$B \to (\rho, \omega)\gamma$ Decays and CKM Phenomenology

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A. Ali and A.P., Eur. Phys. J. **C23** (2002) 89 [hep-ph/0105302]
A. Ali, E. Lunghi, and A.P., Phys. Lett. **B595** (2004) 323 [hep-ph/0405075]

Contents

Introduction

- Interest in $B \to \rho\gamma$ and $B \to \omega\gamma$ decays is motivated by their potential importance for getting an information on the CKM matrix, in particular, to extract $|V_{td}|$ and the angle γ (in the SM, the angle α can be used alternatively because of the unitarity of the CKM matrix)
- Contributions from tree and penguin operators are of comparable size in the decay amplitude and differ by a weak phase, so both branching fractions and different asymmetries are sensitive to the elements of the CKM matrix
- Different approaches have been applied for an analysis of $B \to V\gamma$ decays, where $V = K^*, \rho, \omega$ mesons: generalized QCD factorization, perturbative QCD approach, SCET formalism, etc.
- QCD factorization is one of the most consistent frameworks for a description of exclusive decays; its application to the $B \to V\gamma$ decays and comparison of results with existing experimental data is a main topic of this talk

Effective Electroweak Theory

- Weak interaction phenomena at energies $E \ll M_W, M_Z$ are most conveniently described in the framework of an effective theory
- This theory is derived from the SM by integrating out heavy particles – the top quark, W- and Z-bosons
- Lagrangian density includes all the other quark flavors $q = u, d, s, c, b$

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD\times QED} + \mathcal{L}_{\rm weak}^{b\to d} + \mathcal{L}_{\rm weak}^{b\to s}$$

- Flavour-changing neutral current (FCNC) term $\mathcal{L}_{\rm weak}^{b\to d}$ describes the $b \to d$ transition

$$\mathcal{L}_{\rm weak}^{b\to d} = -\frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(d)} \sum_j C_j(\mu) \mathcal{O}_j(\mu)$$

- $\mathcal{L}_{\rm weak}^{b\to s}$ for $b \to s$ transition can be obtained from $\mathcal{L}_{\rm weak}^{b\to d}$ by replacements:

1. $d \to s$ for the quark fields in all the operators $\mathcal{O}_j(\mu)$
2. $\lambda_d^{(p)} \equiv V_{pb} V_{pd}^* \to \lambda_s^{(p)} \equiv V_{pb} V_{ps}^*$ in the CKM factors
3. $m_d \to m_s$ in the operators $\mathcal{O}_{7\gamma}(\mu)$ and $\mathcal{O}_{8g}(\mu)$

Wilson Coefficients

- Wilson coefficients $C_j(\mu)$ are determined by matching Green's functions of the effective theory and the SM (or its extension) at the electroweak scale $\mu_W = \mathcal{O}(M_W)$
- Application of the Renormalization Group Equation (RGE)

$$\mu \frac{d}{d\mu} C_j(\mu) = \gamma_{kj}(\mu)\, C_k(\mu)$$

allows to evolve $C_j(\mu)$ to the relevant low-energy scale $\mu_b = \mathcal{O}(m_b)$

- Large logarithms $\ln(\mu_W^2/\mu_b^2)$ are resummed from all orders of the perturbation series
- Hierarchy in the Wilson coefficients exists; in the naive dimentional regularization scheme at the next-to-leading logarithmic (NLL) order

$C_1(m_b)$	1.080	$C_3(m_b)$	0.011	$C_7(m_b)$	4.9×10^{-4}
$C_2(m_b)$	-0.177	$C_4(m_b)$	-0.033	$C_8(m_b)$	4.6×10^{-4}
$C_{7\gamma}(m_b)$	-0.317	$C_5(m_b)$	0.010	$C_9(m_b)$	-9.8×10^{-3}
$C_{8g}(m_b)$	0.149	$C_6(m_b)$	-0.040	$C_{10}(m_b)$	1.9×10^{-3}

Operator Basis

- For most phenomenological applications, only operators $\mathcal{O}_j(\mu)$ of the dimension $d = 5$ and $d = 6$ are relevant
- The standard basis of four-fermion operators for the $b \to s$ transition
  - Tree Operators

$$\mathcal{O}_1^{(p)} = (\bar{s}_\alpha p_\alpha)_{V-A} (\bar{p}_\beta b_\beta)_{V-A} \qquad \mathcal{O}_2^{(p)} = (\bar{s}_\alpha p_\beta)_{V-A} (\bar{p}_\beta b_\alpha)_{V-A}$$

  - QCD Penguins

$$\mathcal{O}_3 = (\bar{s}_\alpha b_\alpha)_{V-A} \sum_q (\bar{q}_\beta q_\beta)_{V-A} \qquad \mathcal{O}_4 = (\bar{s}_\alpha b_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V-A}$$

$$\mathcal{O}_5 = (\bar{s}_\alpha b_\alpha)_{V-A} \sum_q (\bar{q}_\beta q_\beta)_{V+A} \qquad \mathcal{O}_6 = (\bar{s}_\alpha b_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V+A}$$

  - Electroweak Penguins

$$\mathcal{O}_7 = (\bar{s}_\alpha b_\alpha)_{V-A} \sum_q \frac{3e_q}{2} (\bar{q}_\beta q_\beta)_{V+A} \qquad \mathcal{O}_8 = (\bar{s}_\alpha b_\beta)_{V-A} \sum_q \frac{3e_q}{2} (\bar{q}_\beta q_\alpha)_{V+A}$$

$$\mathcal{O}_9 = (\bar{s}_\alpha b_\alpha)_{V-A} \sum_q \frac{3e_q}{2} (\bar{q}_\beta q_\beta)_{V-A} \qquad \mathcal{O}_{10} = (\bar{s}_\alpha b_\beta)_{V-A} \sum_q \frac{3e_q}{2} (\bar{q}_\beta q_\alpha)_{V-A}$$

- Electromagnetic and chromomagnetic dipole operators

$$\mathcal{O}_{7\gamma} = \frac{e}{8\pi^2} (\bar{s}_\alpha \sigma^{\mu\nu} [m_b R + m_s L] b_\alpha) F_{\mu\nu} \quad \mathcal{O}_{8g} = \frac{g_s}{8\pi^2} (\bar{s}_\alpha \sigma^{\mu\nu} [m_b R + m_s L] T^A_{\alpha\beta} b_\beta) G^A_{\mu\nu}$$

$B \to V$ Transition Form Factors

The general decomposition of the matrix elements on all possible Lorentz structures admits seven scalar functions (form factors): $V^{(V)}$, $A_i^{(V)}$, $T_i^{(V)}$ ($V = K^*, \rho$) of the momentum squared q^2 transferred from the heavy meson to the light one ($Q = d, s$)

$$\left\langle V(p,\varepsilon^*)|\bar{Q}\,\gamma^\mu b|\bar{B}(P)\right\rangle = \frac{2i\,V^{(V)}(q^2)}{M + m_V}\,\epsilon^{\mu\nu\rho\sigma}\,\varepsilon^*_\nu p_\rho P_\sigma$$

$$\left\langle V(p,\varepsilon^*)|\bar{Q}\,\gamma^\mu\gamma_5 b|\bar{B}(P)\right\rangle = A_1^{(V)}(q^2)\,(M + m_V)\left[\varepsilon^{*\mu} - \frac{(\varepsilon^* q)}{q^2}\,q^\mu\right]$$
$$- A_2^{(V)}(q^2)\,\frac{(\varepsilon^* q)}{M + m_V}\left[P^\mu + p^\mu - \frac{M^2 - m_V^2}{q^2}\,q^\mu\right] + 2m_V\,A_0^{(V)}(q^2)\,\frac{(\varepsilon^* q)}{q^2}\,q^\mu$$

$$\left\langle V(p,\varepsilon^*)|\bar{Q}\,\sigma^{\mu\nu}q_\nu b|\bar{B}(P)\right\rangle = 2\,T_1^{(V)}(q^2)\,\epsilon^{\mu\nu\rho\sigma}\,\varepsilon^*_\nu p_\rho P_\sigma$$

$$\left\langle V(p,\varepsilon^*)|\bar{Q}\,\sigma^{\mu\nu}\gamma_5 q_\nu b|\bar{B}(P)\right\rangle = -i\,T_2^{(V)}(q^2)\,[(M^2 - m_V^2)\,\varepsilon^{*\mu} - (\varepsilon^* q)\,(P + p)^\mu]$$
$$- i\,T_3^{(V)}(q^2)\,(\varepsilon^* q)\left[q^\mu - \frac{q^2}{M^2 - m_V^2}\,(P + p)^\mu\right]$$

Symmetry Relations in $B \to V$ Transitions

The heavy quark symmetry and the behaviour of the final meson in the large energy limit (the large recoil limit) allow to reduce the number of independent form factors to two only. For the case $q^2 = 0$, there are the following form factor relations (terms of order m_V^2/M^2 are neglected):

$$\frac{M\, V^{(V)}(0)}{M + m_V} = \frac{M + m_V}{M}\, A_1^{(V)}(0) = T_1^{(V)}(0) = T_2^{(V)}(0) = \xi_\perp^{(V)}(0)$$

$$\frac{2 m_V}{M}\, A_0^{(V)}(0) = \frac{M + m_V}{M}\, A_1^{(V)}(0) - \frac{M - m_V}{M}\, A_2^{(V)}(0) = T_2^{(V)}(0) - T_3^{(V)}(0) = \xi_\parallel^{(V)}(0)$$

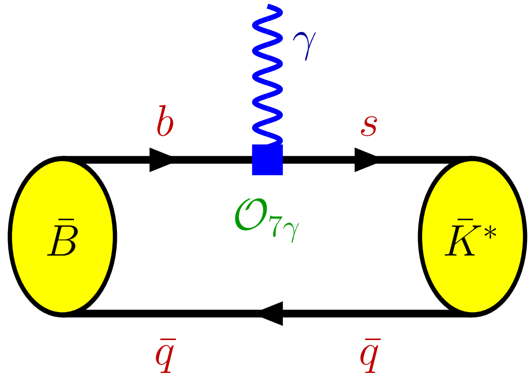
To incorporate QCD corrections, a tentative factorization formula at large recoil and at leading order in the inverse heavy-meson mass was introduced

$$T_1^{(V)}(0) = C_{T_1}^{(V)}\, \xi_\perp^{(V)}(0) + \int dk_+ \int_0^1 du\, \phi_+^{(B)}(k_+)\, \mathcal{T}_{T_1}(k_+, u)\, \phi_\perp^{(V)}(u)$$

- $C_{T_1}^{(V)} = 1 + O(\alpha_s)$ is a renormalization coefficient
- $\mathcal{T}_{T_1}(k_+, u)$ is a hard-scattering kernel calculated in $O(\alpha_s)$
- $\phi_+^{(B)}(k_+)$ and $\phi_\perp^{(V)}(u)$ are leading-twist LCDAs of B and V mesons

$B \to K^*\gamma$ Branching Fraction in LO

- In the leading order, the electromagnetic penguin operator $\mathcal{O}_{7\gamma}$ contributes only in the $B \to K^*\gamma$ amplitude; the form-factor symmetry relation $T_1^{(K^*)}(0) = T_2^{(K^*)}(0)$ should be taken into account



$$M_0^{\rm LO} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* C_7^{\rm eff} \frac{e \bar{m}_b}{4\pi^2} T_1^{(K^*)}(0) \left[(Pq)(e^* \varepsilon^*) - (e^* P)(\varepsilon^* q) + i \, {\rm eps}(e^*, \varepsilon^*, P, q) \right]$$

Here, $P^\mu = p_B^\mu + p_K^\mu$; $q^\mu = p_B^\mu - p_K^\mu$ is the photon four-momentum; e^μ is its polarization vector; ε^μ is the K^*-meson polarization vector

- The branching ratio can be easily obtained and results in the form:

$$\mathcal{B}_{\rm th}^{\rm LO}(B \to K^*\gamma) = \tau_B \frac{G_F^2 |V_{tb} V_{ts}^*|^2 \alpha M^3}{32\pi^4} \bar{m}_b^2(\mu_b) \, |C_7^{\rm eff}(\mu_b)|^2 \, |T_1^{(K^*)}(0, \mu_b)|^2$$

- It is natural to assume the dependence on $\mu_b = \mathcal{O}(m_b)$ of $T_1^{(K^*)}(0, \mu_b)$ for compensating this dependence in the branching ratio, originated by the b-quark mass $\bar{m}_b(\mu_b)$ and the Wilson coefficient $C_7^{\rm eff}(\mu_b)$

$B \to K^* \gamma$ Branching Fraction in NLO

$$\mathcal{B}_{\rm th}(B \to K^* \gamma) = \tau_B \, \frac{G_F^2 \alpha |V_{tb} V_{ts}^*|^2}{32\, \pi^4} \, m_{b,{\rm pole}}^2 \, M^3 \, \left[\xi_\perp^{(K^*)}(0) \right]^2 \left| C_7^{(0){\rm eff}}(\mu) + A^{(1)}(\mu) \right|^2$$

The function $A^{(1)}(\mu)$ includes all the NLO corrections

$$A^{(1)}(\mu) = A_{C_7}^{(1)}(\mu) + A_{\rm ver}^{(1)}(\mu) + A_{\rm sp}^{(1)K^*}(\mu_{\rm sp})$$

- $A_{C_7}^{(1)}(\mu)$ is $\mathcal{O}(\alpha_s)$ correction to the Wilson coefficient $C_7^{\rm eff}(\mu)$
- $A_{\rm ver}^{(1)}(\mu)$ is $\mathcal{O}(\alpha_s)$ correction to the $b \to s \gamma$ vertex
- $A_{\rm sp}^{(1)}(\mu_{\rm sp})$ is the hard-spectator contribution evaluated at an intermediate scale $\mu_{\rm sp} = \sqrt{\mu \Lambda_{\rm H}}$

Can be used to extract the value of $\xi_\perp^{(K^*)}(0)$ from experimental data

Experimental Data on $B \to K^*\gamma$ Decays

Branching ratios (in units of 10^{-6}) [August 2006]

Quantity	BABAR	BELLE	CLEO	Average
$\mathcal{B}(B^+ \to K^{*+}\gamma)$	$38.7 \pm 2.8 \pm 2.6$	$42.5 \pm 3.1 \pm 2.4$	$37.6^{+8.9}_{-8.3} \pm 2.8$	40.3 ± 2.6
$\mathcal{B}(B^0 \to K^{*0}\gamma)$	$39.2 \pm 2.0 \pm 2.4$	$40.1 \pm 2.1 \pm 1.7$	$45.5^{+7.2}_{-6.8} \pm 3.4$	40.1 ± 2.0
$\mathcal{B}(B \to K^*\gamma)$	40.4 ± 2.5	42.8 ± 2.4	43.3 ± 6.2	41.8 ± 1.7
$\mathcal{B}(B \to X_s\gamma)$	$327 \pm 18^{+55}_{-41}$	$355 \pm 32^{+30+11}_{-31-7}$	$321 \pm 43^{+32}_{-29}$	$355 \pm 24^{+9}_{-10} \pm 3$
$R(K^*\gamma/X_s\gamma)$	$0.124^{+0.024}_{-0.019}$	$0.121^{+0.018}_{-0.015}$	$0.135^{+0.033}_{-0.027}$	0.124 ± 0.012

$$\bar{\mathcal{B}}(B \to K^*\gamma) \equiv \frac{1}{2}\left[\mathcal{B}(B^+ \to K^{*+}\gamma) + \frac{\tau_{B^+}}{\tau_{B^0}}\mathcal{B}(B^0 \to K^{*0}\gamma)\right]$$

$$R(K^*\gamma/X_s\gamma) \equiv \frac{\bar{\mathcal{B}}(B \to K^*\gamma)}{\mathcal{B}(B \to X_s\gamma)}$$

Life-time ratio $\tau_{B^+}/\tau_{B^0} = 1.076 \pm 0.008$

Phenomenological Evaluation of $\xi_\perp^{(K^*)}(0)$

- $\mathcal{B}(B^+ \to K^{*+}\gamma)$ $\qquad \xi_\perp^{(K^*)}(0) = 0.262 \pm 0.023$
- $\mathcal{B}(B^0 \to K^{*0}\gamma)$ $\qquad \xi_\perp^{(K^*)}(0) = 0.272 \pm 0.021$
- $R(K^*\gamma/X_s\gamma)$ $\qquad \xi_\perp^{(K^*)}(0) = 0.253 \pm 0.018$

Average $\qquad \bar{\xi}_\perp^{(K^*)}(0) = 0.26 \pm 0.02$

QCD form factor $\qquad \bar{T}_1^{(K^*)}(0, \bar{m}_b) \simeq 1.04\, \bar{\xi}_\perp^{(K^*)}(0) = 0.27 \pm 0.02$

$B \to \rho\gamma$ Branching Fraction

- Ignoring perturbative QCD corrections to amplitudes, the ratio of charged and neutral B-meson decay widths can be written as

$$\frac{\Gamma(B^- \to \rho^-\gamma)}{2\,\Gamma(B^0 \to \rho^0\gamma)} \simeq \left| 1 + \epsilon_A \mathrm{e}^{i\phi_A} \frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*} \right|^2$$

- $\epsilon_A \mathrm{e}^{i\phi_A}$ includes dominant W-annihilation and possible sub-dominant long-distance contributions
- The strong interaction phase ϕ_A disappears in $\mathcal{O}(\alpha_s)$ in the chiral limit and, henceforth, we set $\phi_A = 0$
- Isospin-violating corrections depend on the unitarity triangle angle α

$$\frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*} = -\left| \frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*} \right| \mathrm{e}^{i\alpha} = F_1 + iF_2$$

$B \to \rho\gamma$ Branching Fraction in NLO

- Including the annihilation contribution, the charged-conjugate averaged branching ratio in the NLO is

$$
\begin{aligned}
\bar{\mathcal{B}}_{\rm th}(B^\pm \to \rho^\pm\gamma) &= \tau_{B^+} \frac{G_F^2 \alpha |V_{tb} V_{td}^*|^2}{32\pi^4}\, m_{b,\rm pole}^2\, M^3 \left[\xi_\perp^{(\rho)}(0)\right]^2 \\
&\times \left\{ (C_7^{(0)\rm eff} + A_R^{(1)t})^2 + (F_1^2 + F_2^2)\,(A_R^u + L_R^u)^2 \right. \\
&+ \left. 2F_1 \left[C_7^{(0)\rm eff}(A_R^u + L_R^u) + A_R^{(1)t} L_R^u\right] \right\}
\end{aligned}
$$

- $L_R^u = \epsilon_A\, C_7^{(0)\rm eff}$ detemines the strength of the annihilation contribution
- The amplitude $A^{(1)t}(\mu)$ can be decomposed in three contributing parts

$$
A^{(1)t}(\mu) = A_{C_7}^{(1)}(\mu) + A_{\rm ver}^{(1)}(\mu) + A_{\rm sp}^{(1)\rho}(\mu_{\rm sp})
$$

- In addition to $A^{(1)t}$, the u-quark contribution A^u from the penguin diagrams can no longer be ignored; it also involves hard-spectator corrections

$B \to (\rho, \omega)\gamma$ Branching Fractions in NLO

- For numerical predictions, it is better to use ratio of the $B \to \rho\gamma$ and $B \to K^*\gamma$ decay widths and then connect it with the experimentally measured values of $B \to K^*\gamma$ branching ratios ($S_\rho = 1$ for the $\rho^\pm$-meson and $S_\rho = 1/2$ for the ρ^0-meson)

$$\frac{\overline{\mathcal{B}}_{\rm th}(B \to \rho\gamma)}{\overline{\mathcal{B}}_{\rm th}(B \to K^*\gamma)} = S_\rho \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{(M^2 - m_\rho^2)^3}{(M^2 - m_{K^*}^2)^3} \zeta^2 \left[1 + \Delta R(\rho/K^*)\right]$$

- Similar ratio for $B^0 \to \omega\gamma$ and $B^0 \to K^{*0}\gamma$ decays

$$\frac{\overline{\mathcal{B}}_{\rm th}(B^0 \to \omega\gamma)}{\overline{\mathcal{B}}_{\rm th}(B^0 \to K^{*0}\gamma)} = \frac{1}{2} \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{(M^2 - m_\omega^2)^3}{(M^2 - m_{K^*}^2)^3} \zeta^2 \zeta_{\omega/\rho}^2 \left[1 + \Delta R(\omega/K^*)\right]$$

- The quantity ζ and $\zeta_{\omega/\rho}$ are the ratios of the HQET/LEET form factors; in $SU_{\rm F}(3)$-symmetry limit, ratios $\zeta = 1$ and $\zeta_{\omega/\rho} = 1$
- $SU_{\rm F}(3)$-breaking effects in the QCD form factors $T_1^{(K^*)}(0)$, $T_1^{(\rho)}(0)$, and $T_1^{(\omega)}(0)$ have been evaluated within the QCD sum-rules

$$\zeta = \frac{T_1^{(\rho)}(0)}{T_1^{(K^*)}(0)} \simeq \frac{\xi_\perp^{(\rho)}(0)}{\xi_\perp^{(K^*)}(0)} = 0.86 \pm 0.07 \qquad \zeta_{\omega/\rho} = \frac{T_1^{(\omega)}(0)}{T_1^{(\rho)}(0)} \simeq \frac{\xi_\perp^{(\omega)}(0)}{\xi_\perp^{(\rho)}(0)} = 0.9 \pm 0.1$$

$B \to (\rho, \omega)\gamma$ Branching Fractions in NLO

- Theoretical expression for the dynamical function $\Delta R(\rho/K^*)$

$$\Delta R(\rho/K^*) = 2\epsilon_A F_1 + \epsilon_A^2 (F_1^2 + F_2^2)$$

$$+ \frac{2}{C_7^{(0)\mathrm{eff}}} \mathrm{Re}\left[A_{\mathrm{sp}}^{(1)\rho} - A_{\mathrm{sp}}^{(1)K^*} + F_1 (A^u + \epsilon_A A^{(1)t}) + \epsilon_A (F_1^2 + F_2^2) A^u\right]$$

- Allowed ranges of $\Delta R(\rho/K^*)$ and $\Delta R(\omega/K^*)$ are estimated to be

$$\Delta R(\rho^\pm/K^{*\pm}) = 0.057^{+0.057}_{-0.055} \qquad \Delta R(\rho^0/K^{*0}) = 0.006^{+0.046}_{-0.043}$$

$$\Delta R(\omega/K^{*0}) = -0.002^{+0.046}_{-0.043}$$

Central values of all these functions are close to zero; impact uncertainties $\sim 5\%$ in the ratios

- Main uncertainties in $\Delta R(\rho/K^*)$ come from the variations of CKM angle α and nonperturbative parameters $\xi_\perp^{(\rho)}(0)$ and $\xi_\perp^{(K^*)}(0)$ which can be seen, in particular, for the $B^\pm \to \rho^\pm\gamma$ decay from the table

Contributing Uncertainties in Dynamical Function

Parameter	Value	$\delta R(\rho^\pm/K^{*\pm})$
$\xi_\perp^{(\rho)}(0)$	0.225 ± 0.027	$+0.0356/-0.0280$
α	$(97.3^{+4.5}_{-5.0})^\circ$	$+0.0231/-0.0259$
$\bar{\xi}_\perp^{(K^*)}(0)$	0.262 ± 0.023	$+0.0182/-0.0217$
ϵ_A	0.30 ± 0.07	$+0.0182/-0.0162$
$f_\perp^{(\rho)}(1\ \text{GeV})$	(165 ± 9) MeV	± 0.0143
$a_{\perp 2}^{(K^*)}(1\ \text{GeV})$	0.11 ± 0.09	± 0.0130
$f_\perp^{(K^*)}(1\ \text{GeV})$	(185 ± 10) MeV	± 0.0122
$a_{\perp 2}^{(\rho)}(1\ \text{GeV})$	0.15 ± 0.07	± 0.0118
$\sqrt{z} = m_c/m_b$	0.27 ± 0.06	$+0.0090/-0.0097$
$a_{\perp 1}^{(K^*)}(1\ \text{GeV})$	0.04 ± 0.03	± 0.0056
f_B	(205 ± 25) MeV	± 0.0044
$\lvert V_{ub}/V_{td}\rvert = \sqrt{F_1^2 + F_2^2}$	0.448 ± 0.024	± 0.0034
$\mu/m_{b,\text{pole}}$	$0.5 - 2.0$	± 0.0034
$\lambda_{B,+}^{-1}(1\ \text{GeV})$	(1.79 ± 0.05) GeV^{-1}	± 0.0010
$\sigma_{B,+}(1\ \text{GeV})$	(1.57 ± 0.27)	± 0.0006
$m_{b,\text{pole}}$	(4.65 ± 0.10) GeV	± 0.0001
$\Delta R(\rho^\pm/K^{*\pm})$	$0.057^{+0.057}_{-0.055}$	

$B \to (\rho, \omega)\gamma$ Branching Fractions

- Taking into account the ratio of the CKM matrix elements

$$|V_{td}/V_{ts}| = 0.201 \pm 0.008$$

the branching ratios can be estimated as

$$\bar{\mathcal{B}}_{\rm th}(B^\pm \to \rho^\pm\gamma) = (1.37 \pm 0.26[\rm th] \pm 0.09[\rm exp]) \times 10^{-6}$$
$$\bar{\mathcal{B}}_{\rm th}(\bar{B}^0 \to \rho^0\gamma) = (0.65 \pm 0.12[\rm th] \pm 0.03[\rm exp]) \times 10^{-6}$$
$$\bar{\mathcal{B}}_{\rm th}(\bar{B}^0 \to \omega\gamma) = (0.53 \pm 0.10[\rm th] \pm 0.02[\rm exp]) \times 10^{-6}$$

- In the above estimates, the first error is defined by the uncertainties of the theory and the second is from the direct experimental data on the $B \to K^*\gamma$ branching ratios

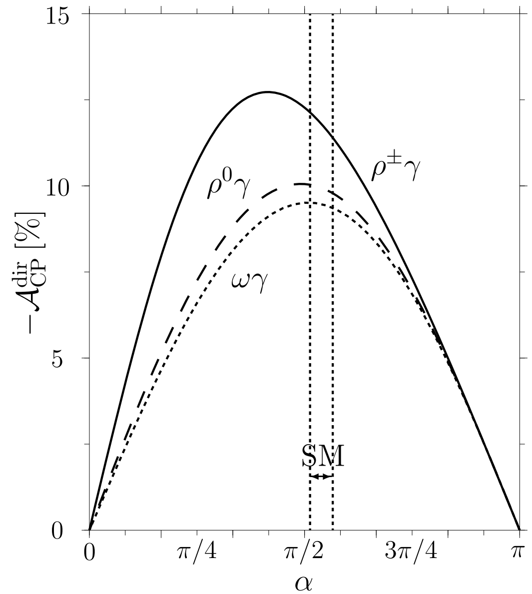
Branching ratios (in units of 10^{-6}) [August 2006]

Mode	BABAR	BELLE	CLEO	Average [HFAG]
$B^+ \to \rho^+\gamma$	$1.06^{+0.35}_{-0.31} \pm 0.09$	$0.55^{+0.42+0.09}_{-0.36-0.08}$	< 13.0	$0.87^{+0.27}_{-0.25}$
$B^0 \to \rho^0\gamma$	$0.77^{+0.21}_{-0.19} \pm 0.07$	$1.25^{+0.37+0.07}_{-0.33-0.06}$	< 17.0	$0.91^{+0.19}_{-0.18}$
$B^0 \to \omega\gamma$	$0.39^{+0.24}_{-0.20} \pm 0.03$	$0.56^{+0.34+0.05}_{-0.27-0.10}$	< 9.2	$0.45^{+0.20}_{-0.17}$

Direct CP-Asymmetry

- CP-asymmetry arises from interference of penguin operator $\mathcal{O}_{7\gamma}$ and four-quark operators $\mathcal{O}_1$ and $\mathcal{O}_2$
- Direct CP-asymmetry in the $B^\pm \to \rho^\pm\gamma$ decay rates

$$\mathcal{A}_{\rm CP}(\rho^\pm\gamma) = \frac{\mathcal{B}(B^- \to \rho^-\gamma) - \mathcal{B}(B^+ \to \rho^+\gamma)}{\mathcal{B}(B^- \to \rho^-\gamma) + \mathcal{B}(B^+ \to \rho^+\gamma)}$$



- Similar defenitions in other two modes $B^0 \to \rho^0\gamma$ and $B^0 \to \omega\gamma$
- SM estimate results

$$\mathcal{A}_{\rm CP}(\rho^\pm\gamma) = (-11.8 \pm 2.9)\%$$

$$\mathcal{A}_{\rm CP}(\rho^0\gamma) = (-9.9^{+3.8}_{-3.4})\%$$

$$\mathcal{A}_{\rm CP}(\omega\gamma) = (-9.5^{+4.0}_{-3.6})\%$$

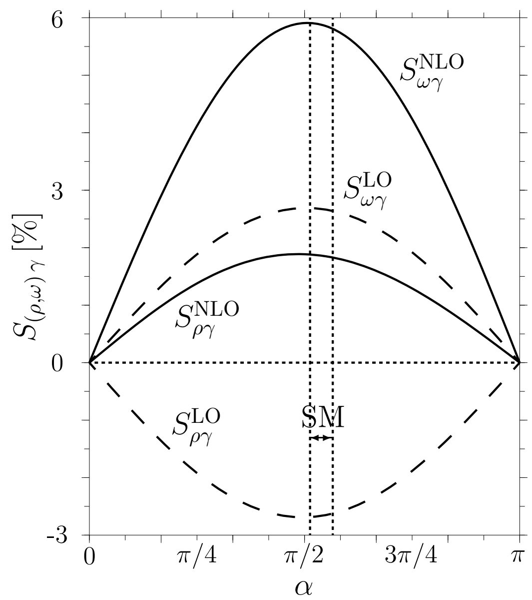
Mixing-Induced CP-Asymmetry

- Time-dependent CP-asymmetry in neutral B-meson decays involves interference of $B^0 - \bar{B}^0$ mixing and decay amplitudes

$$a_{\rm CP}^{\rho\gamma}(t) = -C_{\rho\gamma} \cos(\Delta M_d\, t) + S_{\rho\gamma} \sin(\Delta M_d\, t)$$

- $C_{\rho\gamma} = -\mathcal{A}_{\rm CP}^{\rm dir}(\rho^0\gamma)$
- Mixing-induced CP-asymmetry is

$$S_{\rho\gamma} = \frac{2\,{\rm Im}(\lambda_{\rho\gamma})}{1 + |\lambda_{\rho\gamma}|^2}, \qquad \lambda_{\rho\gamma} \equiv \frac{q}{p}\, \frac{A(\bar{B}_d^0 \to \rho^0\gamma)}{A(B_d^0 \to \rho^0\gamma)}$$



- In the SM, $q/p = {\rm e}^{-2i\beta}$ is pure phase factor to a good approximation
- Similar defenitions can be written for the $B^0 \to \omega\gamma$ decay mode
- SM estimates

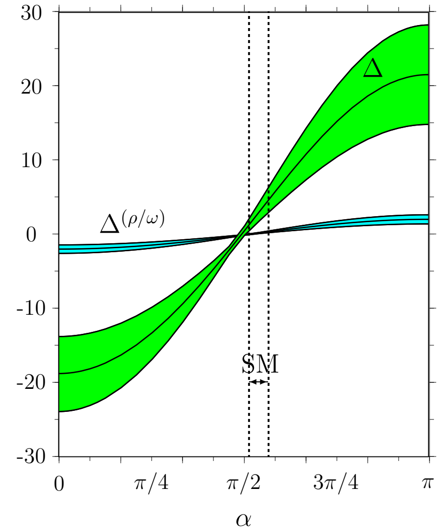
$$S_{\rho\gamma}^{\rm LO} = (-2.7 \pm 0.9)\%, \qquad S_{\rho\gamma}^{\rm NLO} = (1.9^{+3.8}_{-3.2})\%$$

$$S_{\omega\gamma}^{\rm LO} = (+2.7 \pm 0.9)\%, \qquad S_{\omega\gamma}^{\rm NLO} = (5.9^{+4.1}_{-3.5})\%$$

Isospin- and $SU_{\rm F}(3)$-Violating Ratios

- Numerical analysis is peformed for charged-conjugate averaged isospin-violating ratio

$$\Delta = \frac{1}{2}\left[\Delta^{+0} + \Delta^{-0}\right], \quad \Delta^{\pm 0} = \frac{\Gamma(B^\pm \to \rho^\pm \gamma)}{2\Gamma(B^0(\bar{B}^0) \to \rho^0 \gamma)} - 1$$



- SM estimate $\Delta = (2.9 \pm 2.1)\%$
- BABAR measurement $\Delta_{\rm exp} = (-36 \pm 27)\%$
- The ratio $\Delta^{(\rho/\omega)}$ based on neutral B-meson modes can be also of interest

$$\Delta^{(\rho/\omega)} \equiv \frac{1}{2}\left[\Delta_B^{(\rho/\omega)} + \Delta_{\bar{B}}^{(\rho/\omega)}\right]$$

$$\Delta_B^{(\rho/\omega)} \equiv \frac{(M_B^2 - m_\omega^2)^3\, \mathcal{B}(B_d^0 \to \rho^0\gamma) - (M_B^2 - m_\rho^2)^3\, \mathcal{B}(B_d^0 \to \omega\gamma)}{(M_B^2 - m_\omega^2)^3\, \mathcal{B}(B_d^0 \to \rho^0\gamma) + (M_B^2 - m_\rho^2)^3\, \mathcal{B}(B_d^0 \to \omega\gamma)}$$

- SM estimate $\Delta^{(\rho/\omega)} = (11 \pm 11)\%$ mainly determined by $\zeta_{\omega/\rho}$
- HFAG averages result $\Delta_{\rm exp}^{(\rho/\omega)} = (34 \pm 20)\%$
- Both are far from their experimental measurements in the near future

$SU_{\mathrm{F}}(3)$-averaged $B \to (\rho, \omega)\gamma$ Branching Ratio

- This averaging procedure is defined as

$$\bar{\mathcal{B}}[B \to (\rho, \omega)\,\gamma] \equiv \frac{1}{2}\left\{\mathcal{B}(B^+ \to \rho^+\gamma) + \frac{\tau_{B^+}}{\tau_{B^0}}\left[\mathcal{B}(B_d^0 \to \rho^0\gamma) + \mathcal{B}(B_d^0 \to \omega\gamma)\right]\right\}$$

- Combining all the branching fractions together, such an estimate gives

$$\bar{\mathcal{B}}_{\mathrm{th}}[B \to (\rho, \omega)\gamma] = (1.32 \pm 0.25) \times 10^{-6}$$

- Good agreement with experimental measurements within current errors

Branching ratios (in units of 10^{-6}) [August 2006]

Mode	BABAR	BELLE	CLEO	Average [HFAG]
$B \to (\rho, \omega)\gamma$	$1.01 \pm 0.21 \pm 0.08$	$1.32^{+0.34+0.10}_{-0.31-0.09}$	< 14.0	$1.11^{+0.19}_{-0.18}$

Determination of $|V_{td}/V_{ts}|$ from $\bar{R}_{\rm exp}[(\rho,\omega)\,\gamma/K^*\gamma]$

- To extract the value of $|V_{td}/V_{ts}|$ from the $B \to (K^*, \rho, \omega)\,\gamma$ decays, one can use the ratio

$$\bar{R}_{\rm exp}[(\rho,\omega)\gamma/K^*\gamma] = \frac{\bar{\mathcal{B}}_{\rm exp}[B \to (\rho,\omega)\,\gamma]}{\bar{\mathcal{B}}_{\rm exp}(B \to K^*\gamma)} = r_{\rm th}^{(\rho/\omega)} \left|\frac{V_{td}}{V_{ts}}\right|^2 \zeta^2$$

- ζ and $|V_{td}/V_{ts}|$ are treated as free variables
- All other parametric uncertainties are combined in $r_{\rm th}^{(\rho/\omega)}$ error

$$r_{\rm th}^{(\rho/\omega)} = 1.09 \pm 0.06$$

- Recent result $\zeta = 0.86 \pm 0.07$ by Ball and Zwicky can be used

Quantity	BABAR	BELLE	Average [HFAG]
$\bar{R}_{\rm exp}[(\rho,\omega)\gamma/K^*\gamma]$	0.025 ± 0.006	$0.032 \pm 0.008 \pm 0.002$	0.027 ± 0.005
$\lvert V_{td}/V_{ts}\rvert\,\zeta$	$0.151^{+0.017}_{-0.019}$	$0.171^{+0.021}_{-0.024}$	0.156 ± 0.014
$\lvert V_{td}/V_{ts}\rvert$	0.176 ± 0.026	0.199 ± 0.031	0.181 ± 0.022

- For comparison, from the global CKM fit by the CKMfitter Group

$$|V_{td}/V_{ts}| = 0.2011^{+0.0081}_{-0.0065}$$

Summary

- Hard-spectator corrections in $O(\alpha_s)$ and leading order in $\Lambda_{\rm QCD}/M$ to decay amplitudes for $B \to V\gamma$ ($V = K^*, \rho, \omega$) are combined with existing contributions from the vertex corrections and weak-annihilation amplitudes to arrive at NLO expressions for corresponding decay rates
- More reliable theoretical route to calculate $B \to (\rho,\omega)\gamma$ decay rates is via the ratio $\mathcal{B}(B \to (\rho,\omega)\gamma)/\mathcal{B}(B \to K^*\gamma)$ using experimental measurements of $B \to K^*\gamma$ decay rates. Dynamical functions $\Delta R(\rho/K^*)$ and $\Delta R(\rho/K^*)$ are constrained in the ranges: $\Delta R(\rho^\pm/K^{*\pm}) = 0.057^{+0.057}_{-0.055}$, $\Delta R(\rho^0/K^{*0}) = 0.006^{+0.046}_{-0.043}$, and $\Delta R(\omega/K^{*0}) = -0.002^{+0.046}_{-0.043}$, with central values equal to zero to a good approximation. This quantifies the statement that ratios $\mathcal{B}(B \to \rho\gamma)/\mathcal{B}(B \to K^*\gamma)$ and $\mathcal{B}(B \to \omega\gamma)/\mathcal{B}(B \to K^*\gamma)$ are stable against $O(\alpha_s)$ and $1/M$ corrections
- The predicted branching ratios in expected ranges of CKM parameters are in agreement with current experimental data within errors but improvements at higher statistics by both the BABAR and BELLE collaborations are required
- Charged-conjugate average Δ of isospin-violating ratios for $B \to \rho\gamma$ is found to be $\Delta = (3 \pm 2)\%$ in expected ranges of CKM parameters. This prediction quite a bit differs from recent BABAR result $\Delta_{\rm exp} = (-36 \pm 27)\%$ but still consistent with it within errors

Summary (Cont.)

- $SU_{\mathrm{F}}(3)$-violating ratio $\Delta^{(\rho/\omega)}$ is found to be $\Delta^{(\rho/\omega)} = (11 \pm 11)\%$, which is mainly determined by symmetry breaking in $T_1^{(\rho)}(0)$ and $T_1^{(\omega)}(0)$ transition form factors. Using current HFAG averages, the estimate is $\Delta_{\mathrm{exp}}^{(\rho/\omega)} = (34 \pm 20)\%$ which is consistent with predicted value within errors
- Direct CP-asymmetries $\mathcal{A}_{\mathrm{CP}}(\rho\gamma)$ and $\mathcal{A}_{\mathrm{CP}}(\omega\gamma)$ receive contributions from hard-spectator corrections which tend to decrease values estimated from vertex corrections alone. Predicted values of direct CP-asymmetry are sensitive to both the choice of scale μ_b and quark mass ratio $\sqrt{z} = m_c/m_b$ and stay in ranges: $-15\% < \mathcal{A}_{\mathrm{CP}}(\rho^\pm\gamma) < -9\%$, $-13.5\% < \mathcal{A}_{\mathrm{CP}}(\rho^0\gamma) < -6\%$, and $-13.0\% < \mathcal{A}_{\mathrm{CP}}(\rho^0\gamma) < -5.5\%$, clearly indicating a sizable effect
- Mixing-induced CP-asymmetry $S_{\rho\gamma} = (1.9^{+3.8}_{-3.2})\%$, being highly suppressed in the $B^0 \to \rho^0\gamma$ decay due to destructive interference of LO and NLO contributions, gets enhancement $S_{\omega\gamma} = (5.9^{+4.1}_{-3.5})\%$ in the $B^0 \to \omega\gamma$ decay and can reach a level of 10%
- Recent data on $B \to (\rho, \omega)\gamma$ decays allow to determine $|V_{td}/V_{ts}|$ in region which is not yet enough precise but overlapping with the one obtained from global CKM fits