Crossing the cosmological constant barrier in string inspired models

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Abstract

We explore a possibility for the Universe to cross the $w = -1$ cosmological constant barrier for the Dark Energy state parameter. Exact solutions to the Friedmann equations with a string inspired phantom field are constructed. The Universe is considered as a slowly decaying D3-brane, which is described in the string field theory (SFT) framework. The D3-brane dynamics is approximately described by a nonlocal string tachyon interaction and the back reaction of gravity is incorporated in the closed string tachyon dynamics. In a local effective approximation this model contains one phantom component and one usual scalar field.

1 Introduction

Nowadays strings and D-branes theories have found cosmological applications related with the acceleration of the Universe. The combined analysis of the type Ia supernovae, galaxy clusters measurements and WMAP (Wilkinson Microwave Anisotropy Probe) data brings out clearly an evidence for the accelerated cosmic expansion [1, 2, 3]. The cosmological acceleration strongly indicates that the present day Universe is dominated by a smoothly distributed slowly varying cosmic fluid with negative pressure, the so-called dark energy (DE), see [4] and references therein.

To specify a component of a cosmic fluid one usually uses a phenomenological relation between the pressure p and the energy density ϱ corresponding to each component of fluid $p = w\varrho$, where w is the equation-of-state parameter or, for short, the state parameter. We denote as w_{DE} the component with negative w, which corresponds to the DE. Contemporary experiments, including WMAP [3], give strong support that currently the state parameter is close to -1:

$$w_{DE} = -1 \pm 0.1. \tag{1}$$

From the theoretical point of view this domain of w_{DE} covers three essentially different cases: $w_{DE} > -1$, $w_{DE} = -1$ and $w_{DE} < -1$ (see [5] and references therein). Since from the observations there is no barrier between these three possibilities it is interesting to consider models, where these three cases are realized. Under general assumptions it is proved in [6] that within single scalar field models one can realize only one possibility: $w_{DE} \geqslant -1$ (quintessence models) or $w_{DE} \leqslant -1$ (phantom models). It is interesting that the interaction with the Cold Dark Matter does not change the situation and does not remove the cosmological constant barrier [6, 7]. There are several phenomenological models describing the crossing of the cosmological constant barrier [8]. Most of them use more then one scalar field or use a non-minimal coupling with gravity, or modified gravity, in particular via brane-world scenarios. In two-field models

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one of these two fields is a phantom and the other one is an usual field and the interaction is nonpolynomial in general.

The most exciting possibility would be the case $w_{DE} < -1$ corresponding to the so called phantom dominated Universe. In phenomenological models describing this case the weak energy condition $\varrho + p > 0$ is violated and there are problems with stability at classical and quantum levels. Possible way to evade the instability problem for models with $w_{DE} < -1$ is to yield a phantom model as an effective one, which arises from more fundamental theory with normal kinetic term. In particular, if we consider a model with higher derivatives such as $\phi e^{-\Box}\phi$, then in the first nontrivial approximation, $\phi e^{-\Box}\phi \simeq \phi^2 - \phi\Box\phi$, and such a model gives a kinetic term with a ghost sign. It turns out, that such a possibility does appear in the string field theory framework [4] (see also [9]). This model is close in some approximation to a model arising in the string theory, namely in the theory of fermionic NSR string with GSO$-$ sector. According to Sen's conjecture (see [10] for review), the scalar field ϕ is an open string theory tachyon, describing the brane decay. Since the concerned model is a string theory approximation, all stability problems related to a model with higher derivatives are discarded.

2 Action and Equations of Motion

We consider a model of Einstein gravity interacting with a single phantom scalar field ϕ and one standard scalar field ξ in the spatially flat Friedmann Universe. Since these scalar fields are assumed to come from string field theory the string mass M_s and a dimensionless open string coupling constant g_o emerges. In typical cases phantom represents the open string tachyon and the usual scalar field corresponds to the closed string tachyon [4, 11, 12]. The action is

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2M_s^2}R + \frac{1}{g_o^2}\left(+\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g^{\mu\nu}\partial_\mu\xi\partial_\nu\xi - V(\phi,\xi)\right)\right), \qquad (2)$$

where M_P is the reduced Planck mass, $g_{\mu\nu}$ is a spatially flat Friedmann metric

$$ds^2 = -\,dt^2 + a^2(t)\left(dx_1^2 + dx_2^2 + dx_3^2\right).$$

The coordinates (t, x_i) and fields ϕ and ξ are dimensionless. Hereafter we use the dimensionless parameter m_p for short:

$$m_p^2 = \frac{g_o^2 M_P^2}{M_s^2}. \qquad (3)$$

If the scalar fields depend only on time then the equations of motion are as follows

$$H^2 = \frac{1}{3m_p^2}\left(-\frac{1}{2}\dot\phi^2 + \frac{1}{2}\dot\xi^2 + V\right), \qquad (4a)$$

$$\dot H = \frac{1}{2m_p^2}\left(\dot\phi^2 - \dot\xi^2\right), \qquad (4b)$$

$$\ddot\phi + 3H\dot\phi = \frac{\partial V}{\partial\phi}, \qquad (4c)$$

$$\ddot\xi + 3H\dot\xi = -\frac{\partial V}{\partial\xi}. \qquad (4d)$$

Here dot denotes the time derivative and $H \equiv \dot a(t)/a(t)$. Note that of four differential equations (4a)–(4d) only three are independent.

The DE state parameter can be expressed in terms of the Hubble parameter:

$$w_{DE} = -1 - \frac{2}{3}\frac{\dot H}{H^2}. \qquad (5)$$

The crossing of the cosmological constant barrier $w_{DE} = -1$ corresponds to change of sign of $\dot{H}$. The phantom like behavior corresponds to an increasing Hubble parameter. If we know the Hubble parameter as a function of time we can calculate the state parameter w_{DE} without knowledge of potential. It follows from eq. (4b) that if we know the explicit form of fields $\phi(t)$ and $\xi(t)$ and do not know the potential $V(\phi,\xi)$, we can obtain $H(t)$ with accuracy to a constant:

$$H(t) = \frac{1}{2m_p^2}\left(\int^t \dot{\phi}^2(\tau)d\tau - \int^t \dot{\xi}^2(\tau)d\tau\right) + C. \qquad (6)$$

The form of the potential is assumed to be given from string field theory within the level truncation scheme. Usually for a finite order truncation the potential is a polynomial and its particular form depends on the string type. Level truncated cubic OSFT fixes the form of the interaction of local fields to be a cubic polynomial with non-local form-factors. Integrating out low lying auxiliary fields one gets a 4-th degree polynomial [13, 14]. Higher order auxiliary fields may change the coefficients in front of lower terms and produce higher degree polynomials. The Aref'eva DE model [4] assumes that our Universe is a slowly decaying D3-brane and its dynamics is described by the open string tachyon mode and the back reaction of this brane is incorporated in the dynamics of the closed string tachyon. The open string tachyon dynamics is described within a level truncated open string field theory (OSFT). The notable feature of this OSFT description of the tachyon dynamics is a non-local polynomial interaction [10, 13, 14, 15, 16, 17]. It turns out the open string tachyon behavior is effectively described by a scalar field with a negative kinetic term [18].

The scalar field ξ comes from the closed string sector, similar to [19] and its effective local description is given by an ordinary kinetic term [11] and, generally speaking, a non-polynomial self-interaction [20]. An exact form of the open-closed tachyon interaction is not known and we consider the simplest polynomial interaction.

From the string theory we can also assume asymptotic conditions for solutions. To specify the boundary conditions for scalar fields let us recall that we have in mind the following picture. We assume that the phantom field ϕ smoothly rolls from the unstable perturbative vacuum ($\phi = 0$) to a nonperturbative one, say, $\phi = 1$ and stops there. The field ξ corresponds to close string and is expected to asymptotically go to zero in the infinite future. At the same time we can not calculate in the string theory framework coefficients of the potential and the explicit form of solutions. In this paper we show how using the superpotential method one can construct a polynomial potential and exact solutions, which satisfy conditions obtaining in the string theory framework.

Another interesting problem is to find a form of potential and solutions for the given Hubble parameter as a function of time. In this paper we construct toy two-fields model for the Hubble parameter proposed by I.Ya. Aref'eva and A.S. Koshelev [9]. This Hubble parameter corresponds to the DE state parameter, which crosses the cosmological constant barrier infinite number of times.

The existence of a superpotential puts restrictions on the form of the potential. For polynomial potentials these restrictions give relations among coefficients. In this polynomial case we can estimate the behavior of DE state parameter at large times.

3 The Method of Superpotential

We can assume that $H(t)$ is a function (named a superpotential [21]) of $\phi(t)$ and $\xi(t)$:

$$H(t) = W(\phi(t),\xi(t)).$$

This allows us to rewrite (4b) as

$$\frac{\partial W}{\partial \phi}\dot{\phi} + \frac{\partial W}{\partial \xi}\dot{\xi} = \frac{1}{2m_p^2}\left(\dot{\phi}^2 - \dot{\xi}^2\right). \tag{7}$$

System (4) is solved provided the relations

$$\frac{\partial W}{\partial \phi} = \frac{1}{2m_p^2}\dot{\phi}, \qquad \frac{\partial W}{\partial \xi} = -\frac{1}{2m_p^2}\dot{\xi} \tag{8}$$

are satisfied. If this is the case we have the following relation between the potential V and the superpotential W

$$V = 3m_p^2 W^2 + 2m_p^4\left(\left(\frac{\partial W}{\partial \phi}\right)^2 - \left(\frac{\partial W}{\partial \xi}\right)^2\right). \tag{9}$$

This relation gives the potential in terms of W and its first derivatives with respect to ϕ and ξ. Provided the superpotential is given to find a solution of the dynamical system one has to solve the second order system of ordinary differential equations (8).

There are a few ways to use the superpotential method. The standard way [21] is to construct the potential for the solutions given in the explicit form. Note that in distinct of the case of models with one scalar field in two-fields models we can choose a form of the potential $V(\phi, \xi)$. At the same time there exist potentials, which do not correspond to any superpotential. In two fields models the superpotential method gives possibility to find new solutions.

Another way to use the superpotential method is to construct potential and solutions using some properties of them. In particular we will try to find explicit form of solutions with given asymptotic conditions. The superpotential method can be useful also to find solutions and potential, which correspond to given behavior of the Hubble parameter.

The existence of a superpotential puts restrictions on the form of the potential. For polynomial potentials these restrictions give relations among coefficients. In this polynomial case we can estimate the behavior of DE state parameter at large times. We demonstrate that potential obtained by the superpotential method can be changed without changing of the given explicit solutions.

The superpotentials under consideration produce potentials which are rather close to the form of the open-closed tachyon potential for a non-BPS brane. Indeed, within the level truncated string field theory description of a non-BPS D3-brane decay both fields have tachyon mass terms and the interaction is the fourth order polynomial at the lowest levels. A natural deformation of this form of the open-closed string tachyon potential is given by extra sixth order terms. In a non-flat background there is a deformation of the effective local model describing the pure open sector of a non-BPS D3-brane such that the corresponding Friedmann equations have exact solutions [5]. A more straightforward generalization of the model [5] to the case of two fields gives a model with a kink-lump solution. We consider such solutions in the next section.

4 The construction of potential for quintessence and phantom late time behaviors

To demonstrate how we construct potential $V(\phi, \xi)$, which corresponds to the given explicit solutions, let us consider the following kink-lump solution:

$$\phi(t) = \tanh(t) \qquad \text{and} \qquad \xi(t) = \frac{\sqrt{2(1+b)/\omega}}{\cosh(\omega t)}. \tag{10}$$

where b and ω are arbitrary constants. From (6) we obtain

$$H(t) = \frac{1}{6m_p^2}\Big(3\tanh(t) - \tanh^3(t) - 2(1+b)\tanh^3(\omega t)\Big). \tag{11}$$

The corresponding DE state parameter w_{DE} is given by

$$w_{DE} = -1 - 4m_p^2\frac{3 - 3\tanh(t)^2 - 3\tanh(t)^2(1-\tanh(t)^2) - 6(1+b)\omega\tanh(\omega t)^2(1-\tanh(\omega t)^2)}{(3\tanh(t) - \tanh(t)^3 - 2(1+b)\tanh(\omega t)^3)^2}$$

The behavior of the Hubble parameter at large time depends on the parameter b. From the contemporary experimental data it follows that the present date Universe is expanding one that corresponds to $\lim\limits_{t=\infty} H > 0$. This condition is equivalent to $b < 0$. On the other hand, in the past there was eras of the accelerated and decelerated expanding Universe, it means that the Hubble parameter H has to be not a monotonic function and should has an extremum at some point $t_c > 0$.

Let $\omega = 1$, then from (11) we obtain that

$$t_c = \operatorname{arccosh}\left(\pm\frac{\sqrt{2(b+1)(2b+3)}}{2(b+1)}\right). \tag{12}$$

From the condition t_c is a real positive number we obtain the restriction $b > -1$. Eventually, we state that $-1 < b < 0$. For these values of b the Hubble parameter H has one extremum, namely a maximum. It is easy to show that at the large time $w_{DE} > -1$ for $\omega = 1$ and $b > -1$, so we obtain the quintessence like behavior of the Universe.

To obtain the suitable (nonmonotone) Hubble parameter H with the phantom like behavior at the large time ($w_{DE} < -1$) we should choose $\omega \neq 1$ and the corresponding value of b. In particulary, if $\omega = 2$ and $b = -0.01$ then $H(t)$ has extrema in the points $\tilde{t}_{c_1} = 0.3071060782$ and $\tilde{t}_{c_2} = 2.990691130$.

For $\omega = 2$ we obtain

$$\tilde{w}_{DE} \equiv w_{DE}|_{\omega=2} = -1 + 6\left(2\cosh^2(t) - 1\right)^2 \times$$
$$\times\frac{\left(16b\cosh^8(t) + 16(1-b)\cosh^6(t) - 24\cosh^4(t) + 8\cosh^2(t) - 1\right)}{\tanh^2(t)\left(16b\cosh^8(t) - 16b\cosh^6(t) - 4\cosh^2(t) + 1\right)^2}.$$

The DE state parameter $\tilde{w}_{DE}$ has a singularity in the origin and behaves as $-1/t^2$. At the points $\tilde{t}_{c_1}$ and $\tilde{t}_{c_2}$ its value crosses -1.

Using the condition $b < 0$, we find that at late time

$$\tilde{w}_{DE} \approx -1 + \frac{3}{2b\cosh^4(t)} < -1.$$

Thus, we have constructed both a phantom-like (w_{DE} goes to -1 from below) model and a quintessence-like (w_{DE} goes to -1 from above) model.

Let us construct potential, which corresponds to fields (10). The functions ϕ and ξ are solutions of the following differential equations:

$$\dot{\phi} = 1 - \phi^2, \qquad \dot{\xi} = \omega\xi\sqrt{1 - \frac{\xi^2}{B^2}}, \tag{13}$$

where we introduce new variable $B \equiv \sqrt{2(1+b)/\omega}$ for short.

So, we can put

$$\frac{\partial W}{\partial\phi} = \frac{1}{2m_p^2}\Big(1 - \phi^2\Big), \qquad \frac{\partial W}{\partial\xi} = -\frac{\omega\xi}{2m_p^2 B}\sqrt{B^2 - \xi^2} \tag{14}$$

and obtain:

$$H \equiv W = \frac{1}{2m_p^2}\left(\phi - \frac{1}{3}\phi^3 - \frac{\omega}{3B}\sqrt{(B^2-\xi^2)^3}\right) + C, \tag{15}$$

where C is an arbitrary constant. Different values of C correspond to different $V(\phi,\xi)$. In the case $C=0$ from (9) we obtain

$$V = \frac{1}{2}\left(1-\phi^2\right)^2 - \frac{\omega^2}{2B^2}(B^2-\xi^2)\xi^2 + \frac{3}{4m_p^2}\left(\phi - \frac{1}{3}\phi^3 - \frac{\omega}{3B}\sqrt{(B^2-\xi^2)^3}\right)^2. \tag{16}$$

In the case of one field if we know the (phantom) scalar field in the explicit form then we know the superpotential with accuracy to a constant. In the case of two fields we can construct essentially different form of superpotential. Moreover, it is significant that we can construct new potential with the same special solutions without construction of new superpotential. For example, if $\omega = 1$, then the functions (10) satisfy not only system (13) but also the following system of differential equations:

$$\dot{\phi} = b\left(\phi^2 - 1\right) + \frac{1}{2}\xi^2(t), \qquad \dot{\xi}(t) = -\phi(t)\xi(t). \tag{17}$$

The corresponding superpotential and potential are given by

$$\tilde{W} = -\frac{\phi}{6m_p^2}\Big(b\left(3-\phi^2\right) - \frac{3}{2}\xi^2\Big), \tag{18}$$

$$\tilde{V} = \frac{1}{2}\left(b\left(\phi^2-1\right) + \frac{1}{2}\xi^2\right)^2 - \frac{1}{2}\phi^2\xi^2 + \frac{\phi^2}{12m_p^2}\left(b\left(3-\phi^2\right) - \frac{3}{2}\xi^2\right)^2. \tag{19}$$

This example shows that the same functions $\phi(t)$, $\xi(t)$ (and consequently the Hubble parameter $H(t)$, state parameter w_{DE} and deceleration parameter $q(t)$) can correspond to different potentials $V(\phi,\xi)$. So, we conclude that one has a freedom to choose the potential, without changing solutions. Moreover, the solution is not violated if we add to the potential $\tilde{V}$ (or V with $\omega = 1$) a function δV, which is such that δV, $\partial(\delta V)/\partial\phi$ and $\partial(\delta V)/\partial\xi$ are zero on the solution. For example, we can add

$$\delta V = A(\xi,\phi)\left[\phi^2 + \frac{1}{B^2}\xi^2 - 1\right]^2, \tag{20}$$

where $A(\xi,\phi)$ is a smooth function. So, we can obtain potential, which corresponds to exact solutions, but does not correspond to any superpotential.

Note that the superpotential method does not allow to find all possible variants of polynomial potentials. Note that superpotential (15), which can be separated on two summand

$$W(\phi,\xi) = W_1(\phi) + W_2(\xi), \tag{21}$$

does not generate the potential

$$V(\phi,\xi) = V_1(\phi) + V_2(\xi), \tag{22}$$

Such potential can be constructed by the following algorithm:

- From equation (4b) we obtain $\dot{H}(t)$ and, therefore, $H(t)$.
- Substituting the obtained value of $H(t)$ in (4c) and (4d), we obtain $\frac{\partial V}{\partial \phi}$ and $\frac{\partial V}{\partial \xi}$ as functions of t.
- Using condition $\frac{\partial^2 V}{\partial\phi\partial\xi} = \frac{\partial^2 V}{\partial\xi\partial\phi}$, we guess the form of $V(\phi,\xi)$.

- From (4a) we define the constant term in the potential V.

Using this method we have found an another polynomial form of the potential, which corresponds to solutions (10) with $\omega = 1$:

$$\breve{V}(\phi,\xi) = \left(\frac{1}{4M_p^2} - 1\right)\xi^2\phi^2 + \frac{1}{2M_p^2}\phi^2\xi^4 + \frac{1}{2}\xi^2 - \frac{1}{4M_p^2}\xi^4 + \frac{1}{6M_p^2}\xi^6 + C. \qquad (23)$$

5 Construction of new solutions via superpotential method

In previous section we have shown how we can choose potential for given solutions. In this section we demonstrate the possibility to find new exact solutions (may be in quadratures) using superpotential method. Let us solve system (13). The general two-parameter solutions has the form:

$$\phi = \tanh(t - t_0), \qquad \text{and} \qquad \xi(t) = \frac{\sqrt{2(1+b)/\omega}}{\cosh(\omega(t - t_1))}, \qquad (24)$$

where t_0 and t_1 are arbitrary numbers. Note that t_0 and t_1 are complex numbers, therefore, solutions (24) include in particular the function $\phi = \coth(t)$. System (13) gives us only solutions, which are trivial generalization of the initial solutions (10). System (17) has solutions (10) with $\omega = 1$, the solution

$$\phi(t) = -\tanh(b(t - t_0)), \qquad \xi(t) = 0 \qquad (25)$$

and the following solution:

$$\phi(t) = -\frac{\dot{\xi}(t)}{\xi(t)}, \qquad (26)$$

and $\xi(t)$ is defined in quadratures

$$t - t_1 = \pm\int \frac{\sqrt{2}\xi^b(1+b)d\xi}{\xi((1+b)(2(\xi^b)^2 + 2b(\xi^b)^2 + (\xi^b)^2\xi^2 + 2C_1 + 2C_1 b))^{1/2}}, \qquad (27)$$

where C_1 and t_1 are arbitrary constants.

We can conclude that in case of two fields we start with some explicit solutions, construct the potential and obtain new solutions to equations of motion with this potential.

6 Construction of a string inspired potential and exact solutions

The form of the potential is assumed to be given from string field theory within the level truncation scheme. In the present analysis we impose the following restriction on the potential:

- the potential is the sixth degree polynomial:

$$V(\phi,\xi) = \sum_{k=0}^{6}\sum_{j=0}^{6-k} c_{kj}\phi^k\xi^j, \qquad (28)$$

- coefficient in front of 5-th and 6-th powers are of order $1/m_p^2$ and the limit $m_p^2 \to \infty$ gives a nontrivial 4-th degree potential.
- the potential is even: $V(\phi,\xi) = V(-\phi,-\xi)$. It means that if $k+j$ is odd, then $c_{kj} = 0$.
- the function $\phi(t)$ has non-zero asymptotic and $\xi(t)$ has zero asymptotic as $t \to +\infty$.

We also assume that there exists a polynomial superpotential $W(\phi,\xi)$, which determines the potential $V(\phi,\xi)$ by formula (9).

To construct the sixth degree polynomial potential $V(\phi,\xi)$ we should choose $W(\phi,\xi)$ as a third degree polynomial. If all coefficients of $W(\phi,\xi)$ are proportional to $1/m_p^2$, then we obtain that solution does not depend on m_p^2, and coefficient in front of 5-th and 6-th powers are proportional to $1/m_p^2$. To obtain an even potential $V(\phi,\xi)$ we should use an odd superpotential $W(\phi,\xi)$. So, the suitable form of superpotential is as follows:

$$W(\phi,\xi)=\frac{1}{2m_p^2}\left(a_{1,0}\phi+a_{3,0}\phi^3+a_{0,1}\xi+a_{0,3}\xi^3+a_{2,1}\phi^2\xi+a_{1,2}\phi\xi^2\right), \tag{29}$$

where $a_{i,j}$ are constants, which do not depend on m_p^2. The corresponding system of differential equations (8) is as follows:

$$\begin{aligned}\dot\phi &= a_{1,0}+3a_{3,0}\phi^2+2a_{2,1}\phi\xi+a_{1,2}\xi^2,\\ \dot\xi &= -a_{0,1}-3a_{0,3}\xi^2-a_{2,1}\phi^2-2a_{1,2}\phi\xi.\end{aligned} \tag{30}$$

Using asymptotic conditions: $\phi(+\infty)=1$, $\xi(+\infty)=0$, $\dot\phi(+\infty)=\dot\xi(+\infty)=0$ we obtain

$$a_{1,0}=-3a_{3,0}, \qquad a_{0,1}=-a_{2,1}. \tag{31}$$

So, we obtain the following system of equations:

$$\begin{aligned}\dot\phi &= -3a_{3,0}+3a_{3,0}\phi^2+2a_{2,1}\phi\xi+a_{1,2}\xi^2,\\ \dot\xi &= a_{2,1}-3a_{0,3}\xi^2-a_{2,1}\phi^2-2a_{1,2}\phi\xi.\end{aligned} \tag{32}$$

Using (32) we can express $\xi(t)$ via $\phi(t)$ and its derivatives:

$$\xi=\frac{1}{2a_{1,2}(3a_{0,3}-a_{2,1})\dot\phi-2a_{2,1}a_{1,2}^2+18a_{0,3}a_{3,0}a_{1,2}+2(a_{0,3}(6a_{2,1}^2-9a_{1,2}a_{3,0})-a_{2,1}a_{1,2}^2)\phi^2}\times$$
$$\times\left[(6a_{1,2}a_{3,0}+6a_{0,3}a_{2,1}-4a_{1,2}^2)\phi\dot\phi-a_{1,2}\ddot\phi+(18a_{0,3}a_{3,0}a_{2,1}+2a_{2,1}^2a_{1,2}-12a_{1,2}^2a_{3,0})\phi-\right.$$
$$\left.-(2a_{1,2}a_{2,1}^2-18a_{0,3}a_{2,1}a_{3,0}+12a_{1,2}^2a_{3,0})\phi^3\right].$$

At the same time we can not solve the obtained differential equation for $\phi(t)$ without additional assumptions. Really we should fix one of four arbitrary coefficients of the superpotential. The simplest way to do this is to assume explicit form of some combination of functions ϕ and ξ. For example, let us assume that

$$\phi(t)+s\xi(t)=\tanh(\omega t), \tag{33}$$

where s and ω are constants. From (33) we obtain

$$\dot\phi(t)+s\dot\xi(t)=\omega\left(1-(\phi(t)+s\xi(t))^2\right). \tag{34}$$

It gives the following restrictions on the coefficients $a_{i,j}$:

$$a_{3,0}=\frac{3a_{2,1}a_{0,3}+a_{2,1}^2-a_{1,2}^2}{3a_{1,2}}. \tag{35}$$

Parameters s and ω are determined by coefficients of W:

$$\omega=a_{1,2}\frac{3a_{0,3}-a_{2,1}\pm\sqrt{9a_{0,3}^2+6a_{2,1}a_{0,3}+a_{2,1}^2-4a_{1,2}^2}}{3a_{0,3}+a_{2,1}\mp\sqrt{9a_{0,3}^2+6a_{2,1}a_{0,3}+a_{2,1}^2-4a_{1,2}^2}}, \tag{36}$$

$$s = \frac{3a_{0,3} + a_{2,1} \pm \sqrt{9a_{0,3}^2 + 6a_{2,1}a_{0,3} + a_{2,1}^2 - 4a_{1,2}^2}}{2a_{1,2}}. \tag{37}$$

Choosing s one can find exact solutions and analyse the corresponding cosmological consequences. The case $s = 1$ has been considered in [22]. In this case

$$a_{3,0} = \frac{2}{3}a_{2,1} - \frac{1}{3}a_{1,2}, \qquad a_{0,3} = -\frac{1}{3}a_{2,1} + \frac{2}{3}a_{1,2}, \qquad \omega = a_{1,2} - a_{2,1}, \tag{38}$$

and system (33) has the following solutions:

$$\xi(t) = (a_{2,1}t - C_1)\left(\tanh(\omega(t-t_0))^2 - 1\right), \tag{39}$$

$$\phi = \tanh(\omega(t-t_0)) - (a_{2,1}t - C_1)\left(\tanh(\omega(t-t_0))^2 - 1\right). \tag{40}$$

The behavior of the solution depends on the particular values of parameters $a_{2,1}$, t_0 and C_1. It is possible to find particular values of the parameters for which the Hubble parameters H has two extrema and $w_{DE} < -1$ at late time [22].

For $s = 2$ and $s = 3$ we obtain the following solutions:

$$\xi_2(t) = \frac{4(e^{2(2a_{2,1}-a_{1,2})t} - C_1 e^{(a_{2,1}-2a_{1,2})t})}{3(C_1 + e^{2(2a_{2,1}-a_{1,2})t})(e^{(a_{2,1}-2a_{1,2})t} + 1)}. \tag{41}$$

$$\xi_3(t) = \frac{3(-e^{2(a_{1,2}-3a_{2,1})t} + C_1 e^{2(3a_{1,2}-a_{2,1})t/3})}{4(C_1 + e^{2(a_{1,2}-3a_{2,1})t})(e^{2(3a_{1,2}-a_{2,1})t/3} + 1)}. \tag{42}$$

We can conclude that the superpotential method allows to find exact solutions with the given asymptotic properties which correspond to polynomial potential.

7 Two-fields Model with infinity number of the cosmological barrier crossing

From the Cubic Superstring Field Theory I.Ya. Aref'eva and A.S. Koshelev [9] have obtained the model with the following Hubble parameter:

$$H = H_0 + C_H e^{-2rt}\sin(2\nu(t - t_0)), \tag{43}$$

where H_0, C_H, r, ν and t_0 are real constants. Let us construct the two-fields model with the Hubble parameter H in the case $\nu = r$. To consider solutions, which do not depend on m_p^2, we put $C_H = 1/(2m_p^2)$. In this case

$$\dot{H} = -\frac{2r}{2m_p^2}e^{-2rt}(\sin(2rt) - \cos(2rt)) = \frac{r}{m_p^2}e^{-2rt}\left(2\sin(rt)^2 - (\sin(rt) - \cos(rt))^2\right). \tag{44}$$

Using (4b) we can define the following explicit form of solutions:

$$\dot{\phi} = -2\sqrt{r}e^{-rt}\sin(rt), \qquad \dot{\xi} = \sqrt{2r}e^{-rt}(\sin(rt) - \cos(rt)). \tag{45}$$

So,

$$\phi = \frac{1}{\sqrt{r}}e^{-rt}\big(\cos(rt) + \sin(rt)\big), \qquad \xi = -\frac{\sqrt{2}}{\sqrt{r}}e^{-rt}\sin(rt). \tag{46}$$

It is easy to check that

$$\dot{\phi} = \sqrt{2}r\xi, \qquad \dot{\xi} = -\sqrt{2}r\phi + 2r\xi. \tag{47}$$

Let construct the superpotential:

$$\frac{\partial W}{\partial \phi}=\frac{1}{2m_p^2}\sqrt{2}r\xi, \qquad \frac{\partial W}{\partial \xi}=\frac{1}{2m_p^2}\left(\sqrt{2}r\phi-2r\xi\right), \tag{48}$$

so,

$$W=\frac{1}{2m_p^2}\left(\sqrt{2}r\xi\phi-r\xi^2\right)+H_0. \tag{49}$$

And the potential is ($H_0=0$)

$$V=-r^2\left(\xi^2+\phi^2-2\sqrt{2}\phi\xi\right)+\frac{3r^2}{4m_p^2}\left(\sqrt{2}\xi\phi-\xi^2\right)^2. \tag{50}$$

So, we obtain the explicit solutions and polynomial potential, which correspond to the Hubble parameter from the string inspired model with high derivatives.

8 Conclusions

In this paper we have investigated the dynamics of two component DE models, with one phantom field and one usual field. The main motivation for us was a model of the Universe as a slowly decaying D3-brane whose dynamics is described by a tachyon field [4]. To take into account the back reaction of gravity we consider one more scalar field. This scalar field has a usual kinetic term.

Within two component DE models with a general class of interactions we have found conditions that show whether the model is a phantom-like (w_{DE} goes to -1 from below), or it is a quintessence-like (w_{DE} goes to -1 from above). In particular, for the simplest model inspired by a D3-brane we have found that an inclusion of the closed string tachyon drastically changes the late time regime so for the two-component model we have $w_{DE}>-1$ at large time, while in the open string case one has $w_{DE}<-1$.

We have also constructed the two fields model with $w_{DE}<-1$ at late times. We have presented the explicit solution implementing this possibility: $\phi=\tanh(t)$ and $\xi\sim 1/\cosh(2t)$. The corresponding potential can be separated on the forth degree polynomial and gravitational correction which is not a polynomial. In other models gravitational correction is the sixth degree polynomial, the explicit form of open string tachyon is a type of kink, but more complex then $\tanh(t)$. It would be very interesting to find such the lump type field ξ that $w_{DE}<-1$ at late times, $\phi=\tanh(t)$ and the corresponding potential is an even polynomial.

We construct two-fields model with the fourth degree polynomial potential, which corresponds to the Hubble parameter, obtained in the string field theory framework [9].

In this paper we actively have used the superpotential method and have shown that there are new variants to use this method in the case of two fields. We can not only to construct potential for the given solutions, but also to find new solutions.

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