we show that the fundamental constants in model [2] undergo too large variations in comparison with experimental data.

Next, we consider models with stable compactification. It is worth of noting that two particular classes of solutions with the stable compactification of the internal spaces (for models with perfect fluid) were already found in our paper [8]. In the present paper we prove a no-go theorem according to which the models with perfect fluid do not admit the stable compactification in the case of an arbitrary combination of equations of state in the external and internal spaces[1] . There are only two exceptional classes where the stable compactification takes place and these classes exactly coincide with those found in [8]. Hence, these classes entirely exhaust all possibilities for the stable compactification. We construct a particular model which belongs to these classes and has a Friedmann-like behavior for the external space (our Universe) during the radiation and matter dominated stages and late-time acceleration. However, the parameters of model should be fine tuned to get the observable dark energy. The reason of it is simple and rather common for such type of KK models. From the stability condition (condition for the minimum of an effective potential) follows that all parameters of the model have the same order of magnitude as the curvature of the internal space[2]: $R_1 \sim L_F^{-2} \sim 10^{34}$cm^{-2} which is much greater than the observable value of the dark energy[3] $\Lambda_{DE} \sim 10^{-123}\Lambda_{Pl} \sim 10^{-57}$cm^{-2}. So, only with the help of extreme fine tuning, the parameters can adjust to each other in such a way to leave a fraction corresponding to Λ_{DE}.

To avoid the fine tuning problem, we generalize this model to the case of orbifold internal space with branes in fixed points. In the spirit of Universal Extra Dimension (UED) models [12], the Standard Model fields are not localized on the brane but can move in the bulk. We consider flat orbifolds. Therefore, the curvature of the internal space is absent and the rest of parameters can be of the order of Λ_{DE} without fine tuning. However, in this case the effective potential is too flat (mass gravexcitons is very small) to provide necessary constancy of the effective fundamental "constants".

The paper is structured as follows. In section 2, we explain the general setup of our model and give a basic description of KK models with multicomponent perfect fluid. Section 3 is devoted to consideration of a model with dynamical compactification. In spite of the Friedmann-like behavior, it is shown that effective fundamental four-dimensional constants undergo too fast variations. In section 4 we prove the no-go theorem on stable compactification of internal spaces

[1] For a particular case, this theorem was also confirmed in paper [9, 10].

[2] As far as we know, the idea of TeV-scale extra dimensions was first proposed in [11].

[3] The experimental data give an estimate for the dark energy density: $\rho_{DE} \sim 10^{-123}\rho_{Pl} \Longrightarrow \Lambda_{DE} \sim 10^{-123}\Lambda_{Pl}$.

in model with perfect fluid and find only two exceptional classes where such compactification is possible. For these cases, in section 5, we construct a model with conventional cosmology in the external space. Here, the effective 4-D cosmological constant is positive. However, parameters of the model should be fine tuned to get observable dark energy density. In section 6, we generalize the latter model to the case of stable orbifold. It gives us a possibility to avoid the problem of fine tuning. However, the effective potential is not curved enough to provide necessary constancy of the effective fundamental "constants". The main results are summarized in the concluding section 7.

2 General setup

To start with, let us consider a cosmological model with factorizable geometry,

$$\begin{aligned} g &= \overline{g}^{(0)}(x) + \sum_{i=1}^{n} L_{Pl}^2 e^{2\beta^i(\tau)} g^{(i)}(y) \qquad (2.1) \\ &\equiv -e^{2\gamma(\tau)} d\tau \otimes d\tau + L_{Pl}^2 e^{2\beta^0(\tau)} g^{(0)}(\vec{x}) + \sum_{i=1}^{n} L_{Pl}^2 e^{2\beta^i(\tau)} g^{(i)}(y)\,, \end{aligned}$$

which is defined on the manifold (1.1) where, for generality, $M_{D'}$ is also a direct product of n compact d_i-dimensional spaces: $M_{D'} = \prod_{i=1}^{n} \mathcal{M}_i$, $\sum_{i=1}^{n} d_i = D'$. Metric $\overline{g}^{(0)}$ describes the external (our) spacetime $M_0 = \mathbb{R} \times \mathcal{M}_0$ with dimension $D_0 = 1 + d_0 = 4$. In order get effective four-dimensional cosmology in the form of the Friedmann equations, we assume that the factors $\mathcal{M}_i$ are Einstein spaces: $R_{kl}[g^{(i)}] = \lambda^i g^{(i)}_{kl}$, $i = 0, \ldots, n$, $k, l = 1, \ldots, d_i$ and $R[g^{(i)}] = \lambda^i d_i \equiv R_i$. In the case of constant curvature spaces parameters λ^i are normalized as $\lambda^i = k_i(d_i - 1)$ with $k_i = \pm 1, 0$. Quantities $a \equiv L_{Pl} e^{\beta^0}$ and $b_i \equiv L_{Pl} e^{\beta^i}$ $(i = 1, \ldots, n)$ describe scale factors of the external and internal spaces, respectively.

With respect to the internal spaces, there are two possible scenarios: either they are stably compactified at the present time values $b_{(0)i} \equiv L_{Pl} e^{\beta^i_0} = \text{const}$, or there is no such stabilization and b_i remain dynamical functions. In latter case, $\beta^i(\tau)$ define the averaged dynamics of the volume of the internal spaces and $b_{(0)i}$ is just instantaneous values of b_i corresponding to the present time τ_0. For both of these cases, it make sense to consider small inhomogeneous particle-like excitations/fluctuations $\overline{\beta}^i(x)$ over such background (either over constant β^i_0 or over dynamical $\beta^i(\tau)$), which describe massive scalar particles (gravexcitons/radions) developing in the external spacetime [13, 14]. The total volume of the internal spaces at the present time is given by

$$V_{D'} \equiv V_I \times v_0 \equiv \prod_{i=1}^{n} \int\limits_{M_i} d^{d_i} y \sqrt{|g^{(i)}|} \times \left(\prod_{i=1}^{n} e^{d_i \beta^i_0} (L_{Pl})^{D'} \right) = V_I \times \prod_{i=1}^{n} b_{(0)i}^{d_i}. \qquad (2.2)$$

The factor V_I is dimensionless and defined by geometry and topology of the internal spaces.

The action functional for considered multidimensional models reads

$$S = \frac{1}{2\kappa_D^2} \int\limits_M d^D x \sqrt{|g|} \left\{ R[g] - 2\Lambda_D \right\} + S_m\,, \qquad (2.3)$$

where κ_D^2 is D-dimensional fundamental gravitational constant and S_m is an action for bulk matter. In conventional cosmology matter fields are taken into account in a phenomenological way as a perfect fluid with equal pressure in all three special directions. It provides homogeneous (if energy density and pressure depends only on time) and isotropic picture of the Universe.

In multidimensional case we generalize this approach to a $m-$component perfect fluid with energy-momentum tensor

$$T_N^M = \sum_{c=1}^{m} {T^{(c)}}_N^M , \tag{2.4}$$

$${T^{(c)}}_N^M = \mathrm{diag}\,(-\rho^{(c)}(\tau), \underbrace{P_0^{(c)}(\tau), \ldots, P_0^{(c)}(\tau)}_{d_0 \text{ times}}, \ldots, \underbrace{P_n^{(c)}(\tau), \ldots, P_n^{(c)}(\tau)}_{d_n \text{ times}}) . \tag{2.5}$$

The conservation equations we impose on each component separately

$${T^{(c)}}_{N;M}^M = 0. \tag{2.6}$$

Denoting by an overdot differentiation with respect to time τ in metric (2.1), these equations read for the tensors (2.5)

$$\dot{\rho}^{(c)} + \sum_{i=0}^{n} d_i \dot{\beta}^i \left(\rho^{(c)} + P_i^{(c)}\right) = 0 . \tag{2.7}$$

If the pressures and energy density are related via equations of state

$$P_i^{(c)} = \left(\alpha_i^{(c)} - 1\right) \rho^{(c)}, \qquad i = 0, \ldots, n, \quad c = 1, \ldots, m , \tag{2.8}$$

then eq. (2.7) have the simple integrals

$$\rho^{(c)}(\tau) = A^{(c)} a^{-d_0 \alpha_0^{(c)}} \times \prod_{i=1}^{n} b_i^{-d_i \alpha_i^{(c)}} , \tag{2.9}$$

where $A^{(c)}$ are constants of integration.

3 Dynamical compactification: fundamental "constant" variation problem

In this section we investigate a possibility for conventional cosmology in the case of multidimensional models with dynamical behavior of the internal spaces. More precisely, we consider the case of dimensional stabilization when the scale factors of the internal spaces b_i decrease with time.

For simplicity, we consider the case of one internal space: $n = 1$, $b_1 \equiv b$. Perfect fluid is also taken in one component form: $c = 1 \Rightarrow \rho^{(1)} \equiv \rho$, $P_0^{(1)} \equiv P_0$, $P_1^{(1)} \equiv P_1$. Then, the Einstein equations is reduced to 3 differential equations:

$$\kappa_D^2 \rho = \left\{ 3H^2 + \frac{3k_0}{a^2} - \Lambda_D \right\} + \frac{1}{2} d_1 (d_1 - 1) \left(\frac{\dot{b}}{b}\right)^2 + 3 d_1 H \frac{\dot{b}}{b} + \frac{d_1}{2} (d_1 - 1) \frac{k_1}{b^2} , \tag{3.1}$$

$$\kappa_D^2 P_0 = \left\{ -2\frac{\ddot{a}}{a} - H^2 - \frac{k_0}{a^2} + \Lambda_D \right\} - d_1 \frac{\ddot{b}}{b} - \frac{1}{2} d_1 (d_1 - 1) \left(\frac{\dot{b}}{b}\right)^2 - 2 d_1 H \frac{\dot{b}}{b} - \frac{d_1}{2} (d_1 - 1) \frac{k_1}{b^2} \tag{3.2}$$

and

$$\begin{aligned} \kappa_D^2 P_1 &= -3\frac{\ddot{a}}{a} - 3H^2 - \frac{3k_0}{a^2} + \Lambda_D - (d_1 - 1)\frac{\ddot{b}}{b} - \frac{1}{2}(d_1 - 1)(d_1 - 2)\left(\frac{\dot{b}}{b}\right)^2 \\ &- 3(d_1 - 1) H \frac{\dot{b}}{b} - \frac{1}{2}(d_1 - 1)(d_1 - 2)\frac{k_1}{b^2} , \end{aligned} \tag{3.3}$$

where $H \equiv \dot{a}/a$ is the Hubble parameter and dots denote differentiation with respect to synchronous time t (which corresponds to the gauge $\gamma(\tau) = 0 \Rightarrow \tau \equiv t$ in metric (2.1)). This set of equations should be supplemented with eq. (2.7), which results in solution (2.9) in the case of equations of state (2.8): $P_i = (\alpha_i - 1)\,\rho,\ i = 0, 1$.

It worth of noting that eqs. (3.1) and (3.2) have the form of the standard FRW equations if we leave in the rhs only terms in curly brackets. So, we arrive to natural question. Could we rewrite these equations in such a way that they reproduce equations of the conventional cosmology, i.e. the Friedmann equations? On the face of it, the answer is positive. The reason for such answer is based on the following observation made in Ref. [2]: if we suppose that the scale factor of the internal space evolves according to the relation[4]

$$b = \frac{B}{a^q}\,, \quad B \equiv \mathrm{const} \tag{3.4}$$

and parameter q satisfies the condition

$$d_1 q(d_1 q - q - 6) = 0 \tag{3.5}$$

then, for the case of Ricci-flat internal space ($k_1 = 0$), eqs. (3.1)–(3.3) can be correspondingly rewritten as

$$\kappa_0^2 \rho_{(4)} = 3H^2 + \frac{3k_0}{a^2} - \Lambda_D\,, \tag{3.6}$$

$$\kappa_0^2 P_{(4)} = -2\frac{\ddot{a}}{a} - H^2 - \frac{k_0}{a^2} + \Lambda_D \tag{3.7}$$

and

$$\kappa_D^2 P_1 = (d_1 q - q - 3)\,\frac{\ddot{a}}{a} - \frac{1}{2}\left[6 + q(d_1 - 1)(d_1 q - 4)\right] H^2 - \frac{3k_0}{a^2} + \Lambda_D\,. \tag{3.8}$$

Here,

$$\rho_{(4)} \equiv \rho\, V_{d_1}\,, \quad P_{(4)} \equiv \left[P_0 - \frac{d_1 q}{3}(\rho + P_1)\right] V_{d_1} \tag{3.9}$$

are identified with observable four-dimensional energy density and pressure, respectively.

It can be easily seen, that eqs. (3.6) and (3.7) formally reproduce the famous FRW equations (in the presence of the cosmological constant). The third eq. (3.8) can be used to find the pressure along the extra dimensions in term of the scale factor $a(t)$ of the observable Universe. However, there are two main difference between this effective model and the standard FRW universe. First of all, the effective four-dimensional gravitational "constant" κ_0^2 is not a constant but a dynamical function:

$$\kappa_0^2 = \kappa_D^2 / V_{d_1}\,. \tag{3.10}$$

In considered model, the volume of internal space reads

$$V_{d_1} = V_I b^{d_1} = V_I B^{d_1}/a^{d_1 q} \equiv \overline{V}_I a^{-d_1 q}\,. \tag{3.11}$$

It is worth of noting, that eq. (3.5) is necessary condition for a correct normalization of the effective four-dimensional gravitational "constant". Only in this case we can reproduce eq. (3.10). If parameter q does not satisfy condition (3.5), it results in non-correct gauging of κ_0 (see corresponding comments in paper [15]). Eq. (3.5) has two solutions: $q = 0$, which describes the static internal space, and

$$q = \frac{6}{d_1 - 1}\,, \quad d_1 \neq 1\,, \tag{3.12}$$

[4] In this case the internal space decreases with expansion of the external one. So, we obtain a model with dynamical compactification of the extra dimensions. For such models the rate of compactification plays very important role. This point will be discussed at the end of this section.

which corresponds to the dynamical compactification. In this section we shall concentrate on the latter case mentioning briefly the static case at the end of the section.

Second very important difference between our effective model and conventional cosmology consists in the equation of conservation of energy. For our model, eq. (2.7) with the help of eqs. (3.9) can be written in the following form:

$$\frac{d}{dt}\left(a^3\rho_{(4)}\right) + P_{(4)}\frac{d}{dt}\left(a^3\right) = \left(a^3\rho_{(4)}\right)\frac{1}{V_{d_1}}\frac{d}{dt}\left(V_{d_1}\right) = -\left(a^3\rho_{(4)}\right) d_1 q \frac{\dot{a}}{a}\,, \tag{3.13}$$

which, obviously, differs from the standard conservation equation in the FRW Universe by non-zero rhs.

Similar to the FRW Universe, it can be easily shown that eq. (3.7) comes out from eq. (3.6) and (3.13). To prove it, it is necessary to keep in mind that κ_0 is dynamical function (3.10). So, there is no need to solve eq. (3.7).

Let us suppose now that the energy density $\rho_{(4)}$ and pressure $P_{(4)}$ are connected with each other via equation of state:

$$P_{(4)} = (\alpha - 1)\rho_{(4)}\,. \tag{3.14}$$

Then, eq. (3.13) has the following solution:

$$\rho_{(4)} = \rho_0 \left(\frac{a_0}{a}\right)^{3\alpha + d_1 q}\,, \tag{3.15}$$

where ρ_0 is the energy density at the moment when scale factor is a_0. This behavior of the energy density differs from the standard one by additional degree $d_1 q$, which varies from 6 to 12 if dimension d_1 varies from $d_1 \gg 1$ to $d_1 = 2$, respectively. However, it is very important to note, that combination

$$\kappa_0^2 \rho_{(4)} \sim a^{-3\alpha} \tag{3.16}$$

as for the standard cosmology. It follows from the fact that dynamical behavior of $\kappa_0^2 \sim a^{d_1 q}$ exactly compensates additional degree $a^{-d_1 q}$ in expression for $\rho_{(4)}$. As a result, dynamical evolution of the Universe in our model exactly coincides with evolution of the standard FRW Universe. For example, if we take a flat Universe ($k_0 = 0$) without cosmological constant $\Lambda_D = 0$, then the solution of eq. (3.6) has the standard for FRW Universe form:

$$a = \left(\frac{3\alpha}{2} a_* t\right)^{2/(3\alpha)}\,, \tag{3.17}$$

where $a_* \equiv \kappa_D^2 \rho_0 a_0^{3\alpha + d_1 q}/(3\overline{V}_I)$. Hence, the combination $\kappa_0^2 \rho_{(4)} \sim t^{-2}$ has conventional cosmological behavior.

Thus, we recover the standard behavior of the Universe in our multidimensional model due to specific dynamic of the effective four-dimensional gravitational constant $\kappa_0^2 \equiv 8\pi G_4 \Longrightarrow G_4 \sim V_{d_1}^{-1} \sim a^{d_1 q}$ (see eq. (3.10)). Contrary to the Dirac hypothesis on $G_4 \sim t^{-1}$ and negative $\dot{G}_4/G_4 < 0$ [16], in our model G_4 increases with time (if the scale factor a is a growing function of time) and $\dot{G}_4/G_4$ is positive. For example, in the case of solution (3.17) we get

$$G_4 \sim t^{2d_1 q/(3\alpha)} \quad \Longrightarrow \quad \frac{\dot{G}_4}{G_4} = \frac{2d_1 q}{3\alpha}\frac{1}{t} > 0,\ \forall\, \alpha > 0\,. \tag{3.18}$$

It gives for the age of the Universe $t_u \sim 14$Gyr

$$\frac{\dot{G}_4}{G_4} \sim 10^{-10}\mathrm{yr}^{-1}\,. \tag{3.19}$$

There are a number of estimates for possible variations of G_4 (see e.g. [3]-[6] and references therein). Comparison with these estimates shows that our value (3.19) is too large[5]. $|\dot{G}_4/G_4|$ should be smaller by at least two orders of magnitude. Negative sign of $\dot{G}_4/G_4$ is also preferable although positive values are not excluded [4]. In spite of rather large value (3.19), it is worth of noting that some of estimates are not applicable for our model. The reason of it is that the Universe in our model has the same rate of evolution as the conventional Universe. Thus, the time-depending G_4 in our case has no direct effects on the CMB angular power spectrum as well as on the nucleosynthesis.

From other hand, dynamical behavior of the internal spaces can result in variation others fundamental constants. For example, the inclusion in the model of the electromagnetic fields leads to the variation of the fine structure constant: $\alpha_4 = \alpha_D/V_{d_1}$ [4, 7]. Thus, variations of G_4 and α_4 are connected with each other as follows:

$$\frac{\dot{G}_4}{G_4} = \frac{\dot{\alpha}_4}{\alpha_4}\,. \tag{3.20}$$

The observation of quasar absorbtion lines shows that $\dot{\alpha}_4/\alpha_4 \sim 10^{-15}\mathrm{yr}^{-1}$ [18]. Thus, provided that these observational determination of $\dot{\alpha}/\alpha$ is correct, we should also have this estimate for $\dot{G}_4/G_4$. Obviously, this is much less than (3.19). The relatively large value of $\dot{G}_4/G_4$ in our model originates from a high rate of evolution of the internal space: $V_{d_1} \sim a^{-d_1 q} \sim t^{-2d_1 q/(3\alpha)} = t^{-4d_1/[\alpha(d_1-1)]}$. For radiation ($\alpha = 4/3$) and dust ($\alpha = 1$), we have correspondingly $3 < 2d_1 q/(3\alpha) \leq 6$ and $4 < 2d_1 q/(3\alpha) \leq 8$.

The evolution of the internal space can be slow down if parameter $d_1 q \to 0$. However, its value is defined by eq. (3.12) and varies in limits from 6 to 12. We can resolve this problem taking the second solution of eq. (3.5): $q = 0$, which corresponds to the static internal space: $b \equiv$ const. Unfortunately, it is difficult to justify this solution for arbitrary ρ, P_0 and P_1. Other words, solution $b =$ const is, in general, not stable (for details, see the next section). Nevertheless, for some particular cases, stable solutions exist and we discuss them in the next section.

4 Stable compactification: no-go theorem

To investigate the problem of the stable compactification, it is helpful to use the equivalence between the Einstein eqs. (3.1) - (3.3) and the Euler-Lagrange equations for Lagrangian obtained by dimension reduction of the action (2.3) with

$$S_m = -\int\limits_M d^D x \sqrt{|g|}\rho\,, \tag{4.1}$$

where ρ is given by eq. (2.9) (see [1] for details). This equivalence takes place for homogeneous model (2.1). However, we can generalize it to the inhomogeneous case allowing inhomogeneous fluctuations $\overline{\beta}^i(x)$ over stably compactified background $\beta_0^i =$ const:

$$\overline{\beta}^i(x) = \beta^i(x) - \beta_0^i\,, \quad i = 1, \ldots, n\,, \tag{4.2}$$

where coordinates x are defined on the manifold M_0. In this section, $b_{(0)i} = L_{Pl} \exp \beta_0^i$ are treated as the scale factors of the internal spaces stabilized at the present time.

[5]Similar conclusion on too large variations of the 4-D gravitational constant in multidimensional models with ideal perfect fluid and dynamical internal spaces was obtained in paper [17].

Then, after conformal transformation of the external spacetime metric from the Brans-Dicke to the Einstein frame[6]:

$$\overline{g}^{(0)}_{\mu\nu}(x) = \Omega^2 \tilde{g}^{(0)}_{\mu\nu}(x) := \left(\prod_{i=1}^{n} e^{d_i \overline{\beta}^i} \right)^{\frac{-2}{D_0-2}} \tilde{g}^{(0)}_{\mu\nu} \,, \tag{4.3}$$

the dimensional reduction of action (2.3) with the matter term (4.1) results in the following four-dimensional effective theory (see for details, [8, 13, 14]):

$$S = \frac{1}{2\kappa_0^2} \int\limits_{M_0} d^{D_0} x \sqrt{|\tilde{g}^{(0)}|} \left\{ \tilde{R}\left[\tilde{g}^{(0)}\right] - \bar{G}_{ij} \tilde{g}^{(0)\mu\nu} \partial_\mu \overline{\beta}^i \, \partial_\nu \overline{\beta}^j - 2U_{eff} \right\} , \tag{4.4}$$

where $\kappa_0^2 = \kappa_D^2 / V_{D'}$ is effective four-dimensional gravitational constant with $V_{D'}$ from eq. (2.2) (with $b_{(0)i} = L_{Pl} \exp \beta_0^i = \text{const}$) and corresponds to the present day value (i.e. to the Newton gravitational constant). In eq. (4.4), $\bar{G}_{ij} = d_i \delta_{ij} + [1/(D_0 - 2)] d_i d_j$ and

$$U_{eff} = \left(\prod_{i=1}^{n} e^{d_i \overline{\beta}^i} \right)^{-\frac{2}{D_0-2}} \left[-\frac{1}{2} \sum_{i=1}^{n} \tilde{R}_i e^{-2\overline{\beta}^i} + \Lambda_D + \kappa_D^2 \sum_{c=1}^{m} \rho^{(c)} \right] , \tag{4.5}$$

is the effective potential where $\tilde{R}_i := R_i L_{Pl}^{-2} e^{-2\beta_0^i}$ and $\rho^{(c)}$ is defined by eq. (2.9). If we suppose that the external spacetime metric in the Einstein frame has also the FRW form:

$$\tilde{g}^{(0)} = \Omega^{-2} \overline{g}^{(0)} = \tilde{g}^{(0)}_{\mu\nu} dx^\mu \otimes dx^\nu := -e^{2\tilde{\gamma}} d\tilde{\tau} \otimes d\tilde{\tau} + L_{Pl}^2 e^{2\tilde{\beta}^0(x)} g^{(0)} \,, \tag{4.6}$$

which results in the following connection between the external scale factors in the Brans-Dicke frame $a \equiv L_{Pl} e^{\beta^0}$ and in the Einstein frame $\tilde{a} \equiv L_{Pl} e^{\tilde{\beta}^0}$:

$$a = \left(\prod_{i=1}^{n} e^{d_i \overline{\beta}^i} \right)^{-\frac{1}{D_0-2}} \tilde{a} \,, \tag{4.7}$$

then, expression (2.9) for $\rho^{(c)}$ can be rewritten in the form:

$$\kappa_D^2 \rho^{(c)} = \kappa_0^2 \rho^{(c)}_{(4)} \prod_{i=1}^{n} e^{-\xi_i^{(c)} \overline{\beta}^i} \,, \tag{4.8}$$

where

$$\rho^{(c)}_{(4)} = \tilde{A}^{(c)} \tilde{a}^{-d_0 \alpha_0^{(c)}} \,, \quad \tilde{A}^{(c)} = A^{(c)} V_I \prod_{i=1}^{n} b_{(0)i}^{d_i (1 - \alpha_i^{(c)})} \tag{4.9}$$

and

$$\xi_i^{(c)} = d_i \left(\alpha_i^{(c)} - \frac{\alpha_0^{(c)} d_0}{d_0 - 1} \right) . \tag{4.10}$$

It can be easily verified that $\tilde{A}^{(c)}$ has dimension $\text{cm}^{d_0 \alpha_0^{(c)} - D_0}$.

Thus, the problem of stabilization of the extra dimensions is reduced now to search of minima of the effective potential U_{eff} with respect to the fluctuations $\overline{\beta}^i$:

$$\left. \frac{\partial U_{eff}}{\partial \overline{\beta}^k} \right|_{\overline{\beta}=0} = 0 \Longrightarrow \tilde{R}_k = -\frac{d_k}{D_0 - 2} \left[\sum_{i=1}^{n} \tilde{R}_i - 2\Lambda_D \right] + \kappa_0^2 \sum_{c=1}^{m} \rho^{(c)}_{(4)} \left(\xi_k^{(c)} + \frac{2 d_k}{D_0 - 2} \right) , \; k = 1, \ldots, n \,. \tag{4.11}$$

[6]Most easily the analysis of the internal space stabilization can be done in the Einstein frame. For this purpose we perform this conformal transformation. Evidently, if the stabilization takes place in the Einstein frame (i.e. $\overline{\beta}^i = 0$), it also occurs in the BD frame because both of these frames in this case coincide with each other.

The left-hand side of this equation is a constant but the right-hand side is a dynamical function because of dynamical behavior of the effective four-dimensional energy density $\rho^{(c)}_{(4)}$. Thus, we arrived to the following

No-go theorem:

Multidimensional cosmological Kaluza-Klein models with the perfect fluid as a matter source do not admit stable compactification of the internal spaces with exception of two special cases:

$$\text{I.} \quad \alpha_0^{(c)} = 0\,, \quad \forall\, \alpha_i^{(c)}, \quad i = 1, \ldots, n, \quad c = 1, \ldots, m\,. \tag{4.12}$$

$$\text{II.} \quad \xi_i^{(c)} = -\frac{2d_i}{d_0 - 1} \Longrightarrow \begin{cases} \alpha_0^{(c)} = \frac{2}{d_0} + \frac{d_0 - 1}{d_0}\alpha^{(c)}\,, \\ \alpha_i^{(c)} = \alpha^{(c)}, \quad i = 1, \ldots, n, \quad c = 1, \ldots, m\,. \end{cases} \tag{4.13}$$

First case corresponds to vacuum in the external space $\rho^{(c)}_{(4)} = \tilde{A}^{(c)} = \text{const}$ and arbitrary equations of state in the internal spaces. Some bulk matter can mimic such behavior, e.g. vacuum fluctuations of quantum fields (Casimir effect) [13, 19], monopole form fields [13, 20] and gas of branes [21].

In the second case[7], the energy density in the external space is not a constant but a dynamical function with the following behavior[8]:

$$\rho^{(c)}_{(4)}(\tilde{a}) = \tilde{A}^{(c)} \frac{1}{\tilde{a}^{2+(d_0-1)\alpha^{(c)}}} = \tilde{A}^{(c)} \frac{1}{\tilde{a}^{2(1+\alpha^{(c)})}}\bigg|_{d_0=3}\,. \tag{4.14}$$

For example, in three-dimensional external space, such perfect fluid has the form of a gas of cosmic strings for $\alpha^{(c)} = 0$, dust for $\alpha^{(c)} = 1/2$ and radiation for $\alpha^{(c)} = 1$.

5 Late time acceleration of the Universe: fine tuning problem

Let us try now to built more o less viable model with stabilized internal spaces. In our paper [8] were have already investigated Friedmann-like multidimensional cosmological models with the perfect fluid of (4.14). It was shown that one of the necessary conditions for the stable compactification is the negativeness of an effective four-dimensional cosmological constant. However, a negative cosmological constant leads to a deceleration of the Universe instead of an accelerated expansion, as recent observational data indicate [22]. In [23] we already indicated that the effective cosmological constant can be shifted from negative values to positive ones by including into the nonlinear model matter fields, e.g. additional form fields. This hypothesis was confirmed in Ref. [20]. In the present paper, we try to achieve the similar effect combining together cases I. and II. To be more precise, additionally to the the perfect fluid of the type II, we endow our model with the monopole form fields [13, 20]:

$$S_m = -\frac{1}{2}\int_M d^D x \sqrt{|g|} \sum_{i=1}^{n} \frac{1}{d_i!} \left(F^{(i)}\right)^2 = -\int_M d^D x \sqrt{|g|} \sum_{i=1}^{n} \frac{f_i^2}{b_i^{2d_i}}\,, \tag{5.1}$$

[7] In paper [8], it was found that condition (4.13) results in effective potentials (4.5) with separating scale factor contributions from internal and external factor spaces. It was stressed that this separation crucially simplifies the analysis of the internal space stable compactification. We can rewrite (4.13) in the form $2 - d_0\alpha_0^{(c)} + (d_0 - 1)\alpha_i^{(c)} = 0$. In the case $d_0 = 3$, $i = c = 1$, the same expression was found in [9] where it was shown that it provides the static flat internal space in the model with $\Lambda_D = 0$. It can be easily shown that such static solution is unstable.

[8] The corresponding equation of state is: $P^{(c)}_{(4)} = (1/3)(2\alpha^{(c)} - 1)\rho^{(c)}_{(4)}$, where we put $d_0 = 3$.

where $f_i \equiv \mathrm{const}$ are arbitrary constants of integration (free parameters of the model) and for real form fields $f_i^2 > 0$. Comparison of this expression with eqs. (4.1) and (2.9) shows that such monopole form fields are equivalent to n-component perfect fluid with $\alpha_0^{(c)} = 0,\ \alpha_i^{(c)} = 2\delta_i^c, \quad c, i = 1, \ldots, n$, i.e. belong to the case I.

Without loss of prediction power of our conclusions, we can perform our analysis in the case of one internal space $n = 1$. Then, the effective potential for such combined model[9] undergos the following separation:

$$U_{eff} = \underbrace{\left(e^{d_1\overline{\beta}^1}\right)^{-\frac{2}{D_0-2}}\left[-\frac{1}{2}\tilde{R}_1 e^{-2\overline{\beta}^1} + \Lambda_D + \tilde{f}_1^2 e^{-2d_1\overline{\beta}^1}\right]}_{U_{int}(\overline{\beta}^1)} + \underbrace{\kappa_0^2 \sum_{c=1}^{m} \rho_{(4)}^{(c)}(\tilde{a})}_{U_{ext}(\tilde{a})}, \tag{5.2}$$

where $\rho_{(4)}^{(c)}(\tilde{a})$ is defined by eq. (4.14) and $\tilde{f}_1^2 \equiv \kappa_D^2 f_1^2 / b_{(0)1}^{2d_1}$. This separation is the result of cancellation of the prefactor $\exp\left(-2d_1\overline{\beta}^1/(D_0-2)\right)$ in U_{eff} (4.5) with corresponding prefactor in effective 4D energy density form the case II: $\kappa_D^2\rho^c = \kappa_0^2\rho_{(4)}^{(c)}\exp\left(-\xi_1^{(c)}\overline{\beta}^1\right) = \kappa_0^2\rho_{(4)}^{(c)}\exp\left(2d_1\overline{\beta}^1/(D_0-2)\right)$. We will show below, that such separation on the one hand provides a stable compactification of the internal factor space due to a minimum of the first term $U_{int} = U_{int}(\overline{\beta}^1)$ as well as a dynamical behavior of the external factor space due to $U_{ext} = U_{ext}(\tilde{a})$.

First, we investigate the problem of stable compactification of the internal space. It is clear that such stabilization for our model takes place if potential U_{int} has a minimum with respect to fluctuation field $\overline{\beta}^1$:

$$\left.\frac{\partial U_{int}}{\partial \overline{\beta}^1}\right|_{\overline{\beta}^1=0} = 0 \Longrightarrow \frac{D-2}{2d_1}\,\tilde{R}_1 = \Lambda_D + d_0\tilde{f}_1^2\,. \tag{5.3}$$

The value of this potential at the minimum plays the role of effective four-dimensional cosmological constant:

$$\Lambda_{eff} := U_{int}|_{\overline{\beta}^1=0} = -\frac{1}{2}\tilde{R}_1 + \Lambda_D + \tilde{f}_1^2\,. \tag{5.4}$$

With the help of the extremum condition (5.3), Λ_{eff} can be written in the form

$$\begin{aligned}
\Lambda_{eff} &= \frac{D_0-2}{2d_1}\,\tilde{R}_1 - (D_0-2)\tilde{f}_1^2 && (5.5)\\
&= \frac{D_0-2}{D-2}\Lambda_D - \left(\frac{d_0 d_1}{D-2} - 1\right)\tilde{f}_1^2 && (5.6)\\
&= \frac{d_0-1}{d_0}\Lambda_D - \frac{1}{2}\left(1 - \frac{D-2}{d_0 d_1}\right)\tilde{R}_1\,. && (5.7)
\end{aligned}$$

Second derivative of U_{int} in the extremum position reads

$$\begin{aligned}
\left.\frac{\partial^2 U_{int}}{\partial \overline{\beta}^{1\,2}}\right|_{\overline{\beta}^1=0} &= -2\left(\frac{D-2}{D_0-2}\right)^2\tilde{R}_1 + \left(\frac{2d_1}{D_0-2}\right)^2\Lambda_D + \left(\frac{2d_0 d_1}{D_0-2}\right)^2\tilde{f}_1^2 && (5.8)\\
&= \frac{4}{D_0-2}\left[-\frac{1}{2}(D-2)\tilde{R}_1 + 4d_0 d_1^2\tilde{f}_1^2\right] && (5.9)\\
&= \frac{4d_1}{(D_0-2)^2}\left[-(D_0-2)\Lambda_D + (D-2)d_0\left(\frac{d_0 d_1}{D-2} - 1\right)\tilde{f}_1^2\right] && (5.10)\\
&= \frac{4}{(D_0-2)^2}\left[-d_1^2(d_0-1)\Lambda_D + \frac{1}{2}(D-2)^2\left(\frac{d_0 d_1}{D-2} - 1\right)\tilde{R}_1\right]. && (5.11)
\end{aligned}$$

[9] We consider the case where the Casimir energy density is negligible in comparison with the energy density of the form fields.

For stable compactification, this extremum should be a minimum. Then, small fluctuations above it describe minimal scalar field (gravitational excitons [13]) propagated in the external space with the mass squared

$$m^2_{exci} \equiv \frac{D_0-2}{d_1(D-2)} \left.\frac{\partial^2 U_{int}}{\partial \overline{\beta}^{1^2}}\right|_{\overline{\beta}^1=0} > 0\,. \tag{5.12}$$

Additionally, the effective four-dimensional cosmological constant should be positive $\Lambda_{eff} > 0$. This is the necessary condition for the late time acceleration of the Universe in considered model. Both of these conditions (the positiveness of Λ_{eff} and m^2_{exci}) lead to the following inequalities[10]

$$d_1\tilde{f}_1^2 < \ \tfrac{1}{2}\tilde{R}_1 \ < \frac{4d_0d_1}{D-2}\times d_1\tilde{f}_1^2 \tag{5.13}$$

$$\frac{D-2}{D_0-2}\left(\frac{d_0d_1}{D-2}-1\right)\tilde{f}_1^2 < \ \ \Lambda_D \ \ < d_0\times\frac{D-2}{D_0-2}\left(\frac{d_0d_1}{D-2}-1\right)\tilde{f}_1^2 \tag{5.14}$$

$$\frac{1}{2}\frac{D-2}{d_1(d_0-1)}\left(\frac{d_0d_1}{D-2}-1\right)\tilde{R}_1 < \ \ \Lambda_D \ \ < \frac{D-2}{d_1}\times\frac{1}{2}\frac{D-2}{d_1(d_0-1)}\left(\frac{d_0d_1}{D-2}-1\right)\tilde{R}_1\,. \tag{5.15}$$

These inequalities clearly show that the positive minimum takes place only if signs of $\tilde{R}_1$ and Λ_D are positive: $\tilde{R}_1, \Lambda_D > 0$ (for real form fields with $\tilde{f}_1^2 > 0$). Thus, for non-positive $\tilde{R}_1$ and Λ_D the effective cosmological constant is also non-positive[11]. For example, in the case of Ricci-flat ($\tilde{R}_1 = 0$) internal spaces (e.g. $d_1 = 1$) the effective four-dimensional cosmological constant can admit only negative values.

According to the present day observations, our Universe undergoes the late time accelerating expansion due to a dark energy [22]. The origin of the dark energy is the great challenge of the modern theoretical physics and cosmology. The cosmological constant is one of the most probable candidate for it. According to the observations, its value should be $\Lambda_{DE} \sim 10^{-123}\Lambda_{Pl} \sim 10^{-57}\mathrm{cm}^{-2}$. Let us estimate now a possibility for our effective cosmological constant to admit this quantity: $\Lambda_{eff} \sim \Lambda_{DE} \sim 10^{-57}\mathrm{cm}^{-2}$.

It is well known that in KK models the size of extra dimensions at present time should be of order or less than $b_{(0)1} \sim 10^{-17}\mathrm{cm} \sim 1\mathrm{TeV}^{-1}$. In this case $\tilde{R}_1 \sim b^{-2}_{(0)1} \sim 10^{34}\mathrm{cm}^{-2}$. From other hand, inequalities (5.13) - (5.15) show that $\tilde{R}_1, \Lambda_D$ and $\tilde{f}_1^2$ are of the same order of magnitude, i.e. $\tilde{R}_1 \sim \Lambda_D \sim \tilde{f}_1^2 \sim 10^{34}\mathrm{cm}^{-2}$, and have the same sign. Thus, these parameters should be extremely fine tuned (in eq. (5.4)) to compensate each other in such a way that to leave only $10^{-57}\mathrm{cm}^{-2}$. We see two possibilities to avoid this problem. First, the inclusion of different form fields/fluxes may result in a big number of minima (landscape) [24] with sufficient large probability to find oneself in a dark energy minimum. This problem we shall investigate in our forthcoming paper. Second, we can avoid the restriction $\tilde{R}_1 \sim b^{-2}_{(0)1} \sim 10^{34}\mathrm{cm}^{-2}$ if the internal space is Ricci-flat: $\tilde{R}_1 = 0$. For example, $\mathcal{M}_1$ can be an orbifold with branes in fixed points [12]. This model we investigate in the next section.

For masses of gravexcitons we obtain (if $b_{(0)1} \sim 10^{-17}\mathrm{cm} \sim 1\mathrm{TeV}^{-1}$)

$$m_{exci} \sim 1\mathrm{TeV}\,. \tag{5.16}$$

Masses of such heavy gravexcitons are very close to values which do not contradict to the observable data [7, 25]. They decay (e.g. due to reaction $\psi \to 2\gamma$) long before the present time and the effective potential contributes in the form of the effective cosmological constant (5.4).

[10] The inequalities in the lhs result from the condition $\Lambda_{eff} > 0$ applied to eqs. (5.5) -(5.7), whereas the inequalities in the rhs follow from the minimum condition $m^2_{exci} > 0$ applied to eqs. (5.9) - (5.11).

[11] It is worth of noting that expression $[(D-2)/(2d_1)]\,\tilde{R}_1 - \Lambda_D$ should be non-negative for any signs of $\tilde{R}_1$ and Λ_D, as it follows from eq. (5.3).

Let us now turn to the dynamical behavior of the external factor space. We consider zero order approximation, when all excitations are freezed out (or heavy enough to decay before the present time). Because our Universe (external space) is homogeneous and isotropic, functions $\tilde{\gamma}$ and $\tilde{\beta}^0$ depends only on time: $\tilde{\gamma} = \tilde{\gamma}(\tilde{\tau})$ and $\tilde{\beta}^0 = \tilde{\beta}^0(\tilde{\tau})$. Then, the action functional (4.4) (for combined model (5.2)) after dimensional reduction reads:

$$
\begin{aligned}
S &= \frac{1}{2\kappa_0^2}\int\limits_{\tilde{M}_0} d^{D_0}x\sqrt{|\tilde{g}^{(0)}|}\left\{\tilde{R}\left[\tilde{g}^{(0)}\right] - 2U_{eff}\right\} = \\
&= \frac{V_0}{2\kappa_0^2}\int d\tilde{\tau}\left\{e^{\tilde{\gamma}+d_0\tilde{\beta}^0}e^{-2\tilde{\beta}^0}R[g^{(0)}] + d_0(1-d_0)e^{-\tilde{\gamma}+d_0\tilde{\beta}^0}\left(\frac{d\tilde{\beta}^0}{d\tilde{\tau}}\right)^2\right. \\
&\quad \left. -2e^{\tilde{\gamma}+d_0\tilde{\beta}^0}\left(\Lambda_{eff} + \kappa_0^2\sum_{c=1}^{m}\rho_{(4)}^{(c)}\right)\right\} + \frac{V_0}{2\kappa_0^2}d_0\int d\tilde{\tau}\frac{d}{d\tilde{\tau}}\left(e^{-\tilde{\gamma}+d_0\tilde{\beta}^0}\frac{d\tilde{\beta}^0}{d\tilde{\tau}}\right), \qquad (5.17)
\end{aligned}
$$

where $V_0 := \int_{\mathcal{M}_0} d^{d_0}x\sqrt{|g^{(0)}|}$, $\rho_{(4)}^{(c)}$ is defined by eq. (4.14) and usually $R[g^{(0)}] = kd_0(d_0-1)$, $k = \pm 1, 0$. The constraint equation $\partial L/\partial\tilde{\gamma} = 0$ in the synchronous time gauge $\tilde{\gamma} = 0$ yields

$$
\left(\frac{1}{\tilde{a}}\frac{d\tilde{a}}{d\tilde{t}}\right)^2 = -\frac{k}{\tilde{a}^2} + \frac{2}{d_0(d_0-1)}\left(\Lambda_{eff} + \kappa_0^2\sum_{c=1}^{m}\rho_{(4)}^{(c)}(\tilde{a})\right), \qquad (5.18)
$$

which results in

$$
\begin{aligned}
\tilde{t} + const &= \int \frac{d\tilde{a}}{\left[-k + \frac{2\Lambda_{eff}}{d_0(d_0-1)}\tilde{a}^2 + \frac{2\kappa_0^2}{d_0(d_0-1)}\sum_{c=1}^{m}\frac{\tilde{A}^{(c)}}{\tilde{a}^{(d_0-1)\alpha^{(c)}}}\right]^{1/2}}, \\
&= \int \frac{d\tilde{a}}{\left[-k + \frac{\Lambda_{eff}}{3}\tilde{a}^2 + \frac{\kappa_0^2}{3}\sum_{c=1}^{m}\frac{\tilde{A}^{(c)}}{\tilde{a}^{2\alpha^{(c)}}}\right]^{1/2}}, \qquad (5.19)
\end{aligned}
$$

where in the last line we put $d_0 = 3$.

Thus in zero order approximation we arrived at a Friedmann model in the presence of positive cosmological constant $\Lambda_{eff} > 0$ and a multicomponent perfect fluid. It is assumed that Λ_{eff} defines dark energy observed now. The perfect fluid has the form of a dust for $\alpha^{(c)} = 1/2$ and radiation for $\alpha^{(c)} = 1$. As for ordinary matter $\alpha^{(c)} > 0$, the cosmological constant plays a role only for large $\tilde{a}$ and because of the positive sign of Λ_{eff} the Universe undergos the late time acceleration. The inclusion of gravexcitons into consideration (the first order approximation) does not change this picture because gravexcitons with masses (5.16) decay into usual matter before primordial nucleosynthesis [7].

Within the bounds of this scenario, there is also a possibility for a primordial inflation. For this purpose we can consider one component perfect fluid with $\alpha^{(1)} < 0$, e.g. $\alpha^{(1)} = -1/2 \Rightarrow \alpha_0^{(1)} = 1/3$ which describes a frustrated network of domain walls in the external space. It is well known that such perfect fluid results in acceleration of the Universe. For example, in the case $\alpha^{(1)} = -1/2$ the flat Universe ($k = 0$) undergoes the power law inflation at early times: $\tilde{a} \sim \tilde{t}^2$. If domain walls decay into ordinary matter, then the described above Friedmann-like behavior follows the inflation.

6 Stabilization of orbifolds

In this section, to avoid the fine tuning problem, we consider Ricci-flat internal spaces. In this case scalar curvatures of the internal spaces are absent and there is no need for extreme fine tuning of the parameters to get the observable dark energy.

Among such models, Universal extra dimension models (UED) are of special interest [12]. Here, the internal spaces are orbifolds[12] with branes in fixed points. The compactification of the extra dimensions on orbifolds has a number of very interesting and useful properties, e.g. breaking (super)symmetry and obtaining chiral fermions in four dimensions (see e.g. paper by H.-C. Cheng at al in [12]). The latter property gives a possibility to avoid famous no-go theorem of KK models (see e.g. [26]).

In UED models, the Standard Model fields are not localized on the brane but can move in the bulk. Branes in fixed points contribute in action functional (2.3) in the form:

$$\sum_{\substack{fixed\\points}} \int d^4x \sqrt{\overline{g}^{(0)}(x)} L_b \Big|_{\substack{fixed\\point}} , \tag{6.1}$$

where $\overline{g}(x)$ is induced metric (which for our geometry (2.1) coincides with the metric of the external spacetime in the Brans-Dicke frame) and L_b is the matter Lagrangian on the brane. In what follows, we consider the case where branes are only characterized by their tensions $L_{b(k)} = -\tau_{(k)}$, $k = 1, 2, \ldots, m$ and m is the number of branes.

After transformation to the Einstein frame (4.3), the action (6.1) reads

$$\frac{1}{2\kappa_0^2} \int d^4x \sqrt{\tilde{g}^{(0)}(x)} \left(\prod_{i=1}^{n} e^{d_i \overline{\beta}^i} \right)^{-\frac{2}{D_0-2}} \left[-2\kappa_0^2 \sum_{k=1}^{m} \tau_{(k)} \prod_{i=1}^{n} e^{-d_i \overline{\beta}^i} \right] . \tag{6.2}$$

The comparison of this expression with eq. (4.5) shows that branes contribute in the effective potential in the form of one component perfect fluid ($c = 1$) with equations of state: $\alpha_0^{(1)} = 0$, $\alpha_i^{(1)} = 1$, $i = 1, \ldots, n$, i.e. from the case I of the no-go theorem. It means that they contribute only to the U_{int}:

$$U_{int} = \left(e^{d_1 \overline{\beta}^1} \right)^{-\frac{2}{D_0-2}} \left[\Lambda_D + \tilde{f}_1^2 e^{-2d_1 \overline{\beta}^1} - \lambda e^{-d_1 \overline{\beta}^1} \right] , \tag{6.3}$$

where we consider the case of one internal space $i = 1$ and introduce notation $\lambda \equiv -\kappa_0^2 \sum_{k=1}^{m} \tau_{(k)}$.

Obviously, the internal space is stabilized if potential (6.3) has a minimum with respect to $\overline{\beta}^1$. The extremum condition reads:

$$\left. \frac{\partial U_{int}}{\partial \overline{\beta}^1} \right|_{\overline{\beta}^1 = 0} = 0 \Longrightarrow \frac{d_1 D_0}{D_0 - 2} \lambda = \frac{2d_1}{D_0 - 2} \Lambda_D + \frac{2d_1 (D_0 - 1)}{D_0 - 2} \tilde{f}_1^2 . \tag{6.4}$$

The value of this potential at the minimum plays the role of effective four-dimensional cosmological constant:

$$\Lambda_{eff} := U_{int}|_{\overline{\beta}^1 = 0} = \Lambda_D + \tilde{f}_1^2 - \lambda > 0 , \tag{6.5}$$

which we assume to be positive. For the mass of gravexcitons we obtain:

$$m_{exci}^2 \sim \left. \frac{\partial^2 U_{int}}{\partial \overline{\beta}^{1\,2}} \right|_{\overline{\beta}^1 = 0} = \left(\frac{2d_1}{D_0 - 2} \right)^2 \Lambda_D + \left(\frac{2d_1 (D_0 - 1)}{D_0 - 2} \right)^2 \tilde{f}_1^2 - \left(\frac{d_1 D_0}{D_0 - 2} \right)^2 \lambda > 0 . \tag{6.6}$$

It can be easily seen from condition (6.4) and inequality (6.5) that all three parameters are positive[13]: $\tilde{f}_1^2, \Lambda_D, \lambda > 0$. Taking into account inequality (6.6), it can be easily verified that all these parameters have the same order of magnitude:

$$\tilde{f}_1^2 \sim \Lambda_D \sim \lambda \sim \Lambda_{eff} \sim m_{exci}^2 . \tag{6.7}$$

[12]For example, S^1/Z_2 and T^2/Z_2 which represent circle and square folded onto themselves due to Z_2 symmetry.

[13]Thus, from $\lambda > 0$ follows that summarized tensions of branes should be negative.

Therefore, there is no need for fine tuning of parameters to obtain the observable value of dark energy. To get it, it is sufficient to suppose that all these parameters, including Λ_{eff}, are of the order of $\Lambda_{DE} \sim 10^{-123} L_{Pl}$. Thus, we obtain the condition $\Lambda_{eff} \sim \Lambda_{DE} \sim 10^{-57}\mathrm{cm}^{-2}$. From one side, it is natural to assume that parameters of the model have the same order of magnitude. From other side, our model does not answer why this value is equal to $10^{-123} L_{Pl}$. According to the anthropic principle, it takes place because human life is possible only at this value of dark energy.

If we assume that our parameters are defined by $\Lambda_{DE} \sim 10^{-123} L_{Pl}$, then we get for the gravexciton masses $m_{exci} \sim 10^{-33}\mathrm{eV} \sim 10^{-61} M_{Pl}$. These ultra-light particles have a period of oscillations $t \sim 1/m_{\psi} \sim 10^{18}\mathrm{sec}$ which is of order of the Universe age. So, up to now these cosmological gravexcitons did not start to oscillate but slowly move to the position of minimum of the effective potential. In this case it is hardly possible to speak about stabilization of the internal space (the effective potential U_{int} is too flat) and we again arrive to the problem of the fundamental constant variations.

7 Conclusion

In the present paper we considered a possibility for the construction of the conventional cosmology for our observable Universe if underlying theory is multidimensional Kaluza-Klein model. In the spirit of the Friedmann model, our multidimensional models are also endowed with a perfect fluid as a matter source. We investigated two different types of KK models.

For the first class of models, the internal spaces become unobservable due to the dynamical compactification. Following an ansatz proposed in Ref. [2], we obtained effective four-dimensional model which has the form of the Friedmann model with the exception of two important things. First, the effective four-dimensional gravitational "constant" κ_0^2 is a dynamical function and, second, the energy density of effective four-dimensional perfect fluid $\rho_{(4)}$ is not conserved (in usual four-dimensional sense). Accordingly, the energy density $\rho_{(4)}$ behaves differently than in the FRW Universe. However, the combination $\kappa_0^2 \rho_{(4)}$ has exactly the same dependence on the scale factor as in the conventional cosmology. Precisely this combination enters into the Friedmann equations. Thus, the dynamical behavior of the external space in this model exactly coincides with the standard Friedmann model, i.e. here we have the same rate of the evolution. Unfortunately, as our investigation shows, the effective four-dimensional fundamental "constants" (e.g. the gravitational constant and the fine structure constant) undergo too large variations with time in comparison with the observations.

Then, we investigated the problem of the sable compactification of the internal spaces. We proved the no-go theorem which claims that the stable compactification of the internal spaces in KK models is impossible in the general case with arbitrary equations of state in the internal and external spaces. There are only two exceptional classes with appropriate fitted equations of state. From these classes of solutions, we constructed a model with the standard behavior of the FRW Universe for the external space. Moreover, this model is endowed with positive 4-D effective cosmological constant. However, the fine tuning of parameters is required to get the observable dark energy density. To avoid the fine tuning problem, we generalized this model to the case of orbifold internal space with branes in fixed points. In the spirit of the Universal Extra Dimension models, the Standard Model fields are not localized on the brane but can move in all bulk space. We considered Ricci-flat orbifolds. The internal space stabilization condition together with the positivity of the effective 4-D cosmological constant $\Lambda_{eff} > 0$ lead to the conclusion that Λ_{eff} can be of the order of observable dark energy cosmological constant Λ_{DE} without any fine tuning of the parameters. However, in this case the effective potential is too flat (mass gravexcitons is very small) to provide necessary constancy of the effective fundamental "constants".

Acknowledgments
I thank the Organizing Committee of the 14th International Seminar on High Energy Physics "QUARKS-2006" in St. Petersburg for their financial support.

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