Measuring the parameters of supermassive black holes from space

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Abstract

To describe black hole in astrophysics typically astronomers use Newtonian approaches for gravitational field because usually one analyzes processes acting far enough (in Schwarzschild radius units) from black hole horizons. Here we discuss phenomena where we have to use general relativistic approaches to explain present and future observational data like Fe K_α line profiles and shapes of shadows around black holes. Different X-ray missions such as ASCA, XMM-Newton, Chandra etc. discovered features of Fe K_α lines and other X-ray lines as well. Attempts to fit spectral line shapes lead to conclusions that sometimes the profiles line shapes should correspond to radiating regions which are located in the innermost parts of accretion disks where contributions of general relativistic phenomena are extremely important. As an illustration we consider a radiating annulus model to clarify claims given recently by Müller & Camenzind (2004). We discuss properties of highly inclined disks and analyze a possibility to evaluate magnetic fields near black hole horizons. We mention also that shadows could give us another case when one could evaluate black hole parameters (namely, spins, charges and inclination angles) analyzing sizes and shapes shadows around black holes.

We discuss glories (mirages) formed near rapidly rotating Kerr black hole horizons and propose a procedure to measure masses, charges and rotation parameters analyzing these forms of mirages. We also propose to use future radio interferometer RADIOASTRON (Millimetron or MAXIM) facilities to measure shapes of mirages (glories) and to evaluate the black hole spin as a function of the position angle of a distant observer. We propose also a procedure to measure a black hole charge with future space missions.

1 Introduction

Here we discuss samples where we really need general relativistic approaches in the strong gravitational field limit to explain observational data generating by radiation arising in black hole vicinities and typically one could get the data with space missions such as ASCA, RXTE, XMM-Newton, Chandra etc.

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Several years ago it was predicted that profiles of lines emitted by AGNs and X-ray binary systems could have asymmetric double-peaked, double horned or triangular shape according to classification done by Müller and Camenzind [1]. Comprehensive review summarizes the detailed discussion of theoretical aspects of possible scenarios for generation of broad iron lines in AGNs [2] (an influence of microlensing on Fe K_α line shapes and spectra was discussed in [3] but optical depths for the phenomena were calculated in [4–6]). A formation of shadows (mirages) is another sample when general relativistic effects are extremely important and in principle they could be detected with forthcoming interferometrical facilities [7–15] (perspective studies of microlensing with Radioastron facilities were discussed recently [16]). Observations of shadows could give a real chance to observe "faces" of black holes of black holes and confirm general relativity predictions in the framework of strong gravitational filed approach and obtain new constraints on alternative theories of gravity.

2 Toy Model Lessons

Recently Müller and Camenzind [1] presented results of their calculations and classified different types of spectral line shapes and described their origin. In particular, the authors claimed that usually "... triangular form follows from low inclination angles...", "...double peaked shape is a consequence of the space-time that is sufficiently flat. This is theoretically reproduced by shifting the inner edge to the disk outwards... A relatively flat space-time is already reached around 25 r_g..." We clarified their hypothesis about an origin of doubled peaked and double horned line shapes. Based on results of numerical simulations we showed that double peaked spectral lines arise for *almost any* locations of narrow emission rings (annuli) (except closest orbits as we could see below) although Müller and Camenzind [1] suggested that such profiles arise for relatively flat spacetimes and typical radii for emission region about 25 r_g. Using a radiating annulus model we checked the statements and clarified them for the case. We could note here that in the framework of the model we do not use any assumptions about an emissivity law, but we only assume that radiating region is a narrow circular ring (annulus). Thus, below we do not use some specific model on surface emissivity of accretion (we only assume that the emitting region is narrow enough). But general statements (which will be described below) could be generalized on a wide disk case without any problem. We used an approach which was discussed in details in papers [17–37]. The approach was used in particular to simulate spectral line shapes. This approach is based on results of qualitative analysis [38, 39].

Presenting their classification of different types of spectra line shapes Müller and Camenzind [1] noted that double peaked shapes arise usually for emission regions located far enough from black holes. Earlier, we calculated spectral line shapes for annuli for selected radii and distant observer position angles and found an essential fraction of spectral line gallery correspond to double peaked profiles [20]. To check the Müller and Camenzind [1] hypothesis about an origin of double peaked profiles we calculated a complete set of spectral line shapes for emitting annuli. Let us discuss results of our calculations for rapidly rotating black holes (for $a = 0.998$ one could find a detailed description of the calculations in [40–42]). We summarize results of the calculations. As it was shown in the framework of the simple model the double peaked spectral line shape arise almost for all parameters r and a except the case when radii are very small $r \in (0.7, 2)$ and inclination angles are in the band $\theta \in [45^\circ, 90^\circ]$ (for these parameters the spectral line shape has triangular structure). The phenomenon could be easy understood, since for this case the essential fraction of all photons emitted in the opposite direction in respect to emitting segment of annulus is captured by a black hole, therefore a red peak is strongly dumped. For other radii and angles spectral line profiles have double peaked structure. Therefore we clarify the statement Müller and Camenzind [1] that double peaked structure arises if radiation region is far enough. If we assume that there is a weak dependence of emissivity function on radius, then a number of photons characterizes relative intensity in the line (roughly speaking for $r = 0.7$ an intensity (in counts) in 10 times lower then an intensity for $r = 2$) therefore in observations for small radii one could detect only a narrow blue peak but another part of spectra is non-distinguishable from a background. One could note also that for fixed radius there is a strong monotone dependence of intensity on inclination angle (maximal intensity corresponds to photon motion near equatorial plane and only a small fraction of photons reach a distant observer near the polar axis). That is a natural consequence of a photon boost due to a circular motion of emitting fragment of annulus in the equatorial plane and an influence of spin of a rotating black hole. In the framework of the simple model one could understand that sometimes the Fe K_α line has only one narrow peak like in observations of the Seyfert galaxy MCG-6-30-15 by the XMM-Newton satellite [43]. If radiating (or illuminating) region is a narrow annulus evolving along quasi-circular orbits, then initially two peak structure of the spectral line profile transforms in one peaked (triangular) form. Moreover, an absolute intensity in the line is increased for smaller radii since a significant fraction of emitted photons are captured by a black hole during the evolution of emitting region toward to black hole in observations we could detect only narrow blue peak and its height is essentially lower than its height was before for larger radii. Another part of the triangular spectral line shape could be non-distinguishable from a background. A relative low intensity for a triangular spectral line shape could give a narrow single peak structure in observations.

3 Signatures of Accretion Discs with High Inclination

At inclination angles $\theta > 80^\circ$, new observational features of GR could arise. Matt et al. [44] discovered such phenomenon for a Schwarzschild black hole, moreover the authors predicted that their results could be applicable to a Kerr black hole over the range of parameters exploited. The authors mentioned that this problem was not analyzed in detail for a Kerr metric case and it would be necessary to investigate this case. In the detailed consideration [23] we did not use a specific model on surface emissivity of accretion (we only assume that the emitting region is narrow enough), therefore, we checked and confirmed their hypothesis for the Kerr metric case and for a Schwarzschild black hole using other assumptions about surface emissivity of accretion disks. In principle, such a phenomenon could be observed in microquasars and X-ray binary systems where there are neutron stars and black holes with stellar masses. We confirmed also the conclusion that extra peaks are generated by photons which are emitted by the far side of the disk, therefore we have a manifestation of gravitational lensing in the strong gravitational field approach for GR [23]. Some possibilities to observe considered features of spectral line profiles were considered [44]. The authors argued that there are non-negligible chances to observe such a phenomenon in some AGNs and X-ray binary systems. Thus, such properties of spectral line shapes are robust enough with respect to wide variations of rotational parameters of black holes and the surface emissivity of accretion disks as it was predicted [44]. So, their conjecture was confirmed not only for the Kerr black hole case but also for other dependencies of surface emissivity of the accretion disk. We use no astrophysical assumptions about the physical structure of the emission region except the assumption that the region should be narrow enough. Positions and heights of these extra peaks drastically depend on both the radial coordinate of the emitting region (annuli) and the inclination angle. It was found that these extra peaks arise due to gravitational lens effect in the strong gravitational field, namely they are formed by photons with some number of revolutions around black hole. This conclusion is based only on relativistic calculations without any assumption about physical parameters of the accretion disc like X-ray surface emissivity etc. A detailed description of the analysis was given in [23].

4 Magnetic Fields in AGNs and Microquasars

Magnetic fields play a key role in dynamics of accretion discs and jet formation. To obtain an estimation of the magnetic field we simulate the formation of the line profile for different values of magnetic field. As a result we find the minimal B value at which the distortion of the line profile becomes significant. Here we use an approach, which is based on numerical simulations of trajectories of the photons emitted by a hot ring moving along a circular geodesics near black hole, described in [18–20]. The influence of accretion disc model on the profile of spectral line was discussed [31].

Let us discuss possible influence of high magnetic fields on real observational data (see details in [25]). We will try to estimate magnetic fields when one could find the typical features of line splitting from the analysis of the spectral line shape. Further we will choose some values of magnetic field and simulate the spectral line shapes from observational data for these values, assuming that these observational data correspond to an object with no significant magnetic fields. We will try to find signatures of the triple blue peak analyzing the simulated data when magnetic fields are rather high. Assuming that there are no essential magnetic fields (compared to 10^{10} G) for some chosen object (for example, for MCG 6-30-15) we could simulate the spectral line shapes for the same objects but with essential magnetic fields. From results of simulations one can see that classical Zeeman splitting in three components, which can be revealed experimentally today, changes qualitatively the line profiles only for rather high magnetic field. Something like this structure can be detected, e.g. for $H = 1.2 \cdot 10^{11}$ G, but the reliable recognition of three peaks here is hardly possible [25]. It is known that neutron stars (pulsars) could have huge magnetic fields. So, it means that the effect discussed above could appear in binary neutron star systems. The quantitative description of such systems, however, needs more detailed computations. A detailed discussion of the magnetic field influence on spectral line shapes was discussed for flat accretion flows [25, 26] and for non-flat accretion flows [29].

5 Shadows around Black Holes

The full classification of geodesic types for Kerr metric is given in [38]. As it was shown in this paper, there are three photon geodesic types: capture, scattering and critical curve which separates the first two sets. This classification fully depends only on two parameters $\xi = L_z/E$ and $\eta = Q/E^2$, which are known as Chandrasekhar's constants [46]. Here the Carter constant Q is given by

$$Q = p_\theta^2 + \cos^2\theta \left[a^2\left(m^2 - E^2\right) + L_z^2/\sin^2\theta\right], \qquad (1)$$

where $E = p_t$ is the particle energy at infinity, $L_z = p_\phi$ is z-component of its angular momentum, $m = p_i p^i$ is the particle mass. Therefore, since photons have $m = 0$

$$\eta = p_\theta^2/E^2 + \cos^2\theta \left[-a^2 + \xi^2/\sin^2\theta\right]. \qquad (2)$$

We fix a black hole spin parameter a and consider a plane (ξ, η) and different types of photon trajectories corresponding to (ξ, η), namely, a capture region, a scatter region and the critical curve $\eta_{\rm crit}(\xi)$ separating the scatter and capture regions. The critical curve is a set of (ξ, η) where the polynomial $R(r)$ has a multiple root (a double root for this case).

One can therefore calculate the critical curve $\eta(\xi)$ which separates the capture and the scattering regions [38]. We remind that the maximal value for $\eta_{\rm crit}(\xi)$ is equal to 27 and is reached at $\xi = -2a$. Obviously, if $a \to 0$, the well-known critical value for Schwarzschild black hole (with $a = 0$) is obtained.

Thus, at first, we calculate the critical curves for chosen spin parameters a which are shown in Fig. 1. The shape of the critical curve for $a = 0$ (Schwarzschild black hole) is well-known

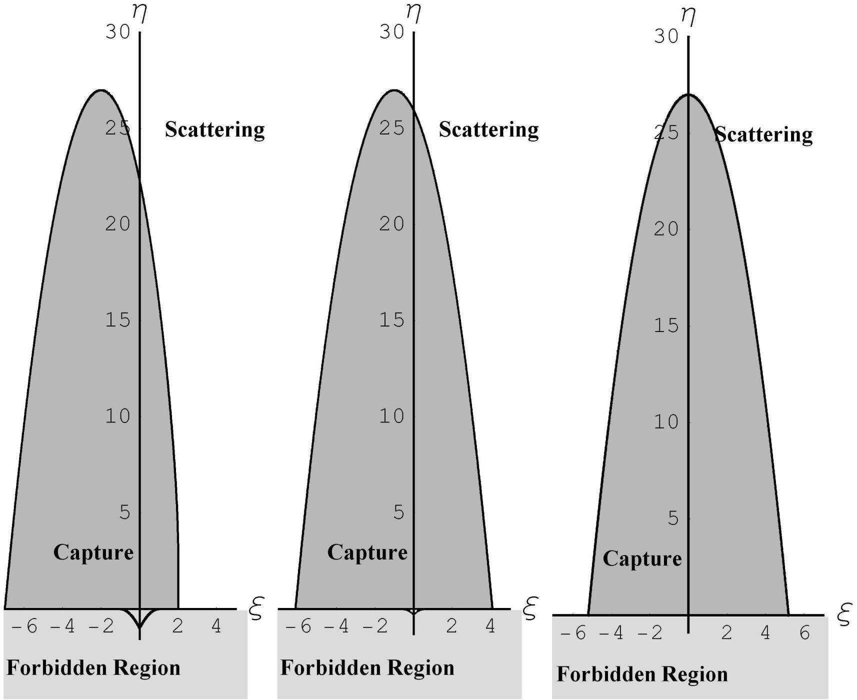

Figure 1: Different types for photon trajectories and spin parameters ($a = 1., a = 0.5, a = 0.$). Critical curves separate capture and scatter regions. Here we show also the forbidden region corresponding to constants of motion $\eta < 0$ and $(\xi, \eta) \bar{\in} M$ as it was discussed in the text.

because for this case we have $\eta_{\rm crit}(\xi) = 27 - \xi^2$ for $|\xi| \leqslant 3\sqrt{3}$, but we show the critical curve to compare with the other cases.

By following this approach we can find the set of critical impact parameters (α, β), for the image (mirage or glory) around a rotating black hole. The sets of critical parameters form caustics around black holes and it is well-known that caustics are the brightest part of each image. We remind that (α, β) parameters could be evaluated in terms of $(\xi, \eta_{\rm crit})$ by the following way [46]

$$\alpha(\xi) = \xi / \sin\theta_0, \tag{3}$$

$$\beta(\xi) = (\eta_{\rm crit}(\xi) + (a^2 - \alpha^2(\xi))\cos^2\theta_0)^{1/2}.$$

Here we will point out general properties of the shadows.

5.1 Equatorial plane observer case

Let us assume that the observer is located in the equatorial plane ($\theta = \pi/2.$). For this case we have from Eqs. (3) and (4)

$$\alpha(\xi) = \xi, \tag{4}$$
$$\beta(\xi) = \sqrt{\eta_{\rm crit}(\xi)}. \tag{5}$$

The maximum impact value $\beta = 3\sqrt{3}$ corresponds to $\alpha = -2a$ and if we consider the extreme spin parameter $a = 1$ a segment of straight line $\alpha = 2, 0 < |\beta| < \sqrt{3}$ belongs to the mirage

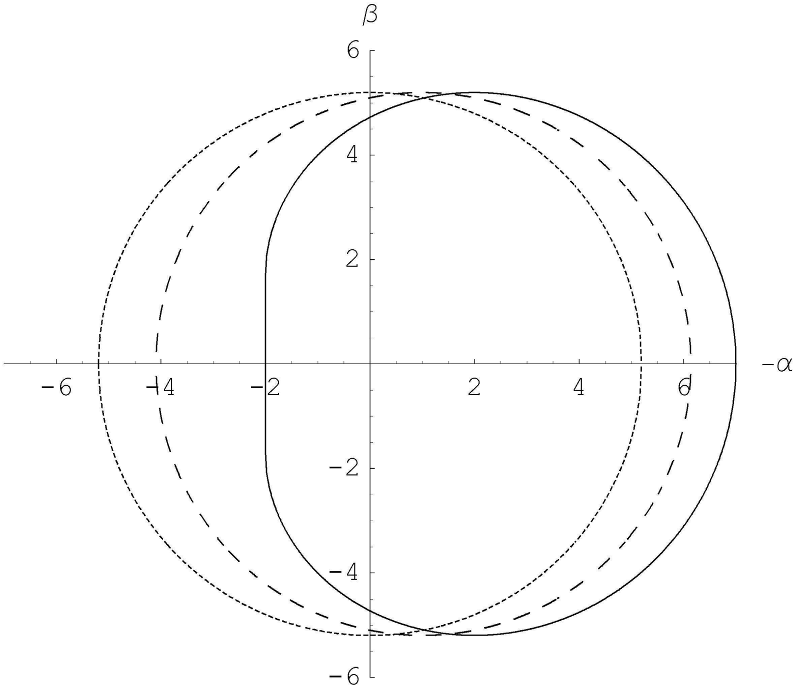

Figure 2: Mirages around black hole for equatorial position of distant observer and different spin parameters. The solid line, the dashed line and the dotted line correspond to $a = 1, a = 0.5, a = 0$, respectively.

(see images in Fig. 2 for different spin parameters). It is clear that for this case one could easy evaluate the black hole spin parameter after the mirage shape reconstruction since we have a rather strong dependence of the shapes on spins. As it was explained earlier, the maximum absolute value for $|\beta| = \sqrt{27} \approx 5.196$ corresponds to $\alpha = -2a$ since the maximum value for $\eta(\xi)$ corresponds to $\eta(-2a) = 27$ as it was found in [38]. Therefore, in principle it is possible to estimate the black hole spin parameter by measuring the position of the maximum value for β, but probably that part of the mirage could be too faint to be detected.

5.2 General case for the angular position of the observer

Let us consider different values for the angular positions of a distant observer $\theta = \pi/2, \pi/3$ and $\pi/8$ for the spin parameter $a = 0.5$ (Fig. 3) and $\theta = \pi/2, \pi/3, \pi/4$ and $\pi/6$ for $a = 1$. (Fig. 4). From these Figures one can see that angular positions of a distant observer could be evaluated from the mirage shapes only for rapidly rotating black holes ($a \sim 1$), but there are no chances to evaluate the angles for slowly rotating black holes, because even for $a = 0.5$ the mirage shape differences are too small to be distinguishable by observations. Indeed, mirage shapes weakly depend on the observer angle position for moderate black hole spin parameters.

5.3 Reissner-Nordström black hole case

"Black holes have no hair": it means that a black hole is characterized by only three parameters ("hairs"), its mass M, angular momentum J and charge Q. Therefore, in principle, charged black holes can form, although astrophysical conditions that lead to their formation may look rather problematic. Nevertheless, one could not claim that their existence is forbidden by theoretical or observational arguments. A detailed description of a possibility to measure a black hole charge with RADIOASTRON interferometer was given in [14].

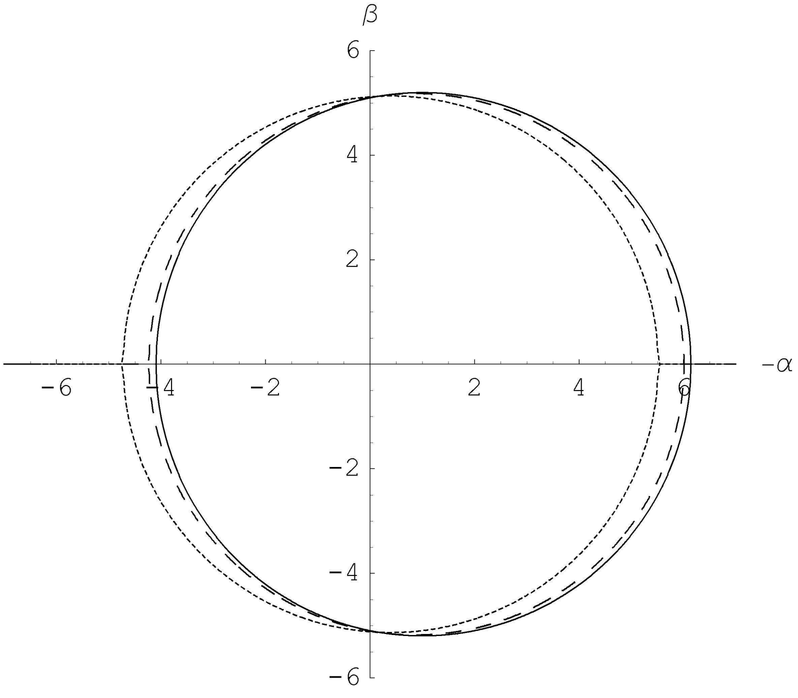

Figure 3: Mirages around black hole for different angular positions of a distant observer and the spin $a = 0.5$. Solid, dashed and dotted lines correspond to $\theta_0 = \pi/2, \pi/3$ and $\pi/8$, respectively.

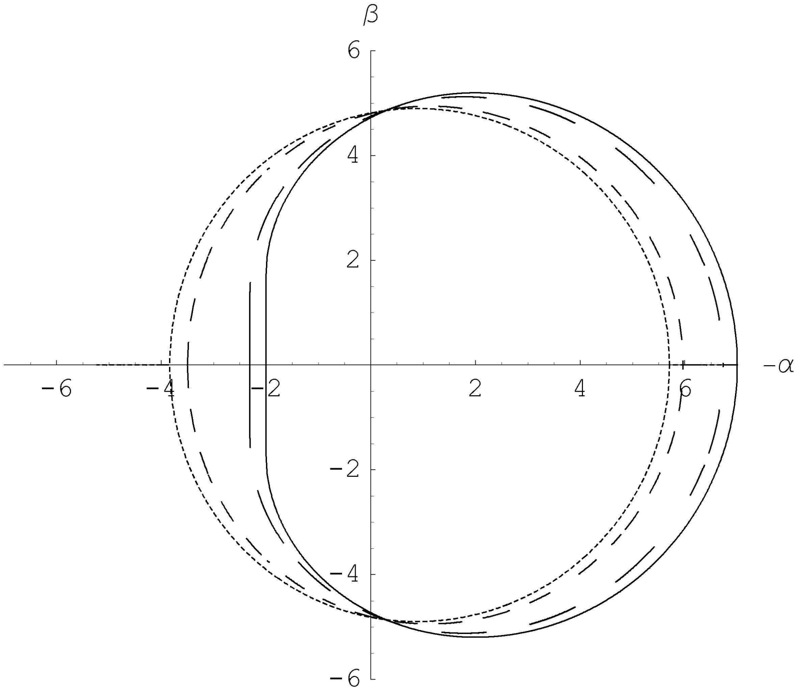

Figure 4: Mirages around black hole for different angular positions of a distant observer and the spin $a = 1$. Solid, long dashed, short dashed and dotted lines correspond to $\theta_0 = \pi/2, \pi/3, \pi/6$ and $\pi/8$, respectively.

Let us consider the problem of the capture cross section of a photon by a charged black hole. It is clear that the critical value of the impact parameter for a photon to be captured by a Reissner - Nordström black hole depends on the multiplicity root condition of the polynomial $R(r)$, i. e. the condition for vanishing discriminant [47–49]. Introducing the notation $\xi^2 = l, Q^2 = q$, we obtain

$$R(r) = r^4 - lr^2 + 2lr - qr. \tag{6}$$

The discriminant Δ of the polynomial $R(r)$ has the form (as it was shown in [48, 49]):

$$\Delta = 16l^3[l^2(1-q) + l(-8q^2 + 36q - 27) - 16q^3].$$

The photon capture cross section for an extreme charged black hole turns out to be considerably smaller than the capture cross section of a Schwarzschild black hole. The critical value of the impact parameter, characterizing the capture cross section for a Reissner - Nordström black hole, is determined by the equation [48, 49]

$$l = \frac{\alpha + \sqrt{\alpha^2 + 64q^3(1-q)}}{2(1-q)}, \tag{7}$$

where $\alpha = 8q^2 - 36q + 27$. Substituting Eq.(7) into the expression for the coefficients of the polynomial $R(r)$ it is easy to calculate the radius of the unstable circular photon orbit (which is the same as the minimum periastron distance). The orbit of a photon moving from infinity with the critical impact parameter, determined in accordance with Eq.(7) spirals into circular orbit.

As it was explained in [7] this leads to the formation of shadows described by the critical value of L_{cr} or, in other words, in the spherically symmetric case, shadows are circles with radii L_{cr}. Therefore, measuring the shadow size, one could evaluate the black hole charge in black hole mass units M.

5.4 Conclusions

Following the idea in [7] we propose to use VLBI technique to observe mirages around massive black holes and in particular, towards the black hole at Galactic Center. The boundaries of the shadows are black hole mirages. We use the length parameter $r_g = \dfrac{GM}{c^2} = 6 \times 10^{11}$ cm for the black hole at Sgr A^* and analytical approach to calculate shadow sizes, as it was explained in the text. By taking the distance of Sgr A^* to be $D_{\rm GC} = 8$ kpc, the length r_g corresponds to angular size ~ 5 μas. Since the minimum arc size for the considered mirages are about at least $4r_g$, the standard RADIOASTRON resolution of about 8 μas is comparable with the required precision. The resolution in the case of the highest orbit and shortest wavelength is ~ 1 μas (see Table 1) good enough to reconstruct the mirage shapes. As can be seen from Fig. 1 and Table 1, it is clear that, in principle, it is possible to evaluate the black hole charge Q by observing the shadow size. The mirage size difference between the extreme charged black hole and Schwarzschild black hole case is about 30% (the mirage diameter for Schwarzschild black hole is about 10.4 and for the extreme charged black hole the diameter is equal to 8 or (in black hole mass units)) and typical angular sizes are about $\sim 52\mu as$ for the Schwarzschild and $\sim 40\mu as$ for the Reissner-Nordström black hole cases, correspondingly. Therefore, for Sgr A^* a charged black hole could be distinguished by Schwarzschild black hole with RADIOASTRON, at least if its charge is close to the maximal value. For stellar mass black holes we need a much higher angular resolution to distinguish charged and uncharged black holes since the typical shadow (mirage) angular sizes are about $2 \times 10^{-5}\mu as$, even for galactic black holes.

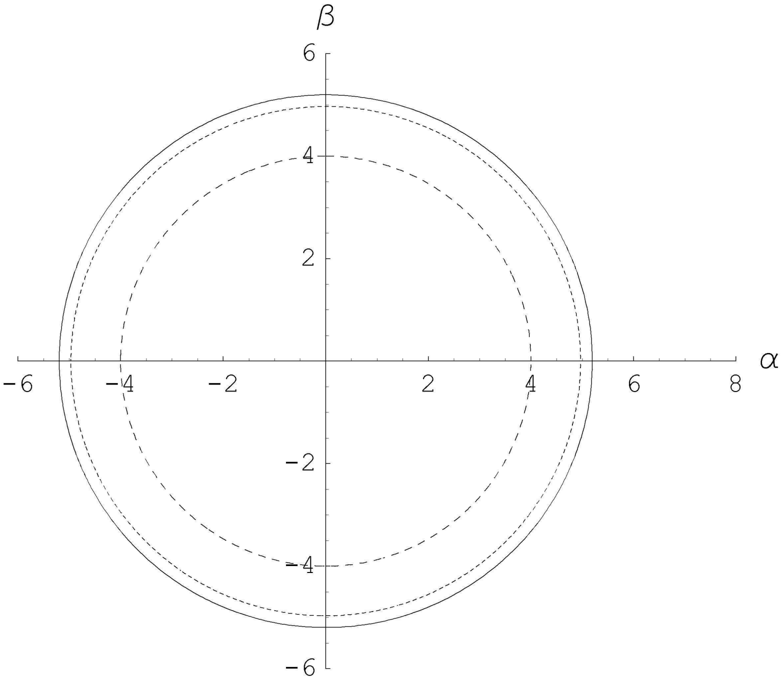

Figure 5: Fig. 5. Shadow (mirage) sizes are shown for selected charges of black holes $Q = 0$ (solid line), $Q = 0.5$ (short dashed line) and $Q = 1$ (long dashed line).

Actually, from the mathematical point of view we proved the following statement: For any positions of source and observer, there is an emitted photon passing close enough to any point of the mirage that can be catched by the observer.

Thus, the mirage is formed by an envelope of a family of photon trajectories or, in other words, by a caustic surface.

The angular resolution of the space RADIOASTRON interferometer will be high enough to resolve radio images around black holes.

In principle, measuring the mirage shapes one could evaluate the black hole mass, inclination angle (e.g. the angle between the black hole spin axis and line of sight) and spin if the black hole distance is known. For example, for the black hole at the Galactic Center the mirage size is $\sim 52\mu as$ for the Schwarzschild case. In the case of a Kerr black hole [7], the mirage is deformed depending on the black hole spin a and on the angle of the line of sight, but its size is almost the same. In the case of a Reissner-Nordström black hole its charge changes the size of the shadows up to 30 % for the extreme charge case. Therefore, the charge of the black hole can measured by observing the shadow size, if the other black hole parameters are known with a sufficient precision. In general, one could say that a measure of the mirage shape (in size) allows to evaluate all the black hole "hairs".

Few years ago the possibility of observing images of distant sources around black holes in the X-ray band was discussed by White [50], Cash et al. [51] by using X-ray interferometer. Indeed, the aim of the MAXIM project is to realize a space based X-ray interferometer capable of observing with angular resolution as small as $0.1\mu as$.

In spite of the difficulties of measuring the shapes of shadow images, to look at black hole "faces" is an attractive challenge since mirages outline the "faces" and correspond to fully general relativistic description of the region nearby the black hole horizon without any assumption about a specific model for astrophysical processes around black holes (of course we assume that there are sources illuminating black hole surroundings). There is no doubt that the rapid growth of observational facilities will give a chance to measure the mirage shapes using not only RADIOASTRON facilities but also other instruments and spectral bands, like the X-ray interferometer MAXIM, the RADIOASTRON mission or other space based interferometers in millimeter and sub-millimeter bands. One could mention here the Astro Space Centre of Lebedev Physics Institute proposed except the RADIOASTRON mission and developed also space based interferometers (Millimetron and Sub-millimetron) for future observations in mm and sub-mm bands. These instruments could be used for the determination of shadow shapes.

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