Conventional Cosmology from Multidimensional models

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Kaluza - Klein models:

$M = M_0 \times M_1$ ⇐ factorizable geometry

M_0: our 4-D space-time

M_1: compact internal space (with radius R)

standard KK:

$S^1, T^2, S^2, \ldots$

Universal Extra Dimensions (UED):

(Appelquist, Cheng, Dobrescu hep-ph/0012100)

orbifolds: $\underbrace{S^1/Z_2\,,\ T^2/Z_2, \ldots}$

circle, square folded once onto itself due to Z_2 symmetry

⇓

a) Lightest KK particles are stable because of KK-parity (translation by πR + flip of sign of odd states)

b) breaking (super) symmetry

! → c) branes in fixed points

All SM fields propagates in M_1

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experiments: $R \lesssim \mathrm{TeV}^{-1}$

⟹ ? I) Conventional cosmology

II) Stabilization of R

$\left(\varkappa_0^2 = \varkappa_D^2 / V_{d_1}\ ;\ \alpha_4 = \alpha_D / V_{d_1}\right)$

The Model

Metric ansatz:

$$g = -e^{2\gamma(\tau)} d\tau \otimes d\tau + \underbrace{\underbrace{L_{pe}^2 e^{2\beta^0(\tau)}}_{\equiv\, a^2(\tau)} g^{(0)}(\vec{x})}_{\bar{g}^{(0)}(x)} + \sum_1^n \underbrace{L_{pe}^2 e^{2\beta^i(\tau)}}_{\equiv\, b_i^2(\tau)} g^{(i)}(y)$$

compact Einstein spaces (e.g. orbifolds)

Action:

$$S = \frac{1}{2\varkappa_D^2} \int d^D x \sqrt{|g|} \left\{ R[g] - 2\Lambda_D \right\} + S_m$$

Perfect fluid EMT:

$$T_N^M = \sum_{c=1}^m T^{(c)}{}_N^M$$

$$T^{(c)}{}_N^M = diag\left(-\rho^{(c)}(\tau), \underbrace{P_0^{(c)}(\tau), \dots, P_0^{(c)}(\tau)}_{d_0 \text{ times}}, \dots, \underbrace{P_n^{(c)}(\tau), \dots, P_n^{(c)}(\tau)}_{d_n \text{ times}}\right)$$

Conservation equation:

$$T^{(c)}{}_{N;M}^M = 0 \Rightarrow \dot{\rho}^{(c)} + \sum_0^n d_i \dot{\beta}^i \left(\rho^{(c)} + P_i^{(c)}\right) = 0$$

$$\Big\Downarrow \quad P_i^{(c)} = \left(\alpha_i^{(c)} - 1\right) \rho^{(c)}$$

$$\rho^{(c)}(\tau) = A^{(c)} a^{-d_0 \alpha_0^{(c)}} \times \prod_{i=1}^n b_i^{-d_i \alpha_i^{(c)}}$$

I. Dynamical compactification

Einstein eqs. ($n, m = 1$; $\rho^{(1)} \equiv \rho$, $P_0^{(1)} \equiv P_0$, $P_1^{(1)} \equiv P_1$; $d_0 = 3$):

$$\varkappa_D^2 \rho = \underbrace{\left\{3H^2 + \frac{3k_0}{a^2} - \Lambda_D\right\}}_{\text{standard FRW}} + \underbrace{O\left(\left(\frac{\dot b}{b}\right)^2, H\frac{\dot b}{b}, \frac{1}{b^2}\right)}_{\text{higher-dimensional corrections}}$$

$$\varkappa_D^2 P_0 = \overbrace{\left\{-2\frac{\ddot a}{a} - H^2 - \frac{k_0}{a^2} + \Lambda_D\right\}} + \overbrace{O\left(\frac{\ddot b}{b}, \left(\frac{\dot b}{b}\right)^2, H\frac{\dot b}{b}, \frac{1}{b^2}\right)}$$

$$\varkappa_D^2 P_1 = \ldots$$

Conventional cosmology conditions:
(Mohammedi, hep-th/0202119)

$b := \frac{B}{a^q}$, $B = \mathrm{const}$

$d_1 q (d_1 q - q - 6) = 0$

$\underbrace{q = 0}_{\text{static}}$ $\quad$ $\underbrace{q = \frac{6}{d_1 - 1}}_{\text{dynamical}}$

$$\left.\begin{aligned} \rho_{(4)} &\equiv \rho \cdot V_{d_1} \\ P_{(4)} &\equiv \left[P_0 - \frac{d_1 q}{3}(\rho + P_1)\right] V_{d_1} \end{aligned}\right\} \text{observable values}$$

$$\varkappa_0^2 \rho_{(4)} = 3H^2 + \frac{3k_0}{a^2} - \Lambda_D \qquad (1)$$

$$\varkappa_0^2 P_{(4)} = -2\frac{\ddot a}{a} - H^2 - \frac{k_0}{a^2} + \Lambda_D \qquad (2)$$

However!:

I. 4-D gravitational "constant" is dynamical value:

$$\varkappa_0^2 = \varkappa_D^2 / V_{d_1}, \quad V_{d_1} = V_I \cdot b^{d_1} = \bar{V}_I \cdot a^{-d_1 q}$$

II. Energy conservation eq. is different:

$$\frac{d}{dt}\left(a^3 \rho_{(4)}\right) + P_{(4)} \frac{d}{dt}\left(a^3\right) = \left(a^3 \rho_{(4)}\right) \frac{1}{V_{d_1}} \frac{d}{dt}\left(V_{d_1}\right) = -\left(a^3 \rho_{(4)}\right) d_1 q \frac{\dot{a}}{a}$$

$$\Big\Downarrow \quad P_{(4)} = (\alpha - 1)\rho_{(4)}$$

$$\rho_{(4)} = \rho_0 \left(\frac{a_0}{a}\right)^{3\alpha + d_1 q}$$

$$\varkappa_0^2 \rho_{(4)} \sim a^{-3\alpha} \Leftarrow \text{conventional behaviour} \quad !$$

$\Rightarrow$ || Dynamical tuning of the gravitational constant

$$\Big\Downarrow$$

"FRW" eq. (1) has the standard solution (standard Friedmann dynamics!):

$$k_0, \Lambda_D = 0 \Rightarrow \quad a = \left(\frac{3\alpha}{2} a_* t\right)^{2/(3\alpha)} \qquad (3)$$

Fundamental constant variation:

$$G_4 = \varkappa_0^2/8\pi \sim a^{d_1 q} \underset{\text{eq.}(3)}{\sim} t^{2d_1 q/(3\alpha)}$$

$$\Downarrow$$

$$\frac{\dot{G}_4}{G_4} = \frac{2d_1 q}{3\alpha}\frac{1}{t} \underset{t \sim \tau_u \sim 14\,\text{Gyr}}{\sim} 10^{-10}\,\text{yr}^{-1}$$

too big !

(experiments: $\dot{G}_4/G_4 < 10^{-11}\,\text{yr}^{-1}$)

Moreover:

Fine structure constant

$$\alpha_4 = \alpha_D / V_{d_1}$$

$$\Rightarrow \quad \frac{\dot{G}_4}{G_4} = \frac{\dot{\alpha}_4}{\alpha_4}$$

Experiments: $\quad \frac{\dot{\alpha}_4}{\alpha_4} \lesssim 10^{-15}\,\text{yr}^{-1}$

4-D eff. fundamental constants undergo too large variations with time.

II. Stable compactification $\left(\beta^i(x) = \beta_0^i + \bar{\beta}^i(x)\right)$

Dimension reduction $\Rightarrow$ 4-D eff. action:

$$S = \frac{1}{2\varkappa_0^2} \int\limits_{M_0} d^{D_0}x \underbrace{\sqrt{|\tilde{g}^{(0)}|}}_{\text{Einstein frame}} \left\{ \tilde{R}[\tilde{g}^{(0)}] - \bar{G}_{ij}\, \tilde{g}^{(0)\mu\nu} \partial_\mu \bar{\beta}^i \partial_\nu \bar{\beta}^i - 2\, U_{eff} \right\}$$

Einstein frame: $\underset{\substack{\uparrow \\ BD}}{\bar{g}^{(0)}_{\mu\nu}} = \left(\prod_{i=1}^n e^{d_i \bar{\beta}^i}\right)^{\frac{-2}{D_0-2}} \underset{\substack{\uparrow \\ E}}{\tilde{g}^{(0)}_{\mu\nu}}$

$BD \equiv E$ if $\bar{\beta}^i \equiv 0$

Effective action:

$$U_{eff} = \left(\prod_{i=1}^n e^{d_i \bar{\beta}^i}\right)^{\frac{-2}{D_0-2}} \left[-\frac{1}{2} \sum_i^n \underbrace{\tilde{R}_i}_{\sim \exp(-2\beta_0^i) = b_{(0)i}^{-2}} e^{-2\bar{\beta}^i} + \Lambda_D + \varkappa_D^2 \sum_{c=1}^m \rho^{(c)} \right]$$

4-D metric $\tilde{g}^{(0)}_{\mu\nu}$ has FRW form with scale factor $\tilde{a}$

$$\rho^{(c)} = A^{(c)} \underset{\substack{\uparrow \\ BD}}{a}^{-d_0\alpha_0^{(c)}} \prod_{i=1}^n e^{-d_i \alpha_i^{(c)} \beta^i} = \underbrace{\tilde{A}^{(c)} \underset{\substack{\uparrow \\ E}}{\tilde{a}}^{-d_0\alpha_0^{(c)}}}_{\equiv\, \rho^{(c)}_{(4)}} \prod_{i=1}^n e^{-\xi_i^{(c)} \bar{\beta}^i}$$

$$a = \prod_{i=1}^n e^{d_i \bar{\beta}^i} \cdot \tilde{a}$$

$$\xi_i^{(c)} = d_i \left(\alpha_i^{(c)} - \frac{\alpha_0^{(c)} d_0}{d_0 - 1} \right)$$

Condition for stabilization:

$$\left. \frac{\partial U_{eff}}{\partial \bar{\beta}^i} \right|_{\bar{\beta}^i = 0} = 0$$

$$\frac{\partial U_{eff}}{\partial \bar{\beta}^i}\bigg|_{\bar{\beta}^i=0} = 0 \Longrightarrow \underset{\substack{\uparrow \\ \text{const}}}{\tilde{R}_k} = -\frac{d_k}{D_0-2}\left[\sum_{i=1}^{n} \tilde{R}_i - 2\Lambda_D\right] + \varkappa_0^2 \sum_{c=1}^{m} \underset{\substack{\uparrow \\ \text{dynamical}}}{\rho_{(4)}^{(c)}} \left(\xi_k^{(c)} + \frac{2d_k}{D_0-2}\right)$$

$$\Downarrow$$

No-go theorem:

Multidimensional cosmological KK models with perfect fluid as a matter source do not admit stable compactification of the internal spaces with exception of 2 special cases:

I. $\underbrace{\alpha_0^{(c)} = 0}_{\Downarrow} \quad \forall\, \alpha_i^{(c)}, \quad i = 1, \dots, n; \quad c = 1, \dots, m$

$\rho_{(4)}^{(c)} = \text{const} \Rightarrow$ vacuum in external space

e.g. Casimir effect, monopole form fields, branes, ...

II. $\xi_i^{(c)} = -\dfrac{2d_i}{d_0-1} \Longrightarrow \begin{cases} \alpha_0^{(c)} = \dfrac{2}{d_0} + \dfrac{d_0-1}{d_0}\alpha^{(c)} \\ \alpha_i^{(c)} = \alpha^{(c)}, \quad i = 1, \dots, n \end{cases}$

$$\Rightarrow \rho_{(4)}^{(c)} \overset{d_0=3}{\sim} 1/\tilde{a}^{\,2(1+\alpha^{(c)})}$$

e.g. gas of cosmic strings $\Leftarrow \alpha^{(c)} = 0$

dust $\Leftarrow \alpha^{(c)} = 1/2$

radiation $\Leftarrow \alpha^{(c)} = 1$

Particular Model ($n=1$):

I + II

monopole form field: 4-D perfect fluid:

$$S_m = -\frac{1}{2}\int d^D x \sqrt{|g|}\,\frac{1}{d_1!}(F)^2 = -\int d^D x \sqrt{|g|}\,\frac{f_1^2}{b_1^{2d_1}}$$

$$\Downarrow$$

$$\alpha_0^{(c)} = 0, \quad \alpha_1^{(c)} = 2\delta_1^{(c)}$$

$$\left(e^{d_1\bar\beta^1}\right)^{-\frac{2}{D_0-2}} \cdot \varkappa_D^2 \rho^{(c)} = \varkappa_0^2 \rho^{(c)}_{(4)}$$

Effective potential:

$$U_{eff} = \underbrace{\left(e^{d_1\bar\beta^1}\right)^{\frac{-2}{D_0-2}}\left[-\frac{1}{2}\tilde R_1 e^{-2\bar\beta^1} + \Lambda_D + \tilde f_1^2 e^{-2d_1\bar\beta^1}\right]}_{U_{int}(\bar\beta^1)} + \underbrace{\varkappa_0^2 \sum_1^m \rho^{(c)}_{(4)}(\tilde a)}_{U_{ext}(\tilde a)}$$

$$\Downarrow \qquad\qquad \Downarrow$$

stable compactification and Λ_{eff}

dynamics of the Universe

Stabilization:

$$\left.\frac{\partial U_{int}}{\partial \bar\beta^1}\right|_{\bar\beta^1=0} = 0 \implies \frac{D-2}{2d_1}\tilde R = \Lambda_D + d_0 \tilde f_1^2$$

4-D eff. cosmological constant:

$$\Lambda_{eff} := \left. U_{int}\right|_{\bar\beta^1=0} = -\frac{1}{2}\tilde R_1 + \Lambda_D + \tilde f_1^2 > 0$$

acceleration of the Universe

Gravexcitons:

$$m^2_{exci} \sim \left.\frac{\partial^2 U}{\partial \bar\beta^{1\,2}}\right|_{\bar\beta^1=0} = -2\left(\frac{D-2}{D_0-2}\right)^2 \tilde R_1 + \left(\frac{2d_1}{D_0-2}\right)^2 \Lambda_D + \left(\frac{2d_0 d_1}{D_0-2}\right)^2 \tilde f_1^2 > 0$$

$$\Rightarrow \tilde f_1^2, \tilde R_1, \Lambda_D > 0 \quad \text{and} \quad \tilde f_1^2 \sim \Lambda_D \sim \tilde R_1 \sim b_{(0)1}^{-2} \sim 10^{34}\,\mathrm{cm}^{-2}$$

$m_{exci} \sim 1\,\mathrm{TeV}$ $\qquad$ $b_{(0)1} \sim 10^{-17}\,\mathrm{cm} \sim \mathrm{TeV}^{-1}$

However:

$$\Lambda_{obs} \sim 10^{-123}\Lambda_{Pe} \sim 10^{-58}\,\mathrm{cm}^{-2}$$

$$\Downarrow$$

$\tilde f_1^2, \Lambda_D, \tilde R_1$ should be extremely fine tuned to compensate each other in such a way that to leave only $10^{-58}\,\mathrm{cm}^{-2}$

Dynamics of the Universe:

$$S_{eff} = \frac{1}{2\varkappa_0^2}\int d^{D_0}x \sqrt{|\tilde{g}^{(0)}|}\left\{\tilde{R}[\tilde{g}^{(0)}] - 2U_{eff}\right\}$$

$$U_{eff} \uparrow \; \Lambda_{eff} + \varkappa_0^2 \sum_{c=1}^{m} \rho_{(4)}^{(c)}$$

$$\Downarrow$$

equation of motion:

$$\left(\frac{1}{\tilde{a}}\frac{d\tilde{a}}{d\tilde{t}}\right)^2 = -\frac{k_0}{\tilde{a}^2} + \frac{2}{d_0(d_0-1)}\left(\Lambda_{eff} + \varkappa_0^2 \sum_1^m \rho_{(4)}^{(c)}(\tilde{a})\right)$$

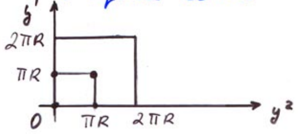
$$\Downarrow$$

$$\tilde{t} = \int \frac{d\tilde{a}}{\left[-k_0 + \frac{\Lambda_{eff}}{3}\tilde{a}^2 + \frac{\varkappa_0^2}{3}\sum_1^m \rho_{(4)}^{(c)} \cdot \tilde{a}^2\right]} \Leftarrow$$ standard FRW equation !

Stabilization of orbifolds:

S^1/Z_2 – 2 fixed points: T^2/Z_2 – 4 fixed points:

↑ square torus



⇒ branes in fixed points:

$$S_b = \sum_{\substack{\text{fixed}\\ \text{points}}} \int d^4x \underbrace{\sqrt{|\bar{g}^{(0)}|}}_{\text{induced metric}} L_b \Big|_{\text{fixed point}}$$

$$L_b \Big|_{\substack{i\text{-th fixed}\\ \text{point}}} = \underbrace{-\tau_i}_{\text{tension of the brane}} = \text{const} \quad , \quad i = 1, \ldots, n$$

Case I: $\alpha_0 = 0, \alpha_1 = 1$.

$$\Rightarrow U_{int} = \left(e^{\bar{\beta}^1}\right)^{\frac{-2d_1}{D_0-2}} \Big[\underbrace{-\tfrac{1}{2}\tilde{R}_1 e^{-2\bar{\beta}^1}}_{=\,0\,!} + \Lambda_D + \tilde{f}_1^{\,2} e^{-2d_1\bar{\beta}^1} - \underbrace{\lambda}_{-\varkappa_0^2 \sum_1^n \tau_i} e^{-d_1\bar{\beta}^1} \Big] \equiv$$

$$\equiv \Lambda_D e^{-A\bar{\beta}^1} + \tilde{f}_1^{\,2} e^{-B\bar{\beta}^1} - \lambda e^{-C\bar{\beta}^1} ,$$

$$A := \frac{2d_1}{D_0-2} \ ; \quad B := \frac{2d_1(D_0-1)}{D_0-2} \ ; \quad C := \frac{d_1 D_0}{D_0-2} \ .$$

Stabilization:

$$\left.\frac{\partial U_{int}}{\partial \bar{\beta}^1}\right|_{\bar{\beta}^1=0} = 0 \Rightarrow \quad C\lambda = A\Lambda_D + B\tilde{f}_1^2$$

4-D effective cosmological constant:

$$\Lambda_{eff} := U_{int}\Big|_{\bar{\beta}^1=0} = \Lambda_D + \tilde{f}_1^2 - \lambda > 0 \quad !$$

Gravexcitons:

$$m_{exci}^2 \sim \left.\frac{\partial^2 U_{int}}{\partial \bar{\beta}^{1\,2}}\right|_{\bar{\beta}^1=0} = A^2\Lambda_D + B^2\tilde{f}_1^2 - C^2\lambda > 0$$

$$\Rightarrow \tilde{f}_1^2, \Lambda_D, \lambda > 0$$

and

$$\underbrace{\tilde{f}_1^2 \sim \Lambda_D \sim \lambda \sim \Lambda_{eff} \sim m_{exci}^2}_{\text{no fine tuning !}}$$

Let

$$\Lambda_{eff} \sim \Lambda_{obs} \sim 10^{-57}\,cm^{-2} \sim (10^{-33}\,eV)^2$$

$$\Rightarrow m_{exci} \sim 10^{-33}\,eV$$