Astronomy meets QCD: cooling constraints for the theories of internal structure of compact objects

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Abstract

We discuss a set of tests which confront observations of cooling compact objects and theories of their thermal evolution. As an example we apply the recently developed Log N-Log S test of compact star cooling theories to hybrid stars with a color superconducting quark matter core, we also apply and discuss other existing tests. While there is not yet a microscopically founded superconducting quark matter phase which would fulfill constraints from cooling phenomenology, we explore the hypothetical 2SC+X phase and show that the magnitude and density-dependence of the X-gap can be chosen to satisfy a set of tests: the temperature – age (T-t) test, the Log N-Log S test, the brightness constraint, and the mass spectrum constraint. Some recent modifications of the population synthesis model used to obtain the Log N-Log S distribution are briefly discussed. In addition, we propose to use the age-distance diagram as a new tool to study the local population of young isolated neutron stars.

1 Introduction

Compact objects known under the common name *neutron stars* belong to the most facinating places in the Universe as the interest of different disciplines of science is focussed on them. In the present contribution we will elucidate how astronomical observations of cooling neutron stars (NSs) provide a tool to study the behaviour of matter at high density. Here astronomy meets QCD.

Sometimes the term *neutron star* is used just as a common name for a large class of objects invented by the fantasy of theorists. They can be "normal" NSs, i.e. compact objects made mainly of hadrons even at the center – *hadron stars*. But there are other interesting possibilities. Central parts of NSs could contain more exotic forms of dense matter like a pion condensate

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or hyperonic matter. Finally, as a result of the deconfinement transition hadrons get dissolved into their quark constituents due to the Pauli principle: compact stars develop a quark matter core. In this case the NS is called a *hybrid star*. Eventually, the whole body (maybe except a tiny envelope) is converted into quark matter. In this case we speak of a *quark star*. So, a large variety of interesting possibilities appears. One of the very few ways to test which particular scenario for the internal structure is realized in Nature is to study the thermal evolution of compact objects. This paper is devoted to the problem of testing theories of NS cooling by confronting results of computations with observational data.

This contribution is mostly based on the paper [1]. Still, some new results and discussions are given. At first, we describe different tests of cooling curves. Then we briefly describe the model for the cooling of hybrid stars to which we intend to apply the tests. In some details we dwell on the population synthesis scenario used here to produce the Log N-Log S distributions. After this we apply the test to a set of cooling curves of hybrid stars. Finally, we discuss further improvements in the scenario and present our conclusions.

2 Tests of cooling curves

Testing the behaviour of matter in different regions of the QCD phase diagram is an extremely important but difficult task. The region corresponding to high density, but low temperature is not accessible to direct studies in terrestrial laboratory experiments. Therefore, astronomical data on the mass-radius relation of NSs, on the gravitational redshift of spectral lines from these objects, etc., can be used together with available data from ground-based experiments to get some constraints on the properties of dense matter [2, 3]. Among astronomical observations those of cooling NSs are most important for studies about physical processes at the deconfinement phase transition in the QCD phase diagram. The idea is to compare calculated cooling curves with data obtained from astronomical observations. In this section we discuss different approaches to do this.

2.1 Age vs. temperature

The most common test is the following. One just selects sources with relatively well known ages and temperatures and confronts data points with theoretical cooling curves, see Fig. 1. Naively it is assumed that if all data points can be covered by cooling tracks then the model is considered being in correspondence with observations.

The main advantages of this test from the point of view of its use by the community are the following two:

1. It is clear and direct.
2. Everybody who calculate the theoretical curves can do it as observational data are available in the literature.

The test is widely used and was very well described many times (see, for example, [4, 5] and references therein). So, we do not give many details. Let us just specify few disadvantages which can be overcome if one uses additional tests and considerations.

A. Well determined temperatures and, especially, ages are known for very few objects. So, statistics is not very large.

B. Even if age and temperature estimates are available, they have significant uncertainties, or they depend on the chosen model.

C. Objects with known temperature and ages form a very non-uniform sample, as they were discovered by different methods with different instruments. Different selection effects are in the game.

D. Mostly, the objects used in the test are younger than 10^5 years.

E. There are some additional pieces of data which are not used in the analysis (like the knowledge about mass distribution, non-detection of some kinds of sources, etc.).

In the following subsections we focus on several additional methods which can help to improve the situation when confronting theory with observations.

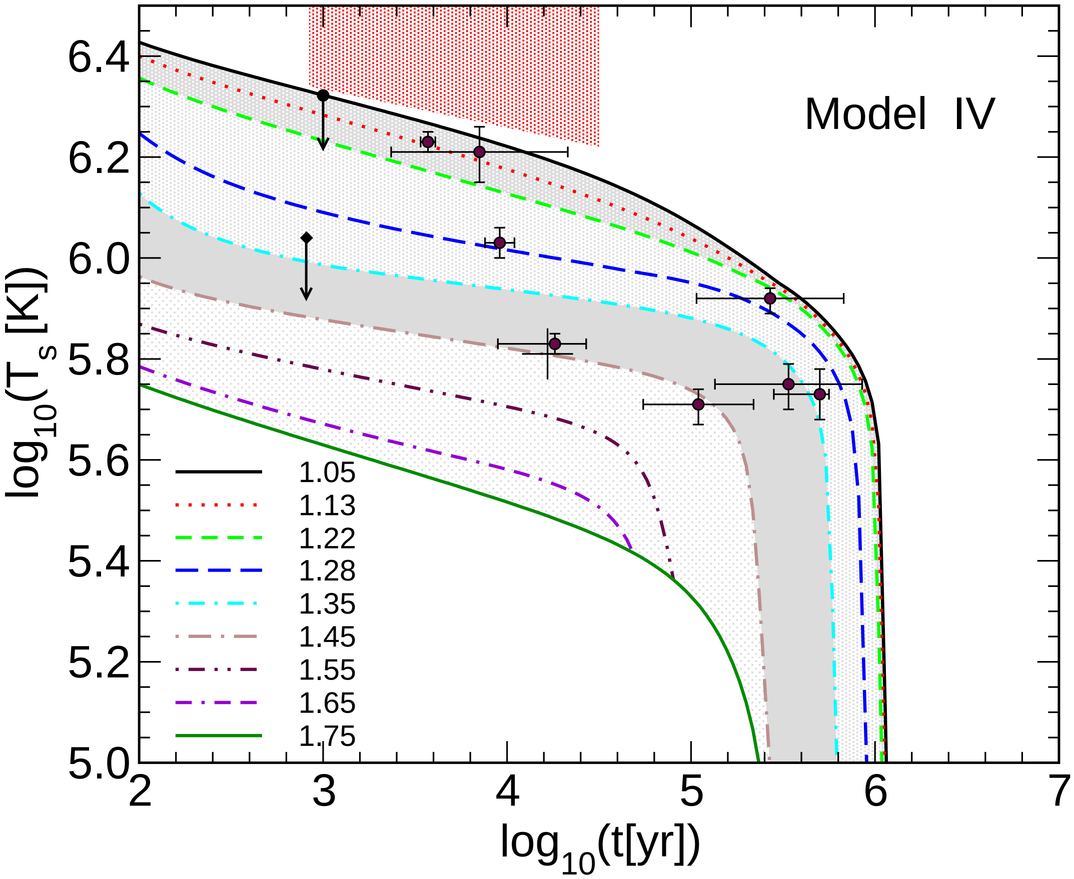

Figure 1: Hybrid star cooling curves for Model IV from [1]. Different lines correspond to compact star mass values indicated in the legend (in units of $M_\odot$), data points with error bars are taken from [4]. The shaded areas corresponds to different bins in the mass spectrum shown in Fig. 2.

2.2 Log N – Log S

The Log N – Log S diagram is a useful tool in astrophysics. Here N represents the number of sources with observed fluxes (at some energy range) larger than S. So, this is an integral distribution, i.e. it always grows towards lower fluxes.

This tool was used several times to study properties of the isolated NSs population. In [6] we proposed to use the Log N – Log S diagram as an additional test of cooling curves of NSs. The idea is to compare the observed Log N – Log S distribution with the calculated in the framework of population synthesis approach (see below), and to extract from this comparison a statement whether the model fits the data.

Our reasoning in favour of the new test is the following:

1. Thanks to the observations made onboard the ROSAT X-ray satellite we have a relatively uniform sample of NSs with detected thermal radiation.

2. The test doesn't require the knowledge of ages, temperature, etc. Only fluxes (which are well determined) and numbers are necessary to use this test.

3. The test is sensitive to older (~ 1 Myr) sources.

4. Most of ingredients of the population synthesis scenario except the cooling curves can be fixed relatively well.

In [6] we were able to demonstrate that the tool really works well as it puts additional constraints to the standard temperature vs. age test.

One of the main disadvantages of the test is that one needs to have a computer code to test a set of cooling curves. The way out can be to develope a web-tool where everyone can download cooling curves and obtain the Log N – Log S distribution for selected parameters of the scenario. We hope to provide such a resource in future. Another disadvantage is related to the precision of a population synthesis model. Not all ingredients are equally well known, and a big piece of astrophysical work has to be done to produce a good model. However, we believe that for young objects in the solar vicinity this problem can be solved, and in our models we are on the right way to do it.

2.3 Brightness constraint

Still, it is interesting to see how the temperature vs. age (T – t) test can be modified, and if it can help to select models successfully without the Log N – Log S test. Such a modification can be very helpful as in order to use the latter test it is necessary to apply some complicated computer code which is not publicly available at the moment. In this and in the following subsections we discuss two possible additional tests.

In [7] it was proposed to take into account the fact that despite many observational efforts very hot NSs (log $T > 6.3$ - 6.4 K) with ages $\sim 10^3 - 10^4$ yrs were not discovered. If they would exist in the Galaxy, then it would be very easy to find them (unless the interstellar absorption prevents us to see a source, but absorption is not equally important in all directions: so there are relatively wide "windows" to observe a significant part of the Galaxy). If we do not see any very hot NSs, then we have to conclude that at least their fraction is very small. This means, that any model pretending to be realistic should not produce NSs with realistic masses with temperatures higher than the observed ones. The region of avoidance can be found in the Fig. 1: this is the hatched trapezoidal area on the top of all curves.

This constraint is very sensitive to the properties of the crust of a NS (see [7] for details). Fitting the crust one can usually find a solution to satisfy the brightness constraint. So, this technique is an useful addition to the standard temperature vs. age test. The usage of only T – t plus the brightness constraint method can lead to a wrong solution as both are not very sensitive to the behaviour of the cooling curves for ages larger $\sim$ few $\times 10^5$ years, and just fitting the crust can result in a solution which can be proven wrong on the basis of the Log N – Log S test when properties of the internal parts of a NS are not properly selected. So, it is important to remember that the Log N – Log S test is not very sensitive to the crust properties. We conclude, that anyway the Log N – Log S test should be used, too, because such complex approach helps to make a more complete testing of cooling curves.

2.4 Mass spectrum constraint

The mass spectrum of NSs is an important ingredient of the population synthesis scenario. Normally, if we consider masses in the range $1\,M_\odot < M < 2\,M_\odot$, lighter stars cool slower. Our estimates of the mass spectrum [8, 6, 1] show that the fraction of NSs with masses larger than $\sim 1.4\,M_\odot$ is very small. This means that more massive objects should not be used to explain observations, especially if we speak about bright or/and typical sources. In particular, close-by young NSs, like Vela, should not be explained as massive stars as this is very improbable that we are lucky to have such a young object (age $\sim$ 10 000 years) so close. As we show in [1] this simple constraint helps to close some models which can successfully pass T-t or/and Log N – Log S tests.

In Fig. 2 darker bins corresponds to more abundant NSs. The same colors are used in Fig. 1. So, ideally, on the T–t plot most of data points should fall inside darker regions. As one can see in Fig. 1, for Model IV this is the case.

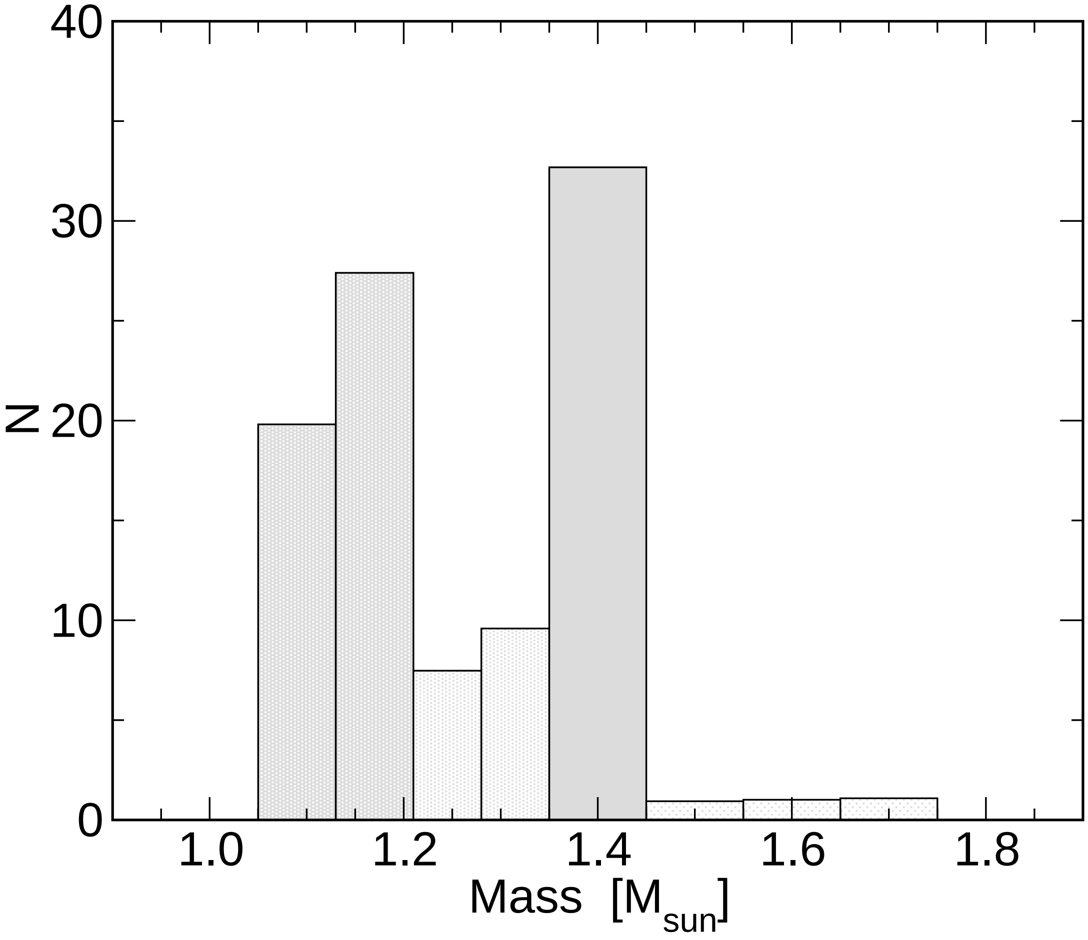

Figure 2: The adopted mass spectrum, binned over eight intervals of different widths. The grey tones for the mass bins encode their abundance and correspond to those used in Fig. 1. Darker bins corresponds to more abundant NSs. On the vertical axis the percentage of NSs in each bin is given.

3 Cooling model for hybrid stars

The details of calculations of the cooling curves used here can be found in [9]. We just briefly outline the main issues.

In the hadronic part of a hybrid star the main processes are the medium modified Urca and the pair breaking and formation processes for our adopted equation of state of hadronic matter. The possibilities of pion condensation and of other so called exotic processes in hadronic matter may be disregarded since these processes have threshold densities at or above the critical density for the occurence of quark matter. For the calculation of the cooling of the quark core in the hybrid star we incorporate the most efficient processes: the quark direct Urca processes on unpaired quarks, the quark modified Urca, the quark bremsstrahlung, the electron bremsstrahlung, and the massive gluon-photon decay. We include the emissivity of the quark pair formation and breaking processes. The specific heat incorporates the quark contribution, the electron contribution and the massless and massive gluon-photon contributions. The heat conductivity contains quark, electron and gluon terms.

The 2SC phase has one unpaired color of quarks (say blue) for which the very effective quark direct Urca process works and leads to a too fast cooling of the hybrid star in disagreement with the data. We have suggested to assume a weak pairing channel which could lead to a small residual pairing of the hitherto unpaired blue quarks. We call the resulting gap Δ_X and show that for a density dependent ansatz:

$$\Delta_{\rm X} = \Delta_0 \exp\left[-\alpha\left(\frac{\mu-\mu_c}{\mu_c}\right)\right] \tag{1}$$

with μ being the quark chemical potential, $\mu_c = 330$ MeV. Here we use different values of α and Δ_0.

The physical origin of the X-gap remains to be identified. It could occur, e.g., due to quantum fluctuations of color neutral quark sextett complexes. Such calculations have not yet been performed with the relativistic chiral quark models.

4 Population synthesis model

Population synthesis is a frequently used technique in astrophysics. It is described, for example, in the review [10] where further references can be found. The idea is to construct an evolutionary scenario for an artificial population of some astronomical objects. The comparison with observations gives the opportunity to test our understanding of evolutionary laws and initial conditions for these sources.

We use the advanced version of the population synthesis scenario introduced in [11]. The main ingredients of the population synthesis model we use are:

- the initial distribution of NSs and their birth rate;
- the velocity distribution of NSs;
- the mass spectrum of NSs;
- cooling curves;
- interstellar absorption;
- properties of the detector.

We assume that NSs are born in the Galactic disc and in the Gould Belt. The disc region is calculated up to 3 kpc from the Sun, and is assumed to be of zero thickness. The birth rate in the disc part of the distribution is taken as 250 NS per Myr. The Gould Belt is modeled as a flat disc-like structure with a hole in the center. The inclination of the Belt relative to the galactic plane is 18°. The NS birth rate in the Belt is 20 per Myr.

The population synthesis code calculates spatial trajectories of NSs with the time step 10^4 yrs. For each point from the set of cooling curves we have the surface temperature of the

NS. Calculations for an individual track are stopped at the age when the hottest NSs reaches the temperature 10^5 K.

As we known the surface temperature, we can calculate the radiation flux in the energy band of interest. In principle, it is possible to take into account atmospheric properties of NSs, as even a tiny atmosphere can influence the spectrum of an object. However, there are many unsolved issues here. A large variety of atmospheric models is discussed, and it is impossible to make a choice in favor of one particular model. On the other hand, for many sources a blackbody fit gives very good results, and so all our calculations are done for pure blackbody spectra.

With the known distance from the Sun and the ISM distribution we calculate the column density, which is necessary to derive absorption as soft X-rays fluxes are significantly absorbed. Finally, on the base of known unabsorbed flux and column density count rates are calculated using the ROSAT response matrix. Results are summarized along each individual trajectory. We calculate 5,000 tracks for each model. The results are then normalized to the chosen NS formation rate (270 newborn NSs per Myr in the whole region of the considered problem).

5 Results of tests when applied to cooling curves of hybrid stars

In this section we present our results for the best model (IV) studied in [1]. The calculated Log N – Log S distributions for different values of parameters are shown in Fig. 3. We used two different values for the size of the Gould Belt (300 and 500 pc), and used two variants of the mass spectrum: the full one and truncated. In Fig. 2 the full spectrum is shown. For the truncated one the contributions of the first two mass bins are added to the third one. The reason for such a modification is related to the fact that it is not clear whether a significant amount of light NSs is formed [12].

This model succesfully passes all four test (T-t, Log N – Log S, mass constraint, brightness constraint). Other three models studied in [1] failed to pass one (or more) of the four tests.

6 Modifications of the population synthesis scenario

Though, the main parameters of the population synthesis scenario for close-by young NSs are known, and we demonstrated that the scenario itself works well, we continue to upgrade the code. Below we briefly discuss the main recent updates made in the model. Some details can also be found in [14].

6.1 Initial spatial distribution

The first thing that can be improved in the model is the initial spatial distribution of NSs, or, better say, distribution of progenitors. Instead of using flat distributions in the galactic disc and in the Belt one can use a realistic distribution of massive stars in the solar vicinity.

In our upgraded model we construct the distribution of progenitors from three parts. Inside 400 pc around the Sun we use actual data on the massive stars distribution obtained by the Hipparcos satellite. 25 out of 270 NS per Myr are assumed to come from this population. Outside this region of up to 3 kpc we use the data on associations of massive stars. Data on 36 most populated associations is used [14]. 245 out of 270 NS per Myr are assumed to come from associations. The remaining 20 NS per Myr are randomly distributed in the plane of the galactic disc between 0.4 and 3 kpc from the Sun, this population represents “field stars” and stars from associations not included explicitly into our model.

Comparison between old and new approaches can be seen in Fig. 4. Three Log N –Log S distributions are calculated for the same cooling curves (Model IV). Two of them are for the

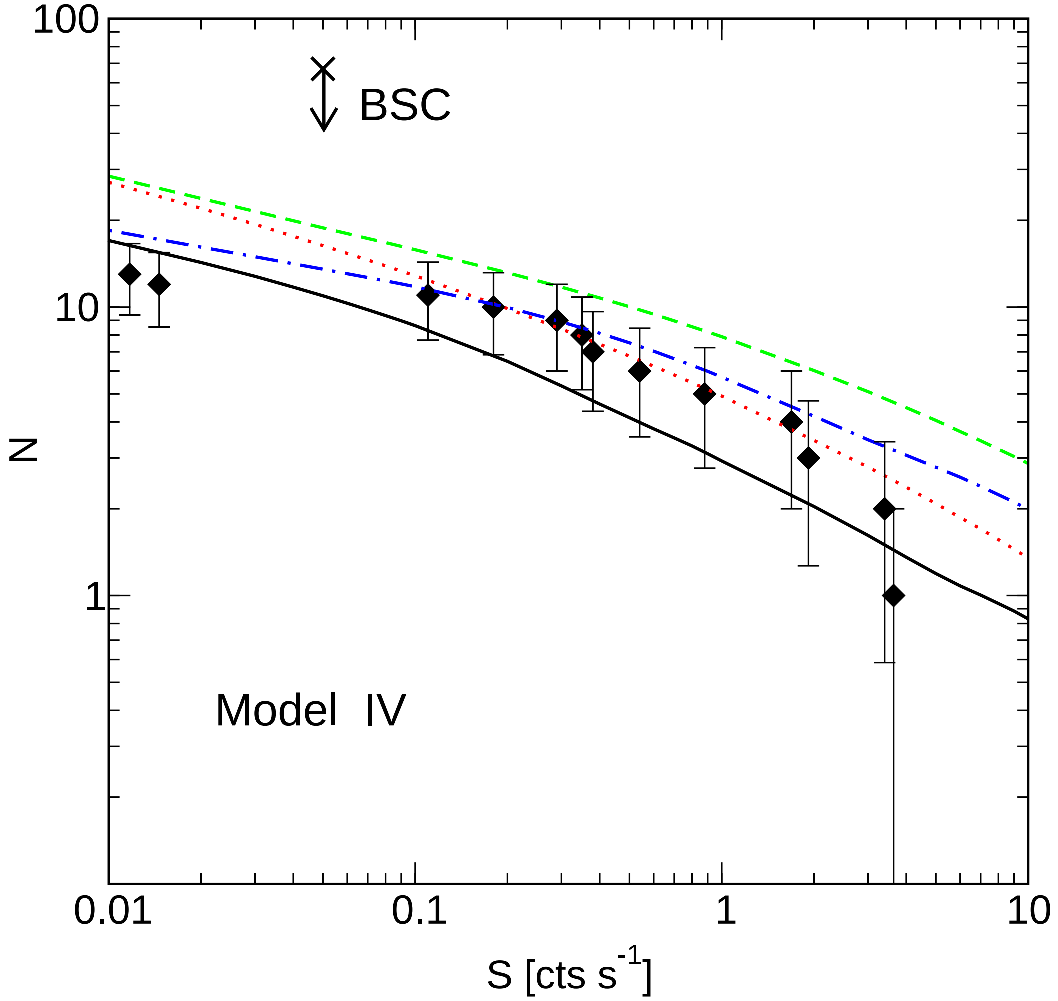

Figure 3: Log N-Log S distribution for Model IV. Four variants are shown: $R_{\rm belt} = 500$ pc and truncated mass spectrum (full line), $R_{\rm belt} = 500$ pc and non-truncated mass spectrum (dotted line), $R_{\rm belt} = 300$ pc and truncated mass spectrum (dash-dotted line), and finally $R_{\rm belt} = 300$ pc (dashed line) for non-truncated mass distribution. BSC corresponds to the limit obtained in [13].

old simple model for two different Belt radii, and the third one is calculated for the new initial distribution model.

6.2 Mass spectrum modifications

The mass spectrum of newborn NSs is not well known. A few observed sources which can potentially help to solve this puzzle are binary radio pulsars, but it is not clear if the mass spectrum for these objects can be applied for the population of isolated (and often radio quiet) NSs. So, we can just try to make a model of a spectrum which is based on available data on progenitors properties and on calculations of stellar evolution and supernova explosions. As computed stellar and supernova models are not very precise one have to try to apply different results of such calculations. In addition to the "old" mass spectrum shown in Fig. 2 we tried a modification. The main difference with the "old" one is that for progenitors with initial masses $> 12\,M_\odot$ we use the relation between progenitor mass and NS mass taken from [15]. The full "new" mass spectrum is shown in Fig. 5. In Fig. 6 we compare Log N – Log S curves for

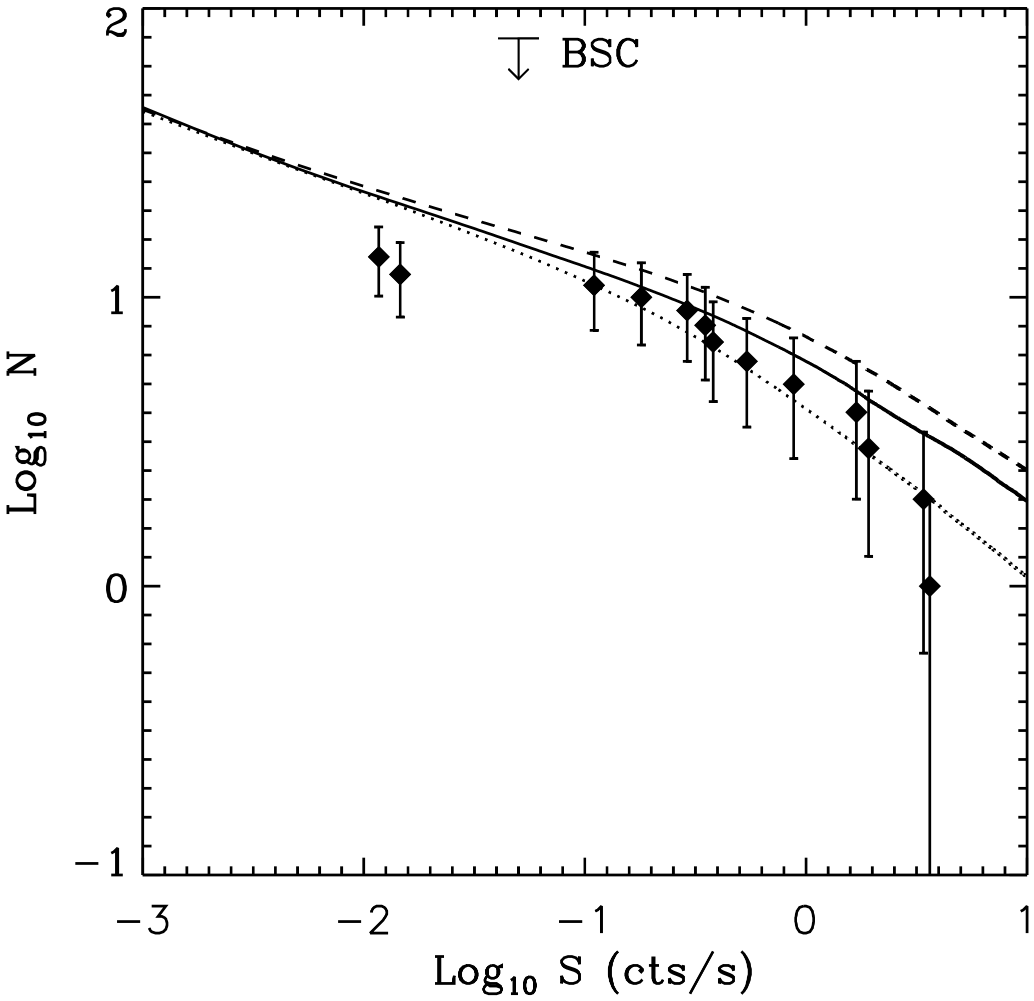

Figure 4: Three curves correspond to different initial distributions. The new distribution (solid curve), and older simple model for two different Belt radii (dashed and dotted curves). The upper curve is for the Belt radius 300 pc, the lower – for 500 pc. BSC corresponds to the limit obtained in [13]. All curves are plotted for "old" (Fig. 2) truncated mass spectrum.

different mass spectra. For an illustration we add a curve for the flat mass spectrum used in [11]. According to this spectrum all masses are equally probable. All these calculations are done for the new initial spatial distribution of progenitors.

Surprisingly, the new mass spectrum (Fig. 5) does not lead to serious modifications of the Log N – Log S distribution. Still, one has to be warned that further investigations of the mass spectrum of newborn isolated NSs are necessary. In particular, it is important to understand how mass can be correlated with other initial parameters of NSs.

6.3 Further improvements

Several other important improvement were made recently in our population synthesis model. In particular, a realistic distribution of the interstellar medium was taken into account. The procedure to calculate the count rate for the ROSAT devices was updated. Finally, we added the possibility to compute the Log N – Log S distribution for the XMM-Newton satellite. These changes will be described elsewhere.

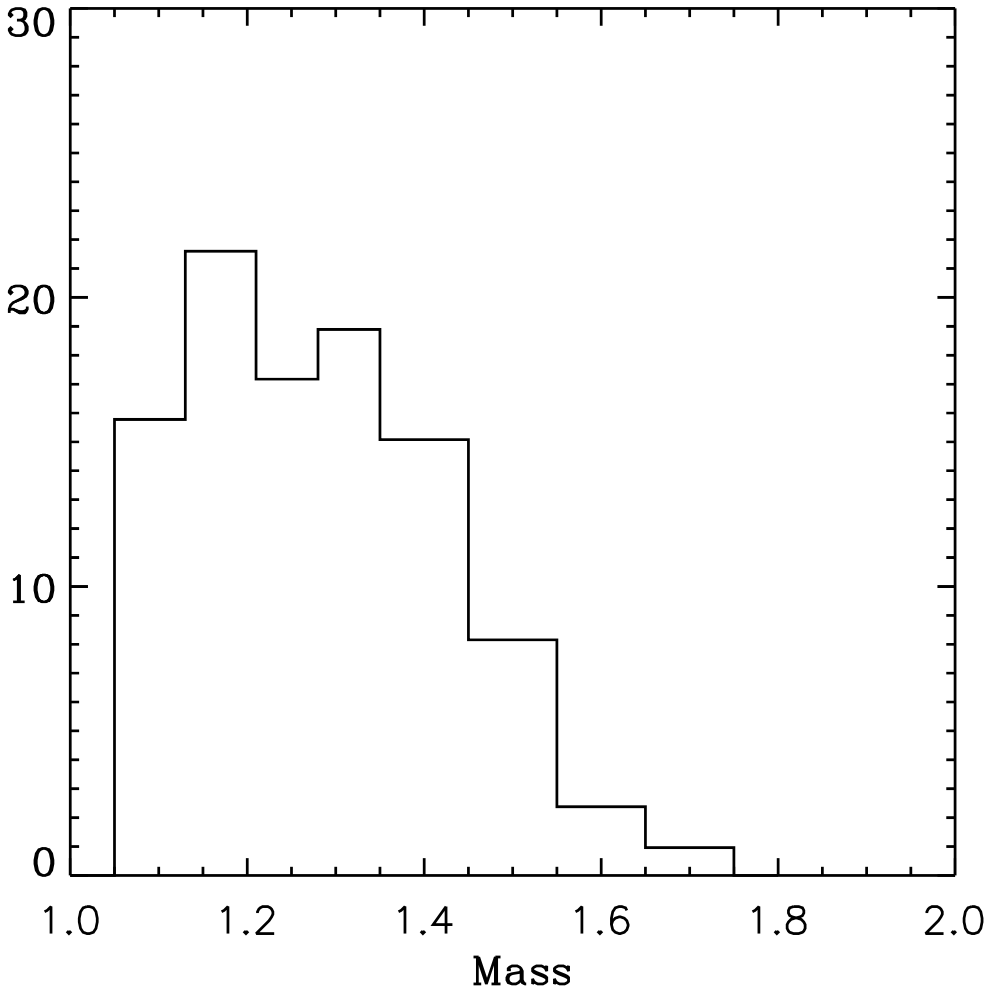

Figure 5: The "new" mass spectrum, binned over eight intervals of different widths. Gravitational is given mass in solar masses.

6.4 Age-Distance diagram

In [16] we introduced the age-distance diagram (ADD) for close-by young isolated NSs. This diagram, as it is clear from its name, represent a plot, where each point corresponds to a star with known age and distance. In the Fig. 7 we show an example from [16].

Five solid lines in Fig. 7 are plotted for 1, 4 13, 20 and 100 sources. Black and empty symbols represents known NSs (of course, as not for all of them ages are known, just a part of the population can be plotted in the graph). The dotted line represents the "visibility" line. This line shows a maximal distance for a given age (or vice versa a maximal age for a given distance) at which a hot (i.e. low-mass) NS can be detected. Here the line is plotted for a very simple model. We just assume some limiting value of unabsorbed flux: 10^{-12} erg cm^{-2} s^{-1}. According to WebPIMMS[1] it corresponds to ~ 0.01 ROSAT PSPC counts per second for $N_H = 10^{21}$ cm^{-2} and a blackbody spectrum with $T = 90$ eV, or to ~ 0.1 ROSAT PSPC counts per second for $N_H = 10^{20}$ cm^{-2} and a blackbody spectrum with $T = 50$ eV. The latter values corresponds to the dimmest source among the Magnificent seven – RX J0420.0-5022; the former to possibly

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[1] http://heasarc.gsfc.nasa.gov/docs/corp/tools.html

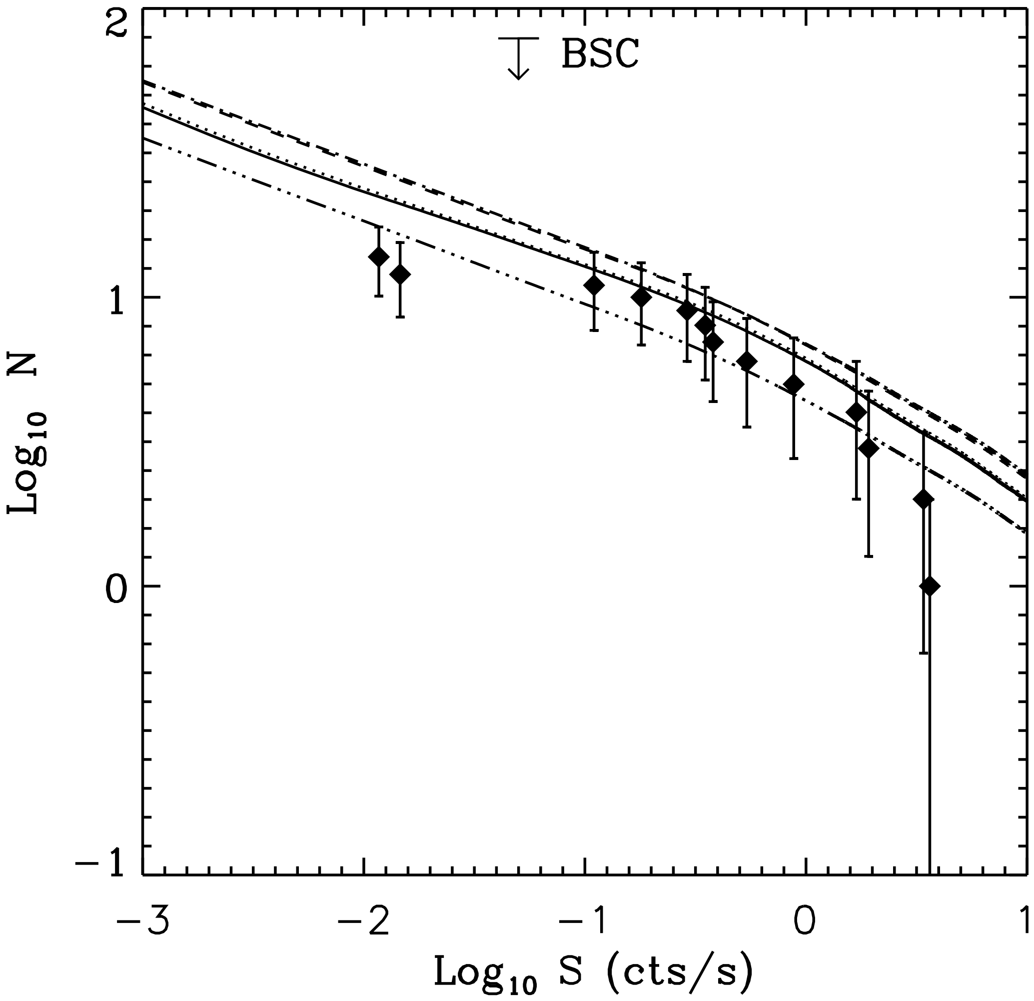

Figure 6: Two upper curves correspond to non-truncated mass spectra (dashed to the "new" and dot-dashed – to the "old"). Two curves in the middle – to truncated spectra (solid – "old", dotted – "new"). The lowest curve is shown for the flat mass spectrum.

detectable hot distant objects. Without any doubt such a simple approach underestimates absorption at large distances. If used for testing cooling curves this line has to be calculated more accurately.

Unfortunately, at the present time ages and distances are known with enough precision only for few young NS in our neighborhood. That is why at the moment the ADD can be just of limited utility. But it seems, that as the data on close-by isolated NSs grows, the ADD can become an additional tool to study this group of sources, especially with population synthesis codes.

The ADD potentially can be used as an additional test for cooling curves. The idea would be to model with a population synthesis code an artificial population and compare it with the observed one using the ADD. When ages and distances for all sources from the Magnificent seven and other close-by young NSs are known it can be done. The advantage of this diagram is that one can add NSs which are unobservable in thermal X-rays. Dark (in soft X-rays) NSs can (and should) also be used to test cooling curves in the framework of population synthesis.

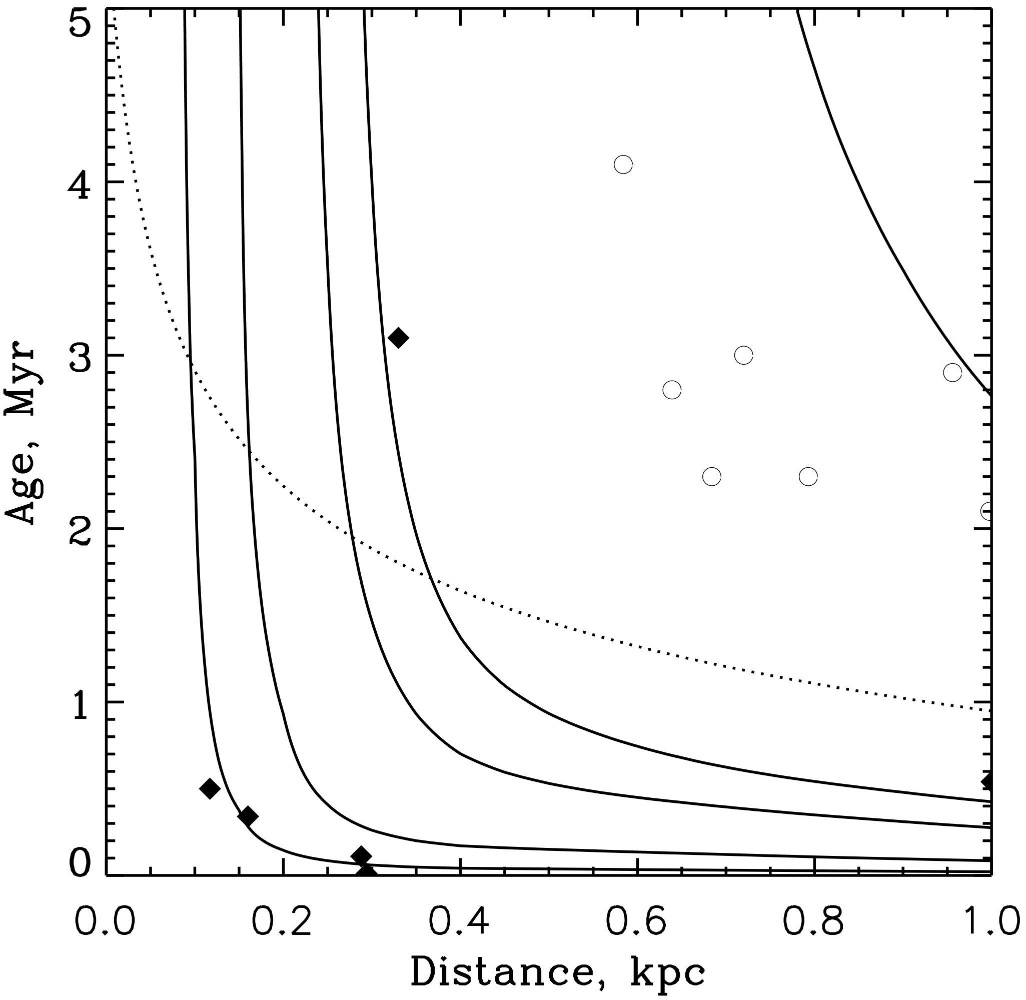

Figure 7: Age-distance diagram for close-by young NSs with realistic birth rate and initial distribution, dynamical effects are taken into account. Filled symbols – sources detected in X-rays, empty symbols – known close-by young NSs (radio pulsars) with non-detected thermal emission.

7 Conclusions

In conclusion, we want to underline that the idea of the conferences like QUARKS-2006 is a very fruitful one, as these meetings are the places where theoretical physicists and astrophysicists can have joint discussions: QCD meets astronomy again. Now it is obvious that there are theoretical models which can be tested only in “celestial laboratories”. Neutron stars definitely are such labs.

Observations of cooling NSs provide a rare opportunity to study particular parts of the QCD phase diagram. We hope that the development of new tools to confront theory of cooling of compact objects with observations can help to achieve progress in both: astrophysics and theoretical physics.

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