Creation from nothing revisited: landscape from cosmological bootstrap

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Abstract

We suggest a novel picture of the quantum Universe. Its creation is described by a *density matrix* which yields an ensemble of universes with the cosmological constant limited to a bounded range $\Lambda_{\rm min} \leq \Lambda \leq \Lambda_{\rm max}$. The domain $\Lambda < \Lambda_{\rm min}$ is ruled out by a cosmological bootstrap requirement (the self-consistent back reaction of hot matter). The upper cutoff results from the quantum effects of vacuum energy and the conformal anomaly mediated by a special ghost-avoidance renormalization. The cutoff $\Lambda_{\rm max}$ establishes a new quantum scale which is determined by the coefficient of the topological Gauss-Bonnet term in the conformal anomaly. This scale is realized as the upper bound — the accumulation point of an infinite sequence of garland-type instantons which constitute the full cosmological landscape. The dependence of the cosmological constant range on particle phenomenology suggests a possible dynamical selection mechanism for the landscape of string vacua.

1 Introduction

The ideas of quantum cosmology [1, 2] and Euclidean quantum gravity [3] are again attracting attention. One of the reasons is that the landscape of string vacua is too big [4] to predict either the observed particle phenomenology or large-scale structure formation within string theory itself. Other methods have to be invoked, at least some of them based on the cosmological wavefunction [5, 6, 7].

This approach is based on the idea of quantum tunneling from the classically forbidden state of the gravitational field. Semiclassically this state is described in terms of the imaginary time, that is by means of the Euclidean spacetime, so that the corresponding amplitudes and probabilities are weighted by the exponentiated Euclidean gravitational action, $\exp(-S_{\rm E})$. The action is calculated on the gravitational instanton – the saddle point of an underlying path integral over Euclidean 4-geometries. This instanton gives rise to Lorentzian signature spacetime by analytic continuation across minimal hypersurfaces. The continuation can be interpreted either as quantum tunneling or as the creation of the Universe from "nothing". Thus, the most probable vacua of the landscape become weighted by the minima of $S_{\rm E}$. This might serve as a method of selecting a vacuum from the enormously big string landscape.

An immediate difficulty with this program arises from the infrared catastrophe of small cosmological constant Λ. The Hartle-Hawking wave function [3], which describes nucleation of

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the de Sitter Universe from the Euclidean 4-dimensional hemisphere, has the form

$$\Psi_{\rm HH} \sim \exp(-S_{\rm E}) = \exp(3\pi/2G\Lambda). \tag{1}$$

This diverges for $\Lambda \to 0$ because of unboundedness of the Euclidean gravitational action. Despite some early attempts to interpret it as the origin of a zero value of Λ [8], this result remains both controversial and anti-intuitive. It disfavors inflation, prefers creation of infinitely large universes, and does not naturally yield the observed dark energy. Apart from the tunneling proposals of [9] which employ an opposite sign in the exponential of (1) and thus open the possibility for opposite conclusions [10], no convincing resolution of this problem has thus far been suggested.

Here we show that Euclidean path integration framework naturally avoids this infrared catastrophe. We attain this result by: i) extending the notion of Hartle-Hawking *pure* state to a density matrix which describes a *mixed* quantum state of the Universe and ii) incorporating the nonperturbative back reaction of hot quantum matter on the instanton background [11]. These extensions seem natural because whether the initial state of the Universe is pure or mixed is a dynamical question rather than a postulate. We address this question below.

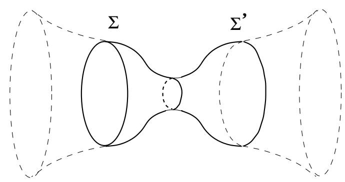

Figure 1: Density matrix instanton. Dashed lines depict the Lorentzian Universe nucleating at minimal surfaces Σ and Σ'.

A density matrix $\rho[\varphi,\varphi']$ is represented in Euclidean quantum gravity [12] by an instanton having two disjoint boundaries Σ and Σ' associated with its two entries φ and φ' (collecting both gravity and matter observables). The instanton interpolates between these, thus establishing mixing correlations, see Fig.1. In contrast, for the density matrix of the pure Hartle-Hawking state the bridge between Σ and Σ' is broken, so that the instanton is a union of two disjoint hemispheres which smoothly close up at their poles (Fig.2) — a picture illustrating the factorization of $\hat\rho = |\Psi_{\rm HH}\rangle\langle\Psi_{\rm HH}|$.

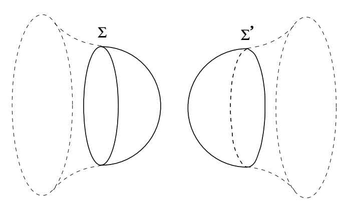

Figure 2: Density matrix of the pure Hartle-Hawking state represented by the union of two vacuum instantons.

The main effect that we advocate here is that thermal fluctuations destroy the Hartle-Hawking instanton and replace it with one filled by radiation. This is already manifest in classical theory of a spatially closed cosmology with the Euclidean FRW metric

$$ds^2 = N^2(\tau)\, d\tau^2 + a^2(\tau)\, d^2\Omega^{(3)} \tag{2}$$

(where $a(\tau)$ is a scale factor and $\Omega^{(3)}$ is a 3-sphere of a unit size). Namely, in the Euclidean Friedmann equation for $a(\tau)$,

$$\frac{\dot a^2}{a^2} = \frac{1}{a^2} - H^2 - \frac{C}{a^4} \tag{3}$$

(we use the gauge $N=1$ and express $\Lambda=3H^2$ in terms of the Hubble constant H), the radiation density term C/a^4 prevents the half-instantons from closing and allows a to vary between two turning points [13, 14]

$$a_\pm=\frac{1}{\sqrt{2}\,H}\,(1\pm\sqrt{1-4CH^2})^{1/2}. \qquad (4)$$

This forces a tubular structure on the instanton which spans at least one period of oscillation between $a_\pm$, provided the constant C (characterizing the amount of radiation) satisfies the bound $4H^2C\leq 1$.

The existence of radiation, in its turn, naturally follows from the partition function of this state. The partition function originates from integrating out the field φ in the coincidence limit $\varphi'=\varphi$. This corresponds to the identification of Σ' and Σ, so that the underlying instanton acquires toroidal topology, see Fig.3. Its points are labeled by the periodically identified Euclidean time, a period being related to the inverse temperature of the quasi-equilibrium radiation. The back reaction of this radiation supports the instanton geometry in which this radiation exists, and we derive the equation which makes this bootstrap consistent.

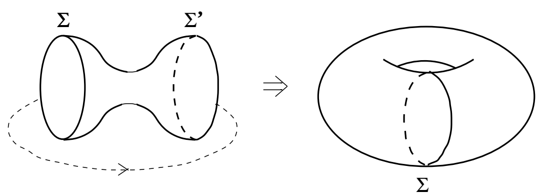

Figure 3: Calculation of the partition function represented by compactification of the instanton to a torus with periodically identified Euclidean time.

Remarkably, when the vacuum energy and conformal anomaly are taken into account this bootstrap yields a set of instantons – a landscape – only in the bounded range of Λ,

$$\Lambda_{\rm min}<\Lambda<\Lambda_{\rm max}. \qquad (5)$$

All values $\Lambda<\Lambda_{\rm min}$ are completely eliminated either because of the absence of instanton solutions or because of their *infinitely large positive* action. A similar situation holds for $\Lambda>\Lambda_{\rm max}$ – no instantons exist there, and the Lorentzian configurations in this overbarrier domain (if any) are exponentially suppressed relative to those of (5). Below we derive these properties and describe the structure of the cosmological landscape inside the domain (5).

2 The density matrix

To quantify the above picture consider the density matrix given by the Euclidean path integral [12]

$$\rho[\,\varphi,\varphi'\,]=e^{\varGamma}\int D[\,g,\phi\,]\,\exp\big(-S_{\rm E}[\,g,\phi\,]\big), \qquad (6)$$

where $S_{\rm E}[\,g,\phi\,]$ is the classical action, and the integration runs over gravitational g and matter ϕ fields interpolating between φ and φ' at Σ and Σ'. The condition ${\rm tr}\,\hat\rho=1$ gives the partition function $\exp(-\varGamma)$ as a similar path integral over periodic fields on the torus with identified boundaries Σ and Σ', $g,\phi\,|_\Sigma=g,\phi\,|_{\Sigma'}$

$$e^{-\varGamma}=\int\limits_{g,\,\phi\,|_\Sigma=\,g,\,\phi\,|_{\Sigma'}}D[\,g,\phi\,]\,\exp\big(-S_{\rm E}[\,g,\phi\,]\big). \qquad (7)$$

The motivation for this definition of the density matrix and its statistical sum can be found in [12]. Here we would only mention that this is a natural generalization of the path integral for the no-boundary wave function of the Universe. Also, it can be regarded as a gravitational generalization of the density matrix of the equilibrium thermodynamical ensemble at finite temperature $T = 1/\beta$, $\hat\rho = \exp\big(\varGamma - \beta\hat H\big)$, for a system with a Hamiltonian operator $\hat H$. Its kernel in the functional coordinate representation $\rho[\varphi,\varphi'] = \langle\varphi\,|\,\hat\rho\,|\,\varphi'\rangle$, similarly to (6), is given by the Euclidean path integral over histories $\phi(\tau)$ in the imaginary time τ, interpolating between $\phi(0) = \varphi'$ and $\phi(\beta) = \varphi$.

The density matrix (6) prescribes a particular mixed quantum state of the system. Its semiclassical calculation yields as a saddle point the configuration of a tubular topology depicted on Fig.1. Its Euclidean part is bounded by two minimal surfaces Σ and Σ'. The analytic continuation across these surfaces yields the ensemble of cosmological models expanding in real Lorentzian time, and this picture can be called as the origin of cosmological thermodynamics via creation from "nothing". Taking the trace of $\hat\rho$ in the normalization condition for the density matrix results in the identification of Σ and Σ' and the toroidal compactification of the instanton depicted on Fig.3. This underlies the semiclassical calculation of the statistical sum and the corresponding Euclidean effective action (7).

The back reaction follows from decomposing $[g,\phi]$ into a minisuperspace $g_0(\tau) = \big(a(\tau), N(\tau)\big)$, and the "matter" sector which includes also inhomogeneous metric perturbations on minisuperspace background $\varPhi(x) = (\phi(x), \psi(x), A_\mu(x), h_{\mu\nu}(x), ...)$. With a relevant decomposition of the measure $D[\,g,\phi\,] = Dg_0(\tau)\times D\varPhi(x)$, the integral for $\varGamma$ takes the form

$$e^{-\varGamma} = \int Dg_0(\tau)\,\exp\Big(-\varGamma[g_0(\tau)]\Big), \tag{8}$$

where $\varGamma[g_0(\tau)]$ is the effective action of quantized matter on the background $g_0(\tau)$,

$$\exp\Big(-\varGamma[g_0(\tau)]\Big) = \int D\varPhi(x)\,\exp\Big(-S_{\rm E}[g_0(\tau),\varPhi(x)]\Big). \tag{9}$$

Our approximation will be to consider it in the one-loop order,

$$\varGamma[g_0] = S_{\rm E}[g_0] + \varGamma_{1-{\rm loop}}[g_0], \tag{10}$$

and handle (8) at the tree level, which is equivalent to solving the *effective equations* for $\varGamma[g_0]$. This incorporates the lowest order back reaction effect of quantum matter on the minisuperspace background.

3 Conformal anomaly and the ghost-avoidance renormalization

To make calculation of $\varGamma[\,g_0\,]$ manageable we restrict attention to conformally-invariant fields as a source of the back reaction. For such fields one can apply the technique of the conformal transformation [15] relating a generic FRW metric (2) rewritten in terms of the conformal time η,

$$ds^2 = a^2(\eta)(d\eta^2 + d^2\Omega^{(3)}), \tag{11}$$

to the metric of the Einstein static Universe of a unit size

$$d\bar s^2 = d\eta^2 + d^2\Omega^{(3)} \tag{12}$$

(these metrics are denoted below as g and $\bar g$). The contribution of this conformal transformation to the effective action, $\varGamma_{1-{\rm loop}}[g]-\varGamma_{1-{\rm loop}}[\bar g]$, is determined by the coefficients of $\Box R$, the Gauss-Bonnet invariant $E = R^2_{\mu\nu\alpha\gamma} - 4R^2_{\mu\nu} + R^2$ and the Weyl tensor term $C^2_{\mu\nu\alpha\beta}$ in the conformal anomaly

$$g_{\mu\nu}\frac{\delta\varGamma_{1-{\rm loop}}}{\delta g_{\mu\nu}} = \frac{1}{4(4\pi)^2}\,g^{1/2}(\alpha\Box R + \beta E + \gamma C^2_{\mu\nu\alpha\beta}). \tag{13}$$

Specifically this contribution can be obtained by the technique of [16]. For a generic conformal transformation relating two metrics, $g_{\mu\nu}(x) = e^{\sigma(x)}\bar g_{\mu\nu}(x)$, it reads

$$\begin{aligned}\varGamma_{1-\rm loop}[g] - \varGamma_{1-\rm loop}[\bar g] &= \frac{1}{2(4\pi)^2}\int d^4x \bar g^{1/2}\left\{\frac{1}{2}\left[\gamma\,\bar C^2_{\mu\nu\alpha\beta} + \beta\left(\bar E - \frac{2}{3}\bar\Box\bar R\right)\right]\sigma\right.\\ &\left.+\frac{\beta}{2}\left[(\bar\Box\sigma)^2 + \frac{2}{3}\,\bar R\,(\bar\nabla_\mu\sigma)^2\right]\right\}\\ &-\frac{1}{2(4\pi)^2}\left(\frac{\alpha}{12}+\frac{\beta}{18}\right)\int d^4x\,\left(g^{1/2}R^2(g) - \bar g^{1/2}R^2(\bar g)\right),\end{aligned} \tag{14}$$

where all barred quantities are calculated with respect to $\bar g_{\mu\nu}(x)$.

For the conformal factor $e^\sigma = a^2(\tau)$ this expression contains higher-derivative terms $\sim \ddot a^2$ which produce ghost instabilities in solutions of effective equations. However, such terms are proportional to the coefficient α which can be put to zero by adding the following *local* counterterm admissible from the viewpoint of general renormalization theory

$$\varGamma_R[g] = \varGamma_{1-\rm loop}[g] + \frac{1}{2(4\pi)^2}\frac{\alpha}{12}\int d^4x\, g^{1/2}R^2(g). \tag{15}$$

Certainly, this additionally spoils conformal invariance of the theory which was anyway irreversibly broken by quantum corrections. Thus it is reasonable to fix this *local* renormalization ambiguity by the additional criterion of the absence of ghosts, what we do here for sake of consistency of the theory at the quantum level.[1]

The contribution $\varGamma_R[g] - \varGamma_R[\bar g]$ to the *renormalized* action then finally reads

$$\varGamma_R[\,g\,] - \varGamma_R[\,\bar g\,] = B\int d\tau\,\left(\frac{\dot a^2}{a} - \frac{1}{6}\frac{\dot a^4}{a}\right), \tag{16}$$

where the parameter B is determined by the coefficient β of the topological Gauss-Bonnet term in the conformal anomaly (13)

$$B = \frac{3}{4}\beta. \tag{17}$$

4 Effective action and vacuum energy of a static Einstein instanton

The static instanton with a period η_0 playing the role of inverse temperature contributes $\varGamma_{1-\rm loop}[\bar g] = E_0\,\eta_0 + F(\eta_0)$. This is a result of a typical thermodynamical calculation, in which the vacuum energy E_0 and free energy $F(\eta_0)$ for bosons and fermions read as

$$E_0 = \pm\sum_\omega\frac{\omega}{2}\,,\quad F(\eta_0) = \pm\sum_\omega \ln\left(1\mp e^{-\omega\eta_0}\right), \tag{18}$$

where the summation runs over field oscillators with energies ω on a unit 3-sphere.

Quartic divergence of the vacuum energy in $\varGamma_{1-\rm loop}[\,\bar g\,]$ constitutes the ultraviolet divergences of the full action $\varGamma_{1-\rm loop}[\,g\,]$. Under a covariant regularization the power and quartic divergences among them are absorbed by the renormalization of the cosmological and Einstein terms, whereas the subtraction of logarithmic divergences yields as a remnant the contribution

[1] This is certainly not an exhaustive solution of the ghost problem in effective equations whose higher-derivative terms still remain in the other sectors of the theory – the graviton sector of transverse-traceless modes, in particular. These sectors, however, are not involved within the minisuperspace FRW metric, and it is suggestive to use this finite ghost-avoidance renormalization as a simple method ultimately eradicating ghosts in the minisuperspace sector.

of a conformal anomaly considered above. For conformal fields, which we consider, the logarithmic divergences are actually zero, because they are given by the sum of integrated Weyl-squared and Euler number terms (γ and β terms of the conformal anomaly (13)). For a conformally flat metric with the torus topology they are both vanishing. Therefore, the regularized one-loop action actually does not have a typical renormalization ambiguity quadratic in the curvature – the term of the same structure as $E_0\,\eta_0$ in $\varGamma_{1-\rm loop}[\bar g]$, $\int d^4x g^{1/2}R^2 \sim \int d\tau a^3/a^4$. Thus, the vacuum energy in an Einstein static spacetime is uniquely calculable. This was independently confirmed by different methods [18] which give

$$E_0 = \frac{1}{960}\times\left\{\begin{array}{c}4\\17\\88\end{array}\right. \tag{19}$$

respectively for scalar, spinor and vector fields. It should be emphasized that this renormalized Casimir energy is *positive* (even for the naively negative vacuum energy of a spinor field $-\sum_n(\omega_n/2)$, [18]).

The ghost-avoidance renormalization (15) should be applied also to $\varGamma_{1-\rm loop}[\bar g]$. This only modifies the value of the vacuum energy E_0 in $\varGamma_{1-\rm loop}[\bar g]$, because $\int d^4x\,\bar g^{1/2}\bar R^2 = 72\,\pi^2\eta_0$. Thus

$$\varGamma_R[\bar g] = C_0\,\eta_0 + F(\eta_0),\quad C_0 \equiv E_0 + \frac{3\,\alpha}{16}. \tag{20}$$

It is remarkable that for all conformal fields of low spins this modified energy reduces to the *one half of the coefficient* B in the conformal part of the total effective action (16)[2]

$$C_0 = \frac{B}{2}. \tag{21}$$

This universal relation between C_0 and $B = 3\beta/4$ can be verified by using the value of the Casimir energy in a static universe (19) and the known anomaly coefficients [17] which for scalar, Weyl spinor and vector fields respectively are equal:

$$\alpha = \frac{1}{90}\times\left\{\begin{array}{c}-1\\-3\\18\end{array}\right.,\quad \beta = \frac{1}{360}\times\left\{\begin{array}{c}2\\11\\124\end{array}\right.. \tag{22}$$

The relation (21) will be very important for the formation of the upper bound of the cosmological constant range (5).

5 Effective Friedmann equation and the cosmological bootstrap

Now we assemble together the classical part of the action, conformal contribution (16) and the action of the static instanton (20). After rewriting the conformal time as a parametrization invariant integral in terms of the lapse N and the scale factor a,

$$\eta_0 = 2\int_{\tau_-}^{\tau_+}\frac{d\tau\,N(\tau)}{a(\tau)}, \tag{23}$$

the total action takes the form

$$\begin{aligned}\varGamma[\,a(\tau),N(\tau)\,] = {}& 2\int_{\tau_-}^{\tau_+} d\tau\Big(-\frac{a\dot a^2}{N} - Na + NH^2a^3\Big)\\ &+2B\int_{\tau_-}^{\tau_+} d\tau\Big(\frac{\dot a^2}{Na} - \frac{1}{6}\,\frac{\dot a^4}{N^3a}\Big)\\ &+B\int_{\tau_-}^{\tau_+} d\tau\,N/a + F\Big(2\int_{\tau_-}^{\tau_+} d\tau\,N/a\Big)\,.\end{aligned} \tag{24}$$

[2]This result implies that the vacuum energy of conformal fields in a static Einstein universe can be universally expressed in terms of the coefficients of the conformal anomaly $m_P^2\,E_0 = 3(2\beta-\alpha)/16$.

Here and below we work in units of the Planck mass $m_P = \sqrt{3\pi/4G}$, and the integration runs between two turning points of the periodic history $a(\tau)$ at $\tau_\pm$ — half a period of the Euclidean time τ.

The effective equation $\delta\varGamma/\delta N(\tau) = 0$ in the gauge $N = 1$ has the form of the Friedmann equation (3) modified by the quantum B-term

$$\frac{\dot a^2}{a^2} + B\left(\frac{1}{2}\frac{\dot a^4}{a^4} - \frac{\dot a^2}{a^4}\right) = \frac{1}{a^2} - H^2 - \frac{C}{a^4}, \tag{25}$$

in which the radiation *constant* C is a *functional* of $a(\tau)$, determined by the *bootstrap* equation

$$C = B/2 + F'(\eta_0), \quad F'(\eta_0) \equiv dF(\eta_0)/d\eta_0, \tag{26}$$

$$\eta_0 = 2\int_{\tau_-}^{\tau_+} d\tau/a(\tau). \tag{27}$$

Here $F'(\eta_0) > 0$ is the thermal energy of a hot gas of particles, which adds to their vacuum energy $B/2$. Thus, the overall back reaction is mediated by the contribution of the radiation-type energy density term and the anomalous quantum B-term. In view of Eqs. (23) and (26) the constant C characterizing the amount of radiation nonlocally depends on the FRW background supported by the radiation itself, and this is the mechanism of the bootstrap we are going to analyze.[3]

The on-shell action $\varGamma_0$ on solutions of (25) can be cast into a convenient form by expressing the combination $-a + H^2a^3$ in (24) in terms of other pieces of the effective Friedmann equation (25). Then, after converting the integral over τ into the integral over a between the turning points $a_\pm$ the on-shell action takes the form

$$\varGamma_0 = F(\eta_0) - \eta_0\frac{dF(\eta_0)}{d\eta_0} + 4\int_{a_-}^{a_+}\frac{da\,\dot a}{a}\left(B - a^2 - \frac{B\dot a^2}{3}\right). \tag{28}$$

The structure of the integral term here will be of crucial importance for the elimination of the infrared catastrophe and formation of the bounded cosmological landscape.

6 Effect of the conformal anomaly and bootstrap: elimination of the infrared catastrophe

Periodic instanton solutions of Eqs.(25)-(27) exist only inside the curvilinear wedge of (H^2, C)-plane between bold segments of the upper hyperbolic boundary and the lower straight line boundary of Fig.4,

$$4CH^2 \leq 1, \;\; C \geq B - B^2H^2, \;\; BH^2 \leq 1/2. \tag{29}$$

Below this domain the solutions are either complex and aperiodic or suppressed by *infinite positive* Euclidean action. Above this domain only Lorentzian (overbarrier) configurations exist, but they are again exponentially damped relative to instantons in (29).

These properties are based on the fact that the turning points of (25) exactly coincide with $a_\pm$ of (4), but the turning point a_- exists only when $a_-^2 \geq B$, because the second term in the equation

$$\dot a^2 = \sqrt{\frac{(a^2-B)^2}{B^2} + \frac{2H^2}{B}\,(a_+^2 - a^2)(a^2 - a_-^2)} - \frac{(a^2-B)}{B}, \tag{30}$$

[3]Note that, contrary to a conventional wisdom, the vacuum energy of quantum fields in a static universe (19) (and its renormalized version (20)) contributes *not* to the cosmological constant, but to the total radiation density. This follows from the structure of the corresponding terms in the effective action discussed in Sect.4 and is also confirmed by the radiation-type equation of state derived for E_0 in [18].

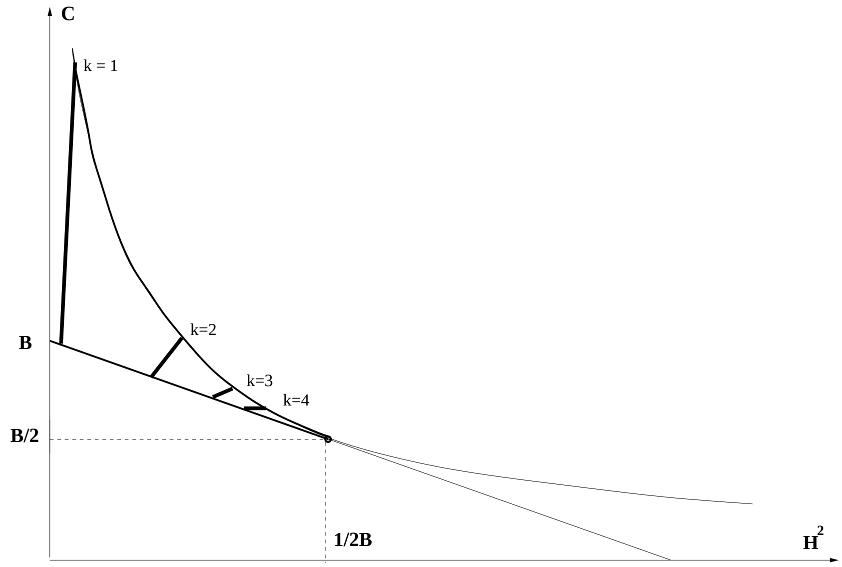

Figure 4: Instanton domain in the (H^2, C)-plane. Garland families are shown for $k = 1, 2, 3, 4$. Their sequence accumulates at the critical point $(1/2B, B/2)$.

which is the corollary of (25), should be negative at a_-. Together with the upper bound on the amount of radiation (existence of solutions in Euclidean time), $4CH^2 \leq 1$, this gives rise to the domain (29). Otherwise, outside of this domain $a(\tau)$ at the contraction phase becomes complex or runs to zero which violates instanton periodicity. In the latter case a smooth Hartle-Hawking instanton with $a_- = 0$ forms and incorporates $\eta_0 \to \infty$ in view of the divergence of the integral (27) at $a(\tau) \to a(\tau_-) = 0$. As a result both $F(\eta_0) \sim -\exp(-\eta_0)$ and $dF(\eta_0)/d\eta_0$ vanish. Therefore, from the bootstrap equation $C = B/2$, and the instanton smoothly closes at $a = 0$, because in view of (30)

$$\dot{a}^2 \to 1 + \sqrt{1 - 2C/B} = 1, \quad a \to 0. \tag{31}$$

These Hartle-Hawking instantons arising outside of the domain (29) are ruled out by the *infinitely large positive* value of their action. For their solutions the action Γ_0 reduces to the last integral term of (28) with the lower limit $a(\tau_-) = 0$. Due to the contribution of the conformal anomaly and in view of (31) its integrand is positive at $a \to 0$, and the integral diverges at the lower limit to $+\infty$. This is the mechanism of how thermal fluctuations destroy the Hartle-Hawking pure state and make a mixed state of the Universe dynamically more preferable [11].

Moreover, inside the range (29) our bootstrap eliminates the infrared catastrophe of $\Lambda \to 0$. Indeed $\eta_0 \to \infty$ as $H^2 \to 0$, so that due to (26) $C \to B/2$, but this is impossible because in view of (29) $C \geq B$ at $H^2 = 0$. Thus, instanton family never hits the C-axes of $H^2 = 0$ and can only interpolate between the points on the boundaries of the domain (29). For a conformal scalar field the numerical analysis gives such a family [11] starting from the lower boundary at

$$H^2 \approx 2.00, \quad C \approx 0.004, \quad \Gamma_0 \approx -0.16, \tag{32}$$

and terminating at the upper boundary at

$$H^2 \approx 13.0, \quad C \approx 0.02, \quad \Gamma_0 \approx -0.09. \tag{33}$$

The upper point describes the static universe filled by a hot radiation with the temperature $T = H/\pi\sqrt{1 - 2BH^2}$, whereas the lower point establishes the lower bound of the Λ-range.

The above results for a scalar field with $B_{\rm scalar} = 1/240$ can also be obtained for other spins. We present them for a vector field with a much bigger value of the constant $B_{\rm vector} = 31/120$.

The lower and upper bounds on the instanton family then read respectively as

$$H^2 \approx 1.06,\ C \approx 0.19,\ \varGamma_0 \approx -0.28, \tag{34}$$
$$H^2 \approx 1.23,\ C \approx 0.20,\ \varGamma_0 \approx -0.23. \tag{35}$$

Together with (32) they give the lower bounds on the instanton range for scalars and vectors. Their comparison shows the tendency of decreasing Hubble scale and growing $-\varGamma_0$ with the growth of spin (or the parameter B).

In fact, the behavior of our bootstrap at $B \gg 1$ can be studied analytically [11]. It corresponds to the decreasing value of the conformal time, $\eta_0 \simeq \pi(4/15\,B)^{1/6}$, and the following asymptotics of the effective action

$$\varGamma_0 \simeq -\frac{2\pi}{3\sqrt{15}}\sqrt{B}, \quad B \gg 1. \tag{36}$$

It is important that $\varGamma_0$ stays negative (even though no infrared catastrophe exists any more) while its absolute value grows with B. This property implies that the growing spin of quantum matter (growing $B = 3\beta/4$, cf. (22)) makes the probability of the underlying instanton higher. This will be important for suppressing the contribution of Lorentzian (overbarrier) configurations in extended formulations of quantum gravity briefly discussed in Conclusions.

The limit of large B is very important. This limit corresponds to strong quantum corrections mediated by a large contribution of the conformal anomaly. The higher the value of B, the stronger it truncates the instanton domain (29) from above, $H^2 < H^2_{\rm max} = 1/2B$. This property will be discussed in Conclusions as a possible mechanism of reducing the landscape of string vacua.

7 Instanton garlands and a new quantum scale

The instanton family of Sect.6 does not exhaust the entire cosmological landscape. Its upper bound follows from the existence of *garlands* that can be obtained by glueing together into a torus k copies of a simple instanton [13, 19]; see Fig.5. Their formalism is the same as above except that the conformal time (27) and the integral term of (28) should be multiplied by k.

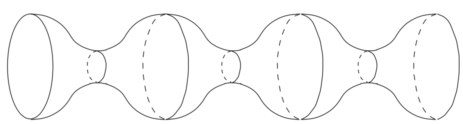

Figure 5: The garland segment consisting of three folds of a simple instanton glued at surfaces of a maximal scale factor.

As in the case of $k = 1$, garland families interpolate between the lower and upper boundaries of (29). In particular, the numerical analysis for $k = 2$ yields the instanton family joining respectively the lower and upper points in the (C, H^2)-plane

$$H^2_{(2)} \approx 45.89,\ C_{(2)} \approx 0.0034,\ \varGamma^{(2)}_0 \approx -0.0113, \tag{37}$$
$$H^2_{(2)} \approx 61.12,\ C_{(2)} \approx 0.0041,\ \varGamma^{(2)}_0 \approx -0.0145. \tag{38}$$

The garlands exist for all k, $1 \leq k \leq \infty$, and their infinite sequence accumulates at the critical point $C = B/2$, $H^2 = 1/2B$, where these boundaries merge, see Fig.4. With $k \to \infty$ the length of the k-th family in the (C, H^2)-plane is getting shorter and shorter and its location closer and closer approaching the critical cusp point. The upper point of each family gives rise to a hot static Universe filled by radiation in the equilibrium state with the temperature $T_{(k)} = H/k\pi\sqrt{1-2BH^2}$ differing from $T = T_{(1)}$ by an extra $1/k$-factor.

The existence of this infinite sequence follows from the behavior of a single fold of the conformal time (27) at $2BH^2 \to 1$. It tends to zero in this limit, so that when multiplied by k it admits the solution of the bootstrap equation for any $k \to \infty$ with $\eta_0^{(k)} = k\eta_0$ slowly growing to infinity. In this limit the sequence of instanton families can be described analytically [11]. It turns out that for large k the conformal time $\eta_0^{(k)} \simeq \ln k^2$, and within the $1/k^2$-accuracy the upper and lower points of each family coincide and read

$$H^2_{(k)} \simeq \frac{1}{2B}\left(1 - \frac{\ln^2 k^2}{2k^2\pi^2}\right), \quad C_{(k)} \simeq \frac{B}{2}\left(1 + \frac{\ln^2 k^2}{2k^2\pi^2}\right), \quad \varGamma_0^{(k)} \simeq -B\,\frac{\ln^3 k^2}{4k^2\pi^2}. \tag{39}$$

Thus, the lengths of instanton families (both in H^2 and C directions) decrease as the second order of the $1/k^2$-expansion, $\Lambda^{(k)}_{\rm max} - \Lambda^{(k)}_{\rm min}, C^{(k)}_{\rm max} - C^{(k)}_{\rm min} \sim 1/k^4$, so that on Fig.4 they fit in ever narrowing wedge near the critical cusp of $C = B/2 = 1/4H^2$. With a growing k, garlands become more and more static and cool down to zero temperature $T_{(k)} \simeq 1/(\sqrt{B}\ln k^2) \to 0$.

It is remarkable that contrary to the tree-level instantons of [19] the garland action is not additive in k, so that as $k \to \infty$ it tends to zero and garlands do not dominate the ensemble. Nevertheless, their existence is very important, because they generate a new quantum scale — the upper bound of the instanton range (5). Their sequence converges to the cold, $T_{(\infty)} = 0$, static instanton with the vanishing action, which realizes this scale as a maximal possible value of the Hubble constant in the instanton landscape

$$H^2_{\rm max} \equiv H^2_{(\infty)} = \frac{1}{2B}. \tag{40}$$

8 Conclusions

Thus, our Universe is created in a hot mixed state, but its evolution does not contradict the large-scale structure formation. After nucleation from the instanton the Universe expands; its radiation dilutes to a negligible density when Λ starts dominating and generates inflation.

The ensemble of universes belongs to a bounded Λ-range (5). Its infrared cutoff is provided by the radiation back reaction and survives even in the classical limit as $B \to 0$. In contrast, the high-energy cutoff (40) is the quantum effect of vacuum energy and the conformal anomaly, which generates a new scale in gravity theory. In view of the relation (17) this scale is determined by the inverse of the coefficient of the topological Gauss-Bonnet term in the conformal anomaly (13),

$$\Lambda_{\rm max} = \frac{2}{\beta}\,m_P^2, \quad m_P^2 \equiv \frac{3\pi}{4G}, \tag{41}$$

and it tends to infinity in the classical limit $\beta \to 0$. In the cosmological landscape this scale is realized as a limiting point of the sequence of garland-type instantons.

We have considered only conformal fields, but we expect that other fields will not qualitatively change the picture, because on quasi-static background they do not differ significantly from their conformal analogues. The value of the vacuum energy $B/2$ in (26) which gives as a lower bound for C exactly the upper boundary of (5), $C = B/2 = 1/4H^2$, is critical. Non-conformal fields are likely to break this relation. Then if $C_0 < B/2$ all garlands survive, though they saturate at $\Lambda_{\rm max}$ with a finite temperature. If $B > C_0 > B/2$, their sequence is truncated at some k. Finally, if $C_0 > B$ the infrared catastrophe occurs again — the $k = 1$ family of instantons hits the C-axes at C_0. Which of these possibilities gets realized is a question deserving further study.

Another open question concerns the normalizability of our partition on the infinite set of instantons. One might think that it is not normalizable because of infinite summation over garland folds k. However, at least naively the total continuous measure of the instanton set in the (H^2, C)-plane is finite because $\sum_k \big(\Lambda^{(k)}_{\rm max} - \Lambda^{(k)}_{\rm min}\big) \sim \sum_k (1/k^4) < \infty$. In order to have a definitive conclusion, though, one must take into account a zero-mode contribution to the actual measure and also estimate preexponential factors. The latter reduce to the quantum mechanical functional determinants of the Hessian of the effective action $\mathit{\Gamma}[\,a(\tau), N(\tau)\,]$, which is a nonlocal operator rather than a differential one. Their calculation is doable and will be reported elsewhere [20].

The boundaries of (5) depend on particle phenomenology. For a single scalar field they are given by Planckian values, $\Lambda_{\rm min} \approx 8.99\, m_P^2$, $\Lambda_{\rm max} = 360\, m_P^2$ and decrease as $1/N$ with a growing number of fields N (in view of the simple scaling of the bootstrap $C \to NC$, $B \to NB$, $F(\eta_0) \to NF(\eta_0)$, $H^2 \to H^2/N$). This justifies a semiclassical expansion for large N and B. Moreover, when ascending the hierarchy of spins, this scaling reduces the domain (5) to a narrow subplanckian range and suggests a long-sought selection mechanism for the landscape of string vacua. Modulo the details of a relevant $4D$-compactification, this mechanism might work as follows. For $B = 3\beta/4$ growing with N and spin, cf. (22), the upper scale (41) decreases towards the increasing phenomenology scale, and coincides with the latter at the string scale $m_{\rm s}^2$ where a positive Λ might be generated by the KKLT or KKLMMT-type mechanism [21]. Our conjecture is that at this scale our bootstrap becomes perturbatively consistent, provided $m_P^2/B = m_{\rm s}^2 \ll m_P^2$, and selects from the string landscape a small subset compatible with the observed particle phenomenology and large-scale structure.

Our results hold within the Euclidean path integral (6) which automatically excludes Lorentzian configurations possibly existing above the upper boundary of (29), $4CH^2 > 1$. However, one can imagine an extended formulation of quantum gravity generalizing (6) to a wider path integration domain. Our conclusions nevertheless remain true. Indeed, according to (36) the effective action scales as $\mathit{\Gamma}_0 \sim -\sqrt{B}$, $B \gg 1$, and because it is *negative* our landscape at the scale $m_{\rm s}$ is weighted by $\exp(\#\sqrt{B}) {=} \exp(\#\, m_P/m_{\rm s}) \gg 1$. Therefore it strongly dominates over Lorentzian configurations, the amplitudes of the latter being $O(1)$ in view of their pure phase nature. Thus, our results look robust against possible generalizations of Euclidean quantum gravity.

This is how the cosmological landscape emerges from "nothing" and perhaps tames its string counterpart, provided some like it hot.

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