Entropy Growth and the Equation of State

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DESY

0. Overview

- 1. Entropy and Gravity
- 2. The Second Law in Cosmological Situations
- 3. Constraints on Dark Energy
- 4. Summary and Conclusions

Entropy and Gravity

Black Holes and Entropy

- Black holes are rather dumb:

  Their memory is limited to mass, charge and angular momentum.
- They do not remember the details of their origin!

- Mass is concentrated in a point.

→ Seems very ordered!

→ 0 Entropy???

A violation of the 2. Law?

- According to the
  2. Law of Thermodynamics:
  Entropy always increases!

- Throw entropic matter into BH

➡ Vanishes

➡ Violation of the 2. Law????

Area Law

- The horizon area of black holes always increases!

  (in classical gravitational systems)
- E.g. when matter is dropped into the black hole.

➡ Entropy of BH ~ Horizon Area

$$S_{\rm BH} = 2\pi A_{\rm BH} M_{\rm P}^2$$

Generalized 2. Law

- In systems with ordinary matter and black holes

Generalized 2. Law (GSL)

$$dS = dS_{\mathrm{mat}} + dS_{\mathrm{H}} \geq 0$$

A Note on Entropy Bounds

- The GSL cannot work if an arbitrary amount of entropy can be stored in a system with fixed size and energy.

$$\Rightarrow \quad S_{\rm sys} \leq 2\pi A_{\rm sys}$$

Covariant entropy bound

$$S_{\rm sys} \leq 2\pi E_{\rm sys} L_{\rm sys}$$

Bekenstein bound

Cosmological

Situations

Problems

- Space is not asymptotically flat.
- Typically non stationary situations!
- Global quantities like total energy and entropy are hard to define!

Which Horizon? Which Region?

- „Horizons“:
  - Future event horizon
  - Apparent/Hubble horizon
  - Boundary of the causal region

- Regions:
  - Spacelike
  - Null „from the middle“
  - Null „from the surface“

An Example

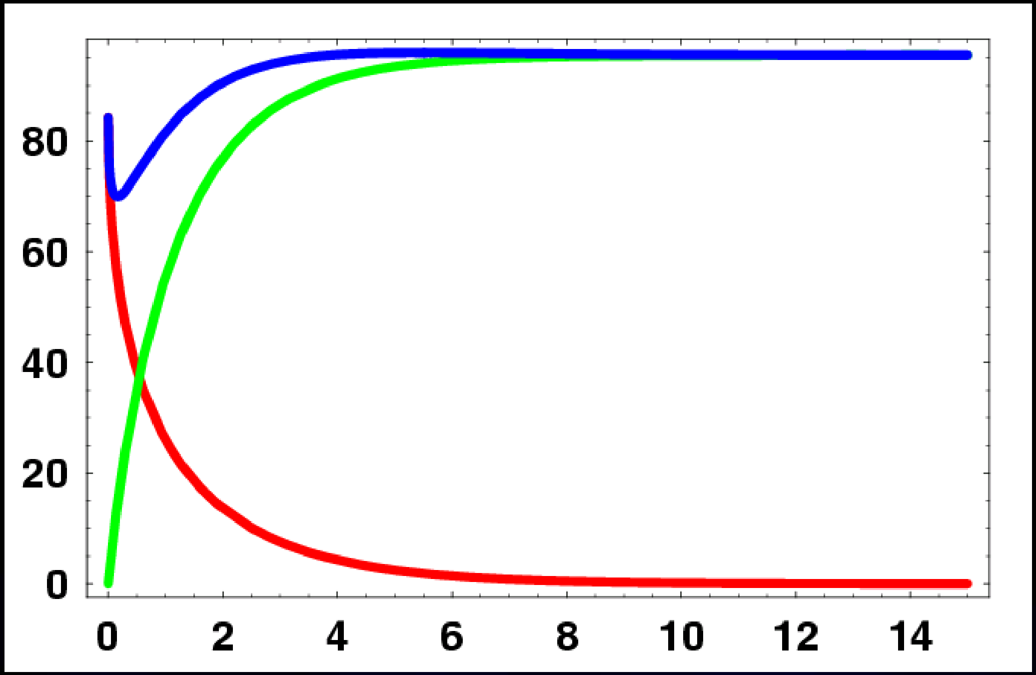
- Flat universe with Λ and radiation

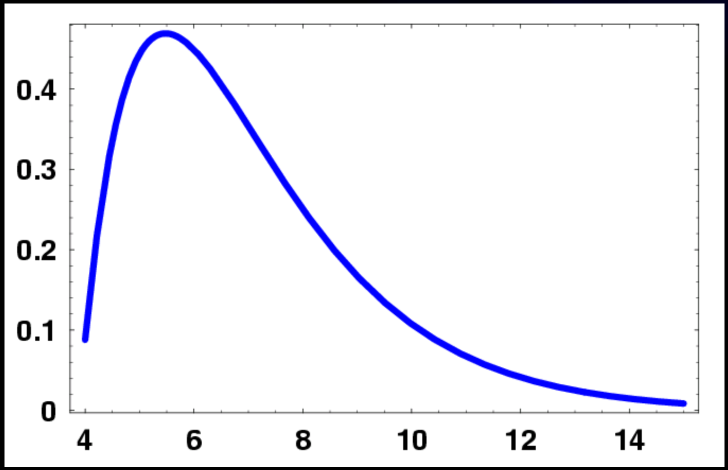
$$S_{\rm H} = 2\pi A_{\rm H}(t) = 8\pi^2 D_{\rm H}^2(t).$$

$$S_{\rm R} = \frac{4}{3}\pi D_{\rm H}^3(t) s_{\rm R}(t) = \frac{16}{9}\pi\sigma T_0^3 \frac{D_{\rm H}^3(t)}{a^3(t)}.$$

$$D_{\rm H}(t) = a(t)\int_t^\infty \frac{dt'}{a(t')}.$$

An Example





An Example

Message: GSL seems to work!

Constraints on Dark Energy

Idea: (pure dark energy universe)

- Roughly speaking:

$$D_{\rm H}(t) = a(t)\int_t^\infty \frac{dt'}{a(t')} \sim \frac{1}{H}$$

- Use Friedmann equation $3H^2 = \rho$

→ If ρ increases

→ D_H decreases

→ S_H decreases → Trouble!

And the Equation of State...

- Friedmann equation II:

$$\dot{\rho} = -3H\rho(1+w)$$

➡ $\dot{\rho} > 0 \quad \text{for} \quad w < -1$

➡ **Phantom Universes may be in trouble!!**

Real Universe

- There is additional matter and radiation!

$$\rho_{\rm tot} = \rho_{\rm DE} + \rho_{\rm mat} + \rho_{\rm rad}$$

$$w_{\rm eff} = -1 - \frac{\dot{\rho}_{\rm tot}}{3H\,\rho_{\rm tot}}$$

- Matter and radiation carry additional entropy!
- But $S_{mat}+S_{rad} \sim 10^{88+11} <<<< S_H \sim 10^{120}$

→ $S_{mat}+S_{rad}$ negligible

Assumption

- For the moment we assume that the dark energy does not carry entropy!

- This seems reasonable:
  - A homogeneous scalar field has zero entropy
  - An adiabatically expanding perfect fluid has $\frac{S}{V} \sim \frac{1}{a^3(t)}$

Constant equation of state

- For DE with constant EOS w<-1/3 we will have at late times

$$\rho_{\rm DE} \gg \rho_{\rm mat} + \rho_{\rm rad}$$

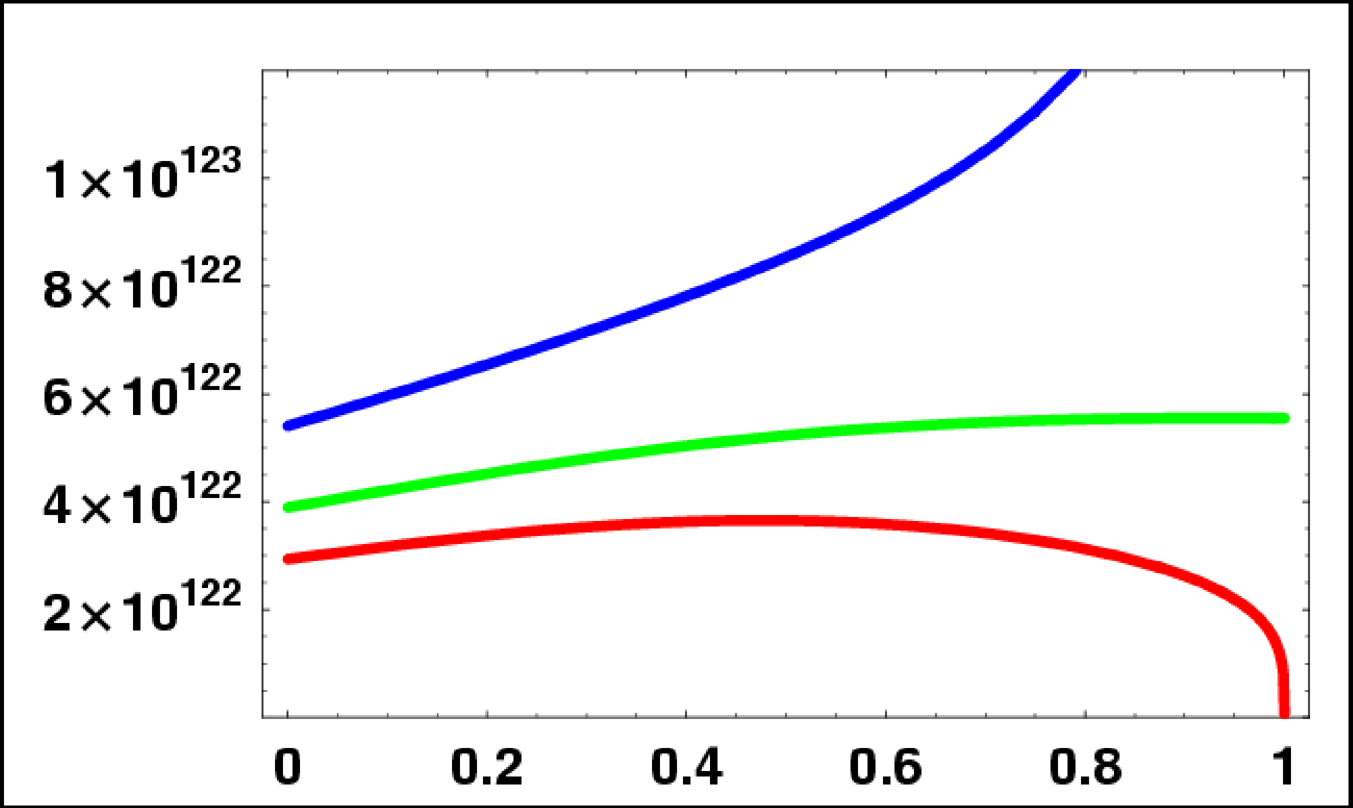
- Then $w_{\rm eff} \approx w_{\rm DE}$

and $\dot{D}_{\rm H} = \left(\dot{\frac{const}{H}}\right) \sim (1 + w_{\rm DE})$

➡ $\dot{S}_{\rm H} \geq 0$ ⬌ $w \geq -1$

➡ Phantom universes with constant EOS will violate GSL!

No Big Rip



Near Big Rip $S_{tot} \to 0$

➡ Big Rip violates GSL!

Commercials

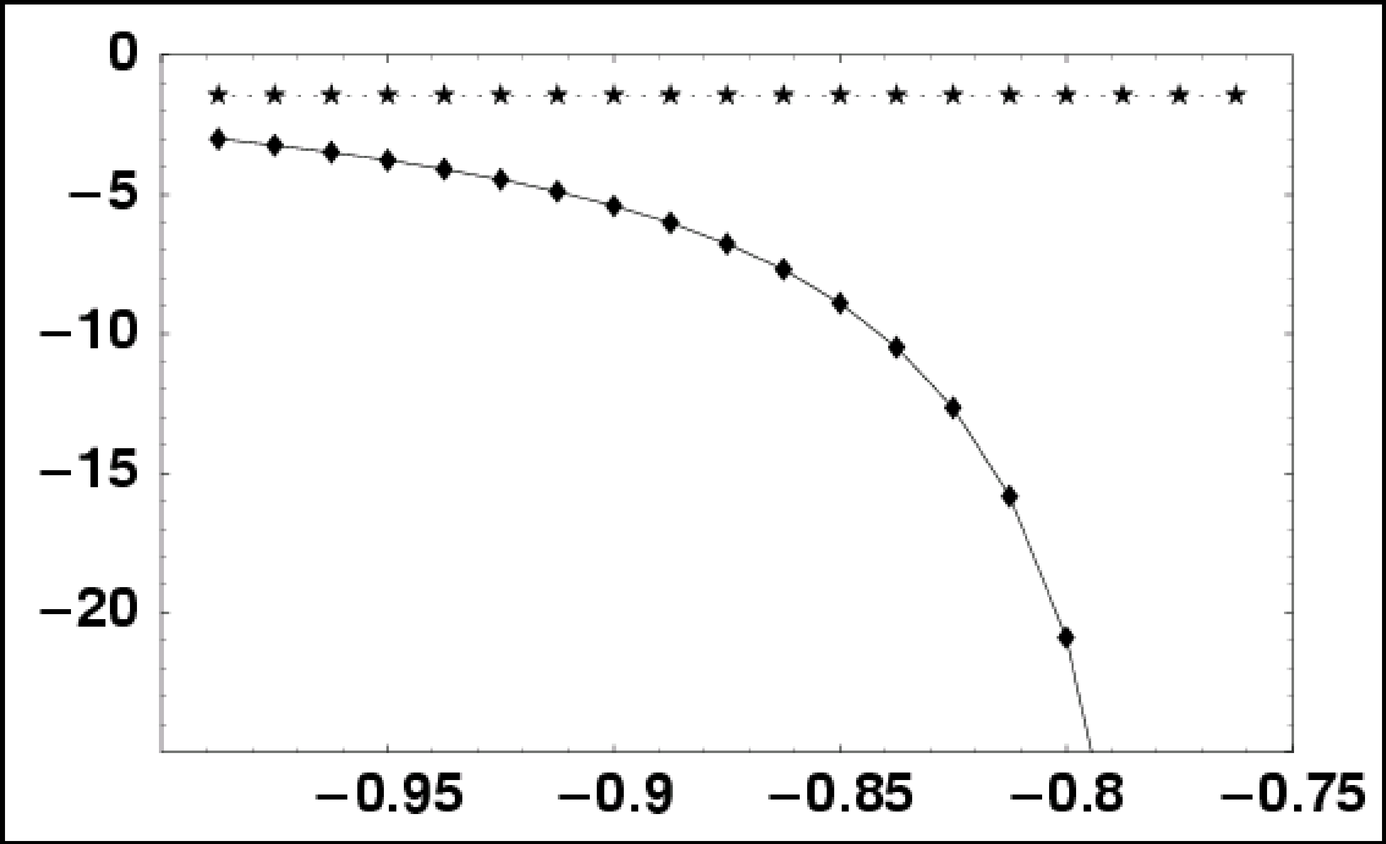
Your Universe is constantly loosing Entropy?

Your Universe ends in a Big Rip?

Use GSL!

The entropic Rip Stop!

$w \geq -1$ since today



If w switches to $w \geq -1$ today
The GSL still provides a constraint on w!

Summary and Conclusions

Conclusions

- In systems with gravitational interactions horizons contribute to the entropy.
- A generalized second law of thermodynamics may hold.
- The GSL is in conflict with big rip situations.
- It places bounds on the equation of state
- Viscosity is a possible but unlikely way out

Viscosity

A way out?

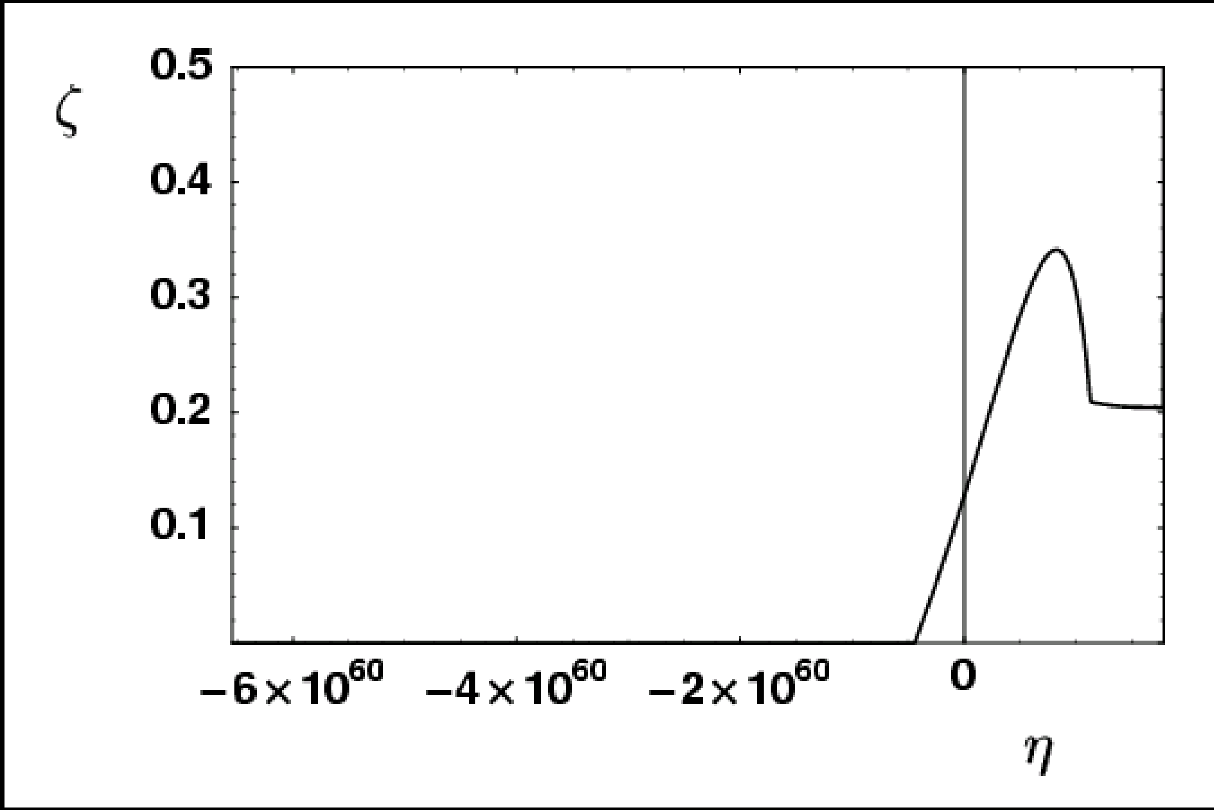
$S_{DE} \neq 0$

- So far we assumed that the dark energy is a scalar field or an adiabatically expanding perfect fluid.

- Bulk viscosity leads to entropy production!

$$\frac{1}{V}\frac{dS}{dt} = \frac{9H^2\alpha\rho}{T} = \frac{3H\zeta\rho}{T}$$

Replenishing entropy



Natural η: 10^{-60}

Possible. But Huge viscosity is needed!