The Higgs in the Sky:

production of gravitational waves during a first order phase transition

work done in collaboration with

C. Delaunay, G. Servant

to appear soon

see also

C.G., G. Servant, J. Wells

hep-ph/0407019 = PRD71, 036001 (2005)

Christophe Grojean

CERN-TH & SPhT - CEA Saclay

(Christophe.Grojean@cern.ch)

GW and the ElectroWeak Phase Transition

GW interact very weakly and are not absorbed

direct probe of physical process of the very early universe

possible cosmological sources:

inflation, vibrations of topological defects, excitations of xdim modes, 1^{st} order phase transitions...

ElectroWeak Phase Transition (if 1^{st} order)

typical freq. ~ (size of the bubble)$^{-1}$ ~ (fraction of the horizon size)$^{-1}$

$$@\, T = 100 \text{ GeV}, \qquad H = \sqrt{\frac{8\pi^3}{45}}\frac{T^2}{M_{Pl}} \sim 10^{-15} \text{ GeV}$$

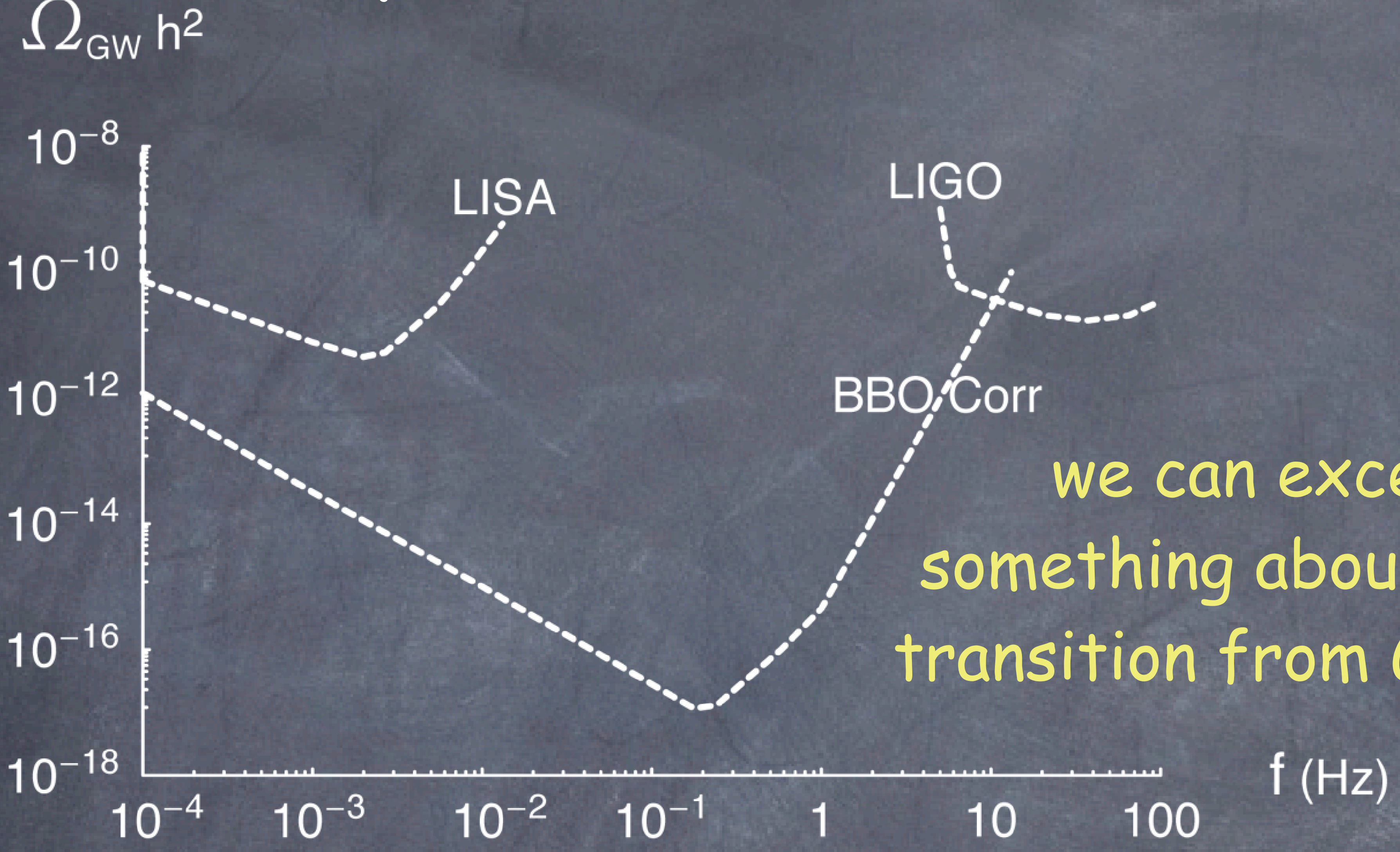
redshifted freq.

~ today ~

$$f \sim \# \frac{2\cdot 10^{-4} \text{ eV}}{100 \text{ GeV}}\, 10^{-15} \text{ GeV} \sim \# \, 10^{-5} \text{ Hz}$$

The GW spectrum from a 1^{st} order electroweak PT is peaked around the milliHertz frequency

Why should you be excited about mHZ freq.?



we can except to learn something about the EW phase transition from GW experiments

- test of the dynamics of the phase transition
  (quite important to analyze models of EW baryogenesis!)
- reconstruction of the Higgs potential / study of new models of EW symmetry breaking (little Higgs, gauge-Higgs, composite Higgs, Higgsless...)

complementary to collider informations

A not so new subject...

- Early 90's, M. Turner and his students studied the production of GW produced by bubble collisions. Not much attention since the LEP data excluded a 1st order phase transition within the SM.

Kosowsky, Turner, Watkins'92
Kamionkowski, Kosowsky, Turner '94

- '01-'02: Kosowsky et al. and Dolgov et al. computed the production of GW from turbulence ⇛ stronger signal. Application to the (N)MSSM where a 1st order phase transition is still plausible.

Kosowsky, Mack, Kahniashvili'02
Dolgov, Grasso, Nicolis'02
Caprini, Durrer '06

In this talk:

⇛ Model-independent analysis for detectability of GW from 1st order phase transitions

Grojean, Servant

⇛ GW from H^6 induced 1st order EW phase transition

Delaunay, Grojean, Servant, Wells

A two parameter problem...

A 1st order phase transition proceeds by nucleation of bubbles

kinetic energy of bubbles is transferred to GW either by

- bubble collisions
- injection of energy into the plasma fluid
  (creating a homogeneous, isotropic, fully developed and stationary turbulent regime).

Need to move large mass rapidly ⇛ detonation regime: bubble walls propagate faster than the speed of sound

the GW background is controlled by two quantities

$$\alpha \sim \frac{\text{false vacuum energy density} = \text{latent heat}}{\text{plasma thermal energy density}}$$

$$\beta \sim \text{rate of time variation of the nucleation rate } \Gamma \ (\Gamma = \Gamma_0 e^{-\beta t})$$
$$\sim (\text{duration of transition})^{-1}$$

The stronger is the transition,
the larger is α and the smaller is β

Estimate of the GW energy density

energy density at the emission time

$$\rho^\star_{GW} \sim \frac{E^\star_{GW}}{\text{volume}} \sim \frac{\mathcal{P}^\star_{GW} \times \beta^{-1}}{(\text{bubble wall speed} \times \beta^{-1})^3}$$

quadrupole formula
for power of GW emission

$$\mathcal{P}^\star_{GW} = \frac{1}{5M_{Pl}^2} \langle \left(\dddot{Q}_{ij}^{TT} \right)^2 \rangle$$

Q_{ij}^{TT} quadrupole moment of T_{ij}^{TT}

$$\dddot{Q}_{ij}^{TT} \sim \frac{\text{mass of system in motion} \times (\text{size of system})^2}{(\text{time scale of system})^3} \sim \frac{\text{kinetic energy}}{\text{time scale of system}}$$

$$E_{\text{kin}} \sim \kappa \alpha \rho_{\text{rad}} \times \text{volume}$$

κ : fraction of vacuum energy which goes into kinetic energy of bulk motion of the fluid (as opposed to heating)

$$\Omega^\star_{GW} = \frac{\rho^\star_{GW}}{\rho_{\text{tot}}}$$

with $\rho_{\text{tot}} = (1+\alpha)\rho_{\text{rad}}$ $\quad H_\star^2 = \frac{\rho_{\text{tot}}}{M_{Pl}^2}$

$$\Omega^\star_{GW} = \kappa^2 \, v_b^3 \, \frac{\alpha^2}{(1+\alpha)^2} \frac{H_\star^2}{\beta^2}$$

v_b
bubble wall speed

Detonation Regime

- bubble wall speed

speed of sound

(weak phase transition)

$$1/\sqrt{3} \leq v_b = \frac{1/\sqrt{3} + \sqrt{\alpha^2 + 2\alpha/3}}{1+\alpha} \leq 1$$

speed of light

(strong phase transition)

Steinhardt'82

- fraction of vacuum energy into kinetic energy

numerical estimate

increases with α varies between 0 and 1

$$\kappa \sim \frac{1}{1+.715\,\alpha}\left(.715\,\alpha + \frac{4}{27}\sqrt{\frac{3\,\alpha}{2}}\right)$$

Kamionkowski, Kosowsky, Turner '94

- characteristic velocity of the eddies in the plasma

(turbulent regime)

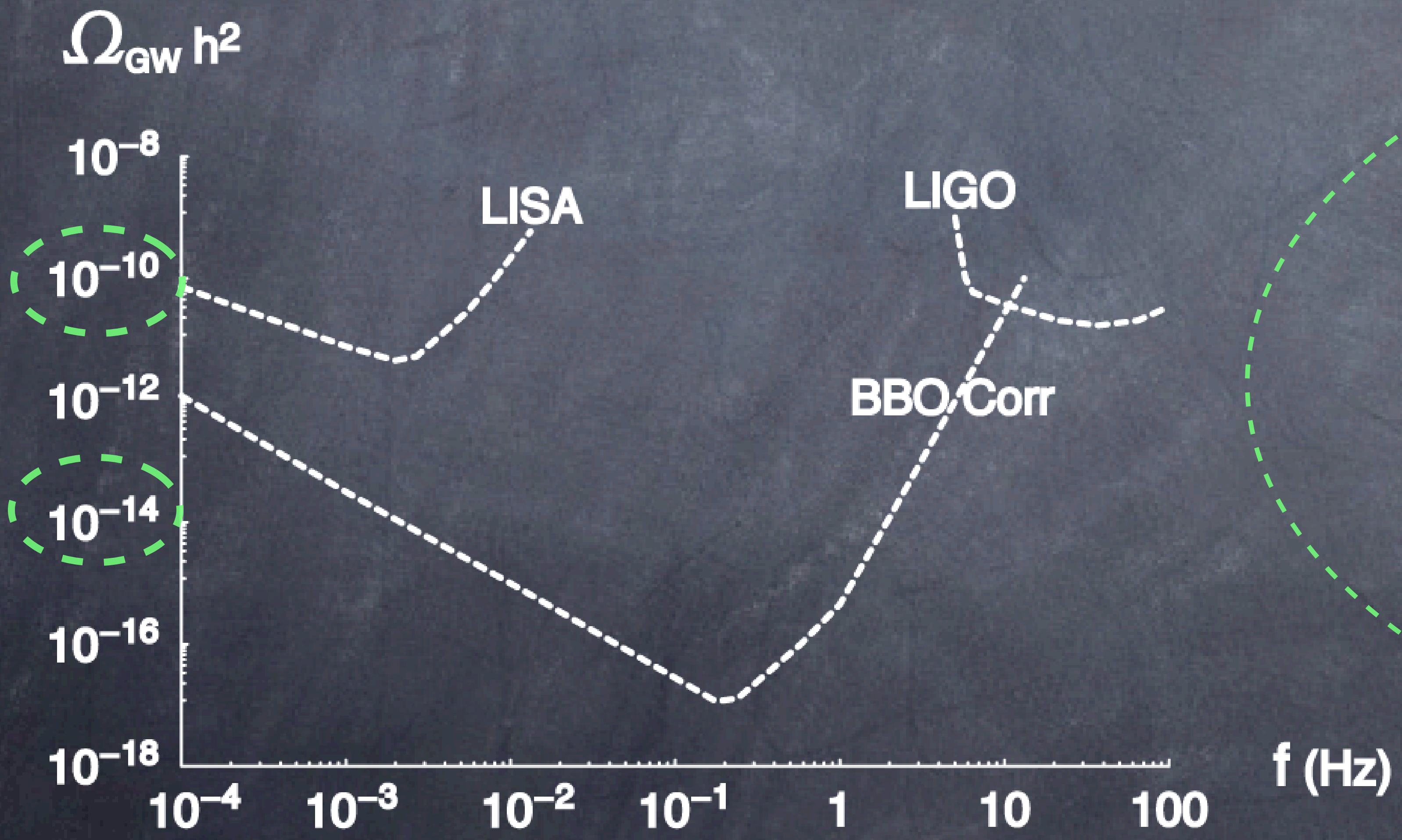
$$u_s = \sqrt{\frac{\kappa\alpha}{4/3+\kappa\alpha}} \leq v_b$$

Nicolis '02

GW energy density today

$$\Omega^\star_{GW} \xrightarrow{\text{redshift}} \Omega_{GW} = \left(\frac{a_\star}{a_0}\right)^4 \left(\frac{H_\star}{H_0}\right)^2 \Omega^\star_{GW} \sim 2{\cdot}10^{-5} h^{-2} \left(\frac{100}{g_\star}\right)^{1/3} \Omega^\star_{GW}$$

$$H_0 \sim h \times 2{\cdot}10^{-42}\,\text{GeV}$$



$$\Omega^\star_{GW}$$

had to be quite big to get an observable signal today

$\Omega^\star_{GW} \gtrsim 10^{-6}$ for LIGO/LISA

$\Omega^\star_{GW} \gtrsim 10^{-12}$ - 10^{-9} for BBO

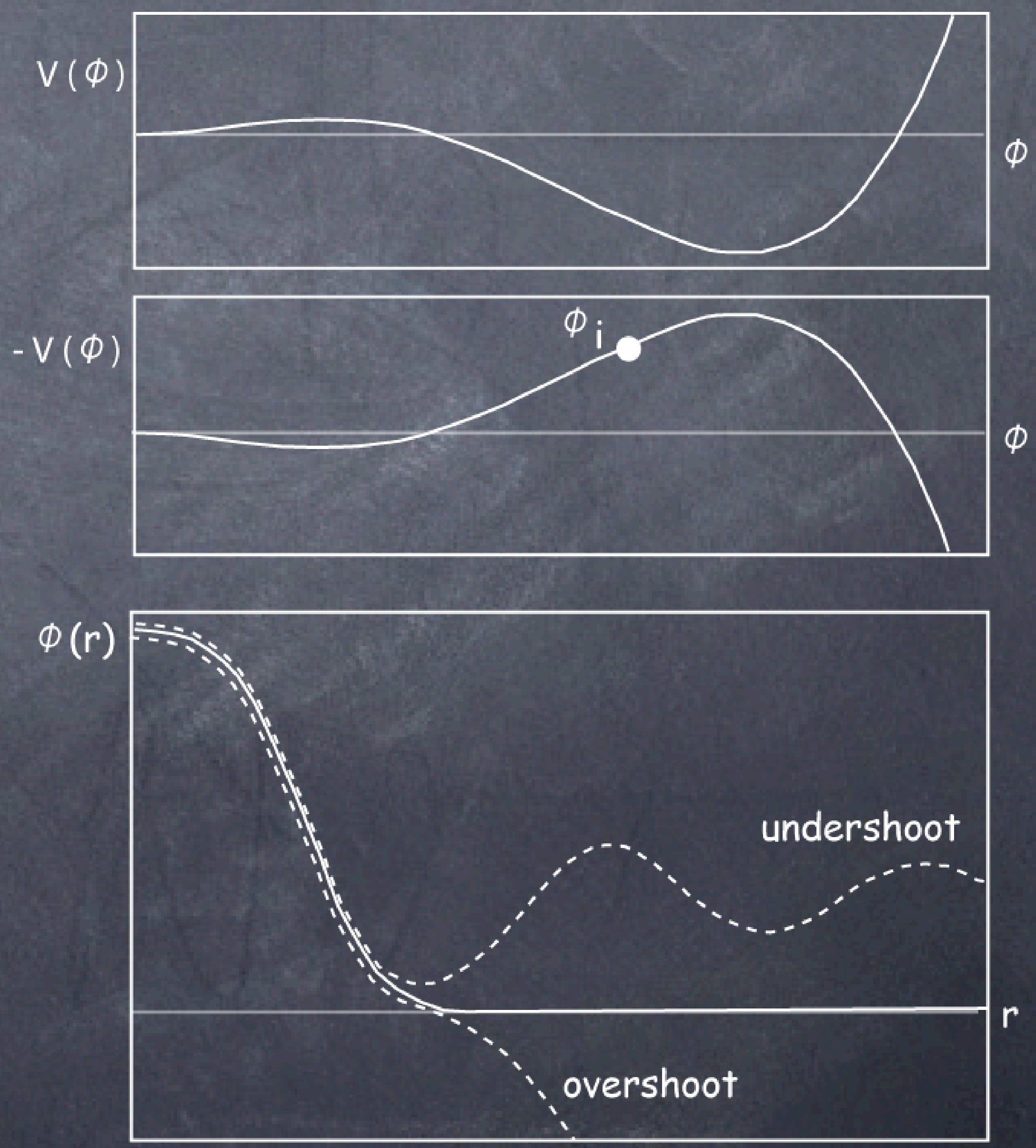
How to compute α and β/H

the numerical computation of α of β/H is quite involved

- need to compute the nucleation temperature

- find critical bubbles

$$S_3(T) = 4\pi \int dr\, r^2 \left[\frac{1}{2}\left(\frac{d\phi_b}{dr}\right)^2 + V(\phi_b, T)\right]$$: euclidean action for a critical bubble



Overshooting-undershooting method to search for the bounce solution

$$\frac{d^2\phi_b}{dr^2} + \frac{2}{r}\frac{d\phi_b}{dr} - \frac{\partial V}{\partial \phi_b} = 0$$

$$\left.\frac{d\phi_b}{dr}\right|_{r=0} = 0 \quad , \quad \phi_b|_{r=\infty} = 0$$

Nucleation occurs when the probability
for the nucleation of 1 bubble
per 1 horizon volume is ~ O(1)

⇒ translates into $S_3(T_*)/T_* \simeq 140$

How to compute α and β/H

once you know the nucleation temperature

α is the energy difference between the false and the true vacuum

β/H is the related to the derivative of the euclidean action

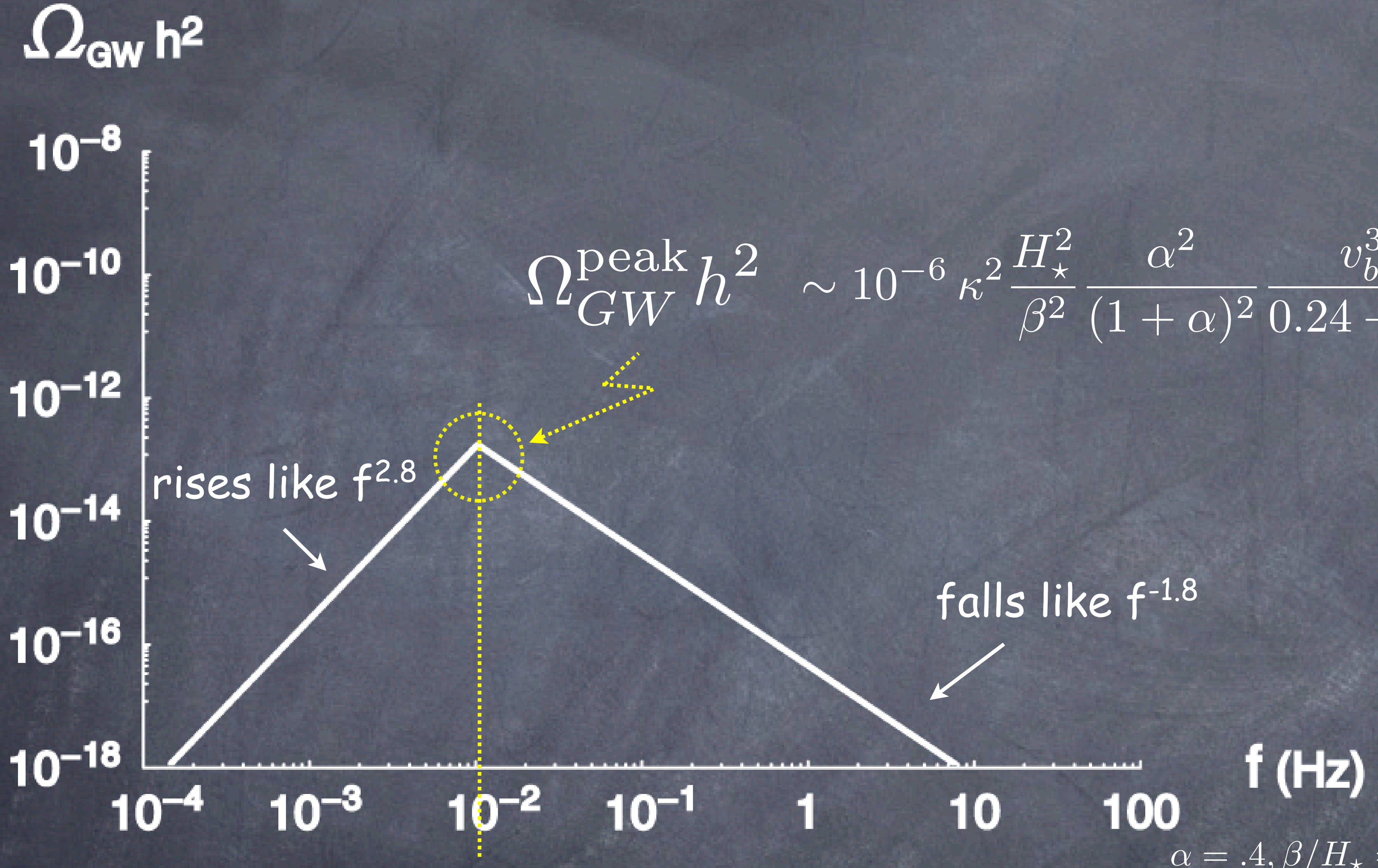
$$\Gamma \propto e^{-S_3(T)/T}$$

the peak frequency of the GW signal (at the time of emission) is

$$\frac{f_*}{H_*} = T\frac{d}{dT}\left(\frac{S_3}{T}\right) \sim \frac{S_3}{T} \sim \ln\frac{m_{Pl}}{T_*}$$

typically, β/H ~ $\frac{f_*}{H_*} \sim 200$

Complete Spectrum: Collision



$$\Omega_{GW}^{\rm peak} h^2 \sim 10^{-6}\,\kappa^2 \frac{H_\star^2}{\beta^2} \frac{\alpha^2}{(1+\alpha)^2} \frac{v_b^3}{0.24+v_b^3} \left(\frac{100}{g_\star}\right)^{1/3}$$

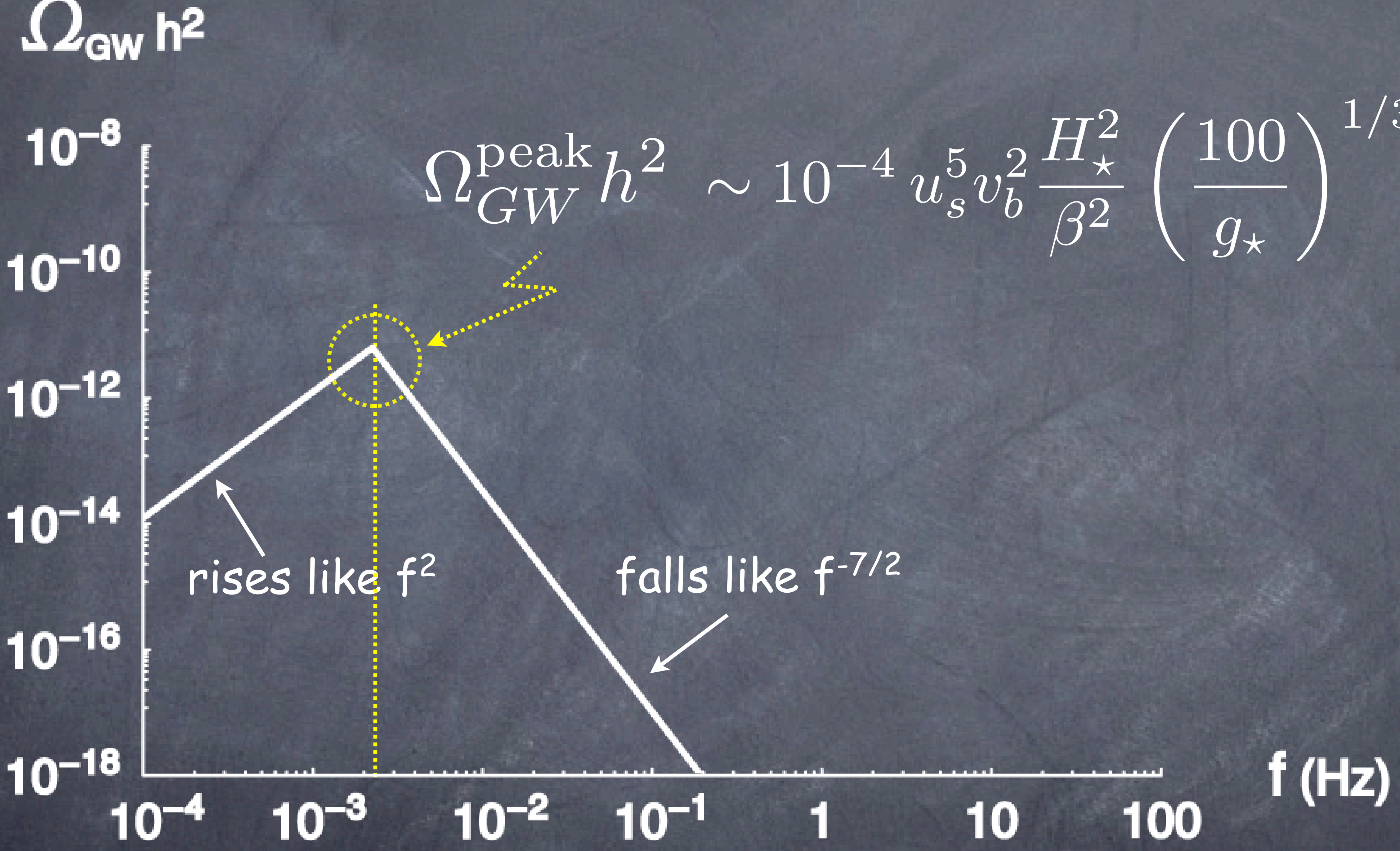
$\alpha = .4, \beta/H_\star = 200, T = 1\ \mathrm{TeV}, g_\star = 100$

$$f_{\rm peak} \simeq 5{\cdot}10^{-6}\, \frac{\beta}{H_\star} \frac{T_\star}{100\,\mathrm{GeV}} \left(\frac{g_\star}{100}\right)^{1/6}\ \mathrm{Hz}$$

Kamionkowski, Kosowsky, Turner ’94

Complete Spectrum: Turbulence

standard Navier-Stokes turbulence, with an
ordinary Kolmogorov energy spectrum



$$\Omega_{GW}^{\text{peak}} h^2 \sim 10^{-4}\, u_s^5 v_b^2 \frac{H_\star^2}{\beta^2} \left(\frac{100}{g_\star}\right)^{1/3}$$

rises like f^2

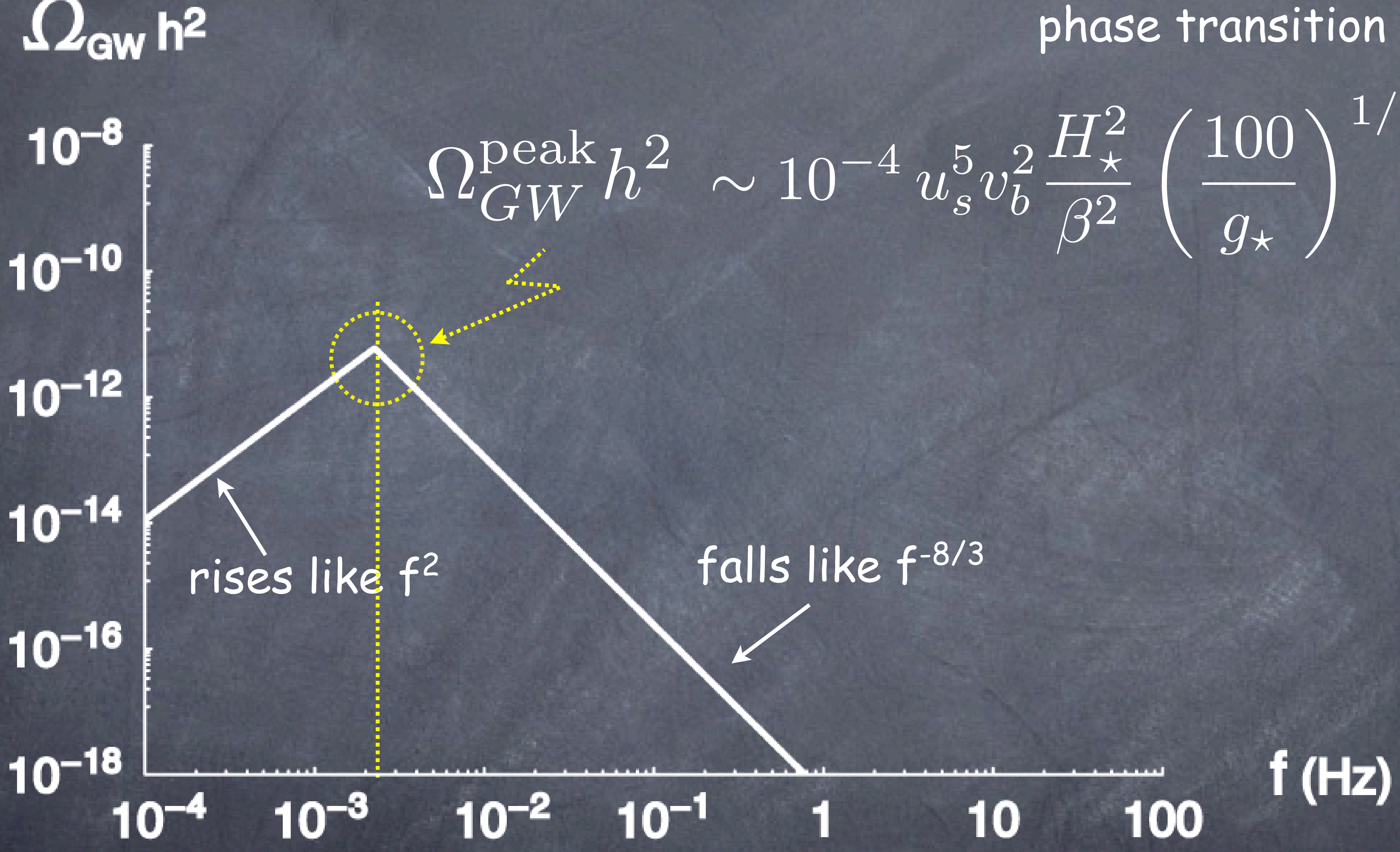
falls like $f^{-7/2}$

$\alpha = .4, \beta/H_\star = 200, T = 1 \text{ TeV}, g_\star = 100$

$$f_{\text{peak}} \simeq 3{\cdot}10^{-6} \frac{u_s}{v_b} \frac{\beta}{H_\star} \frac{T_\star}{100\,\text{GeV}} \left(\frac{g_\star}{100}\right)^{1/6} \text{Hz}$$

Complete Spectrum: MHD Turbulence

MHD effects due to the presence of large magnetic fields produced before or during the phase transition

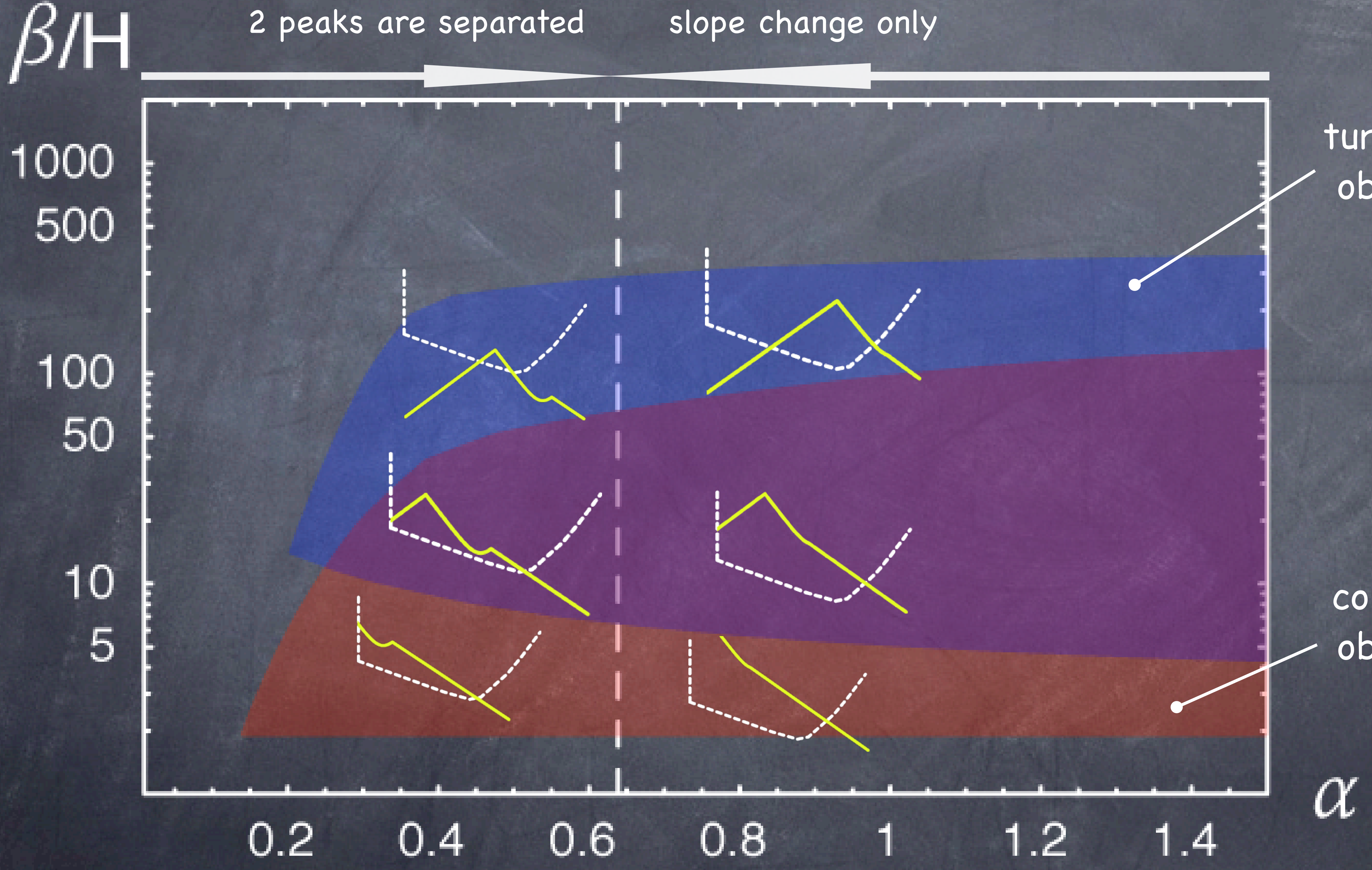


$$\Omega_{GW}^{\rm peak} h^2 \sim 10^{-4}\, u_s^5 v_b^2 \frac{H_\star^2}{\beta^2} \left(\frac{100}{g_\star}\right)^{1/3}$$

$\alpha = .4, \beta/H_\star = 200, T = 1 \text{ TeV}, g_\star = 100$

$$\mathrm{f}_{\rm peak} \simeq 3{\cdot}10^{-6}\, \frac{u_s}{v_b} \frac{\beta}{H_\star} \frac{T_\star}{100\,\mathrm{GeV}} \left(\frac{g_\star}{100}\right)^{1/6} \ \mathrm{Hz}$$

Scanning the (α, β/H) plane

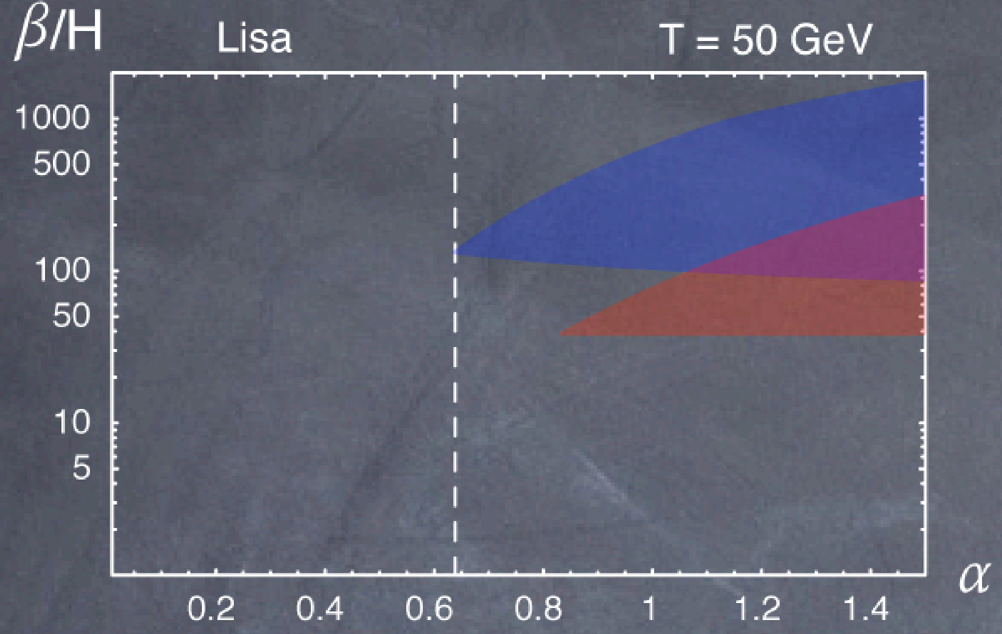


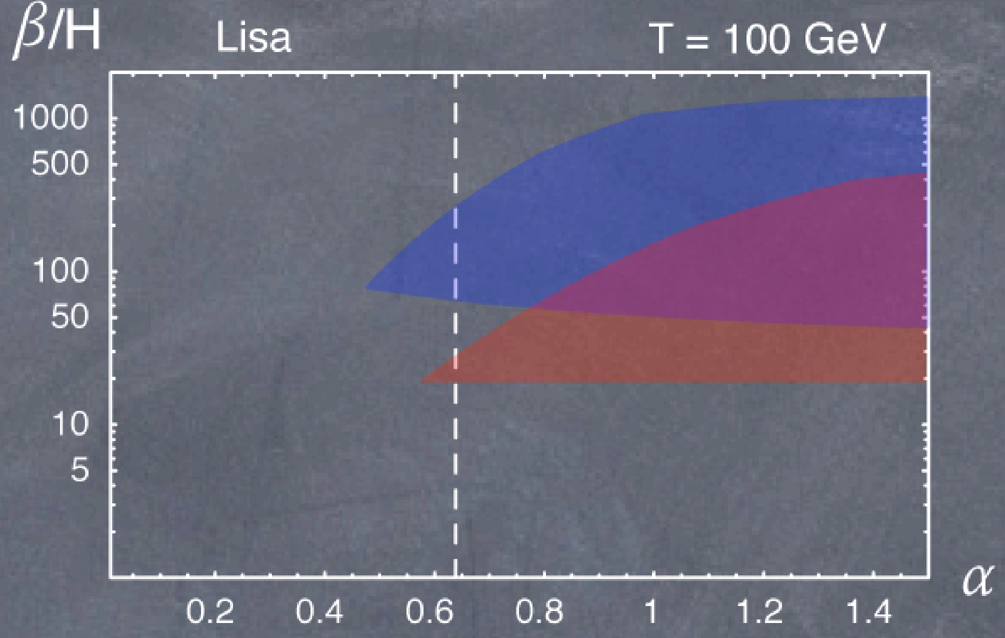
turb. peak is observable

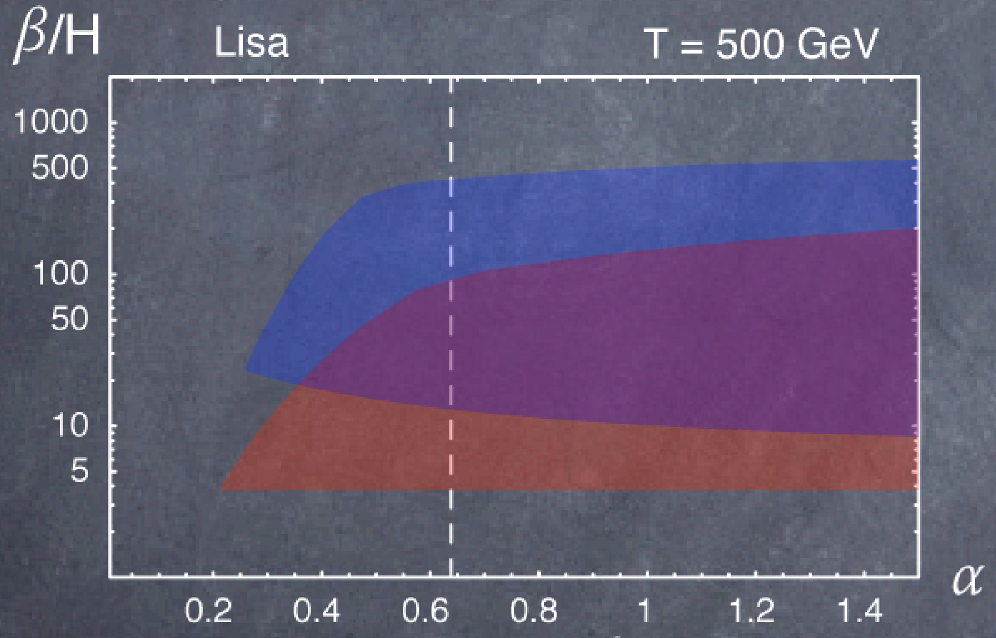
coll. peak is observable

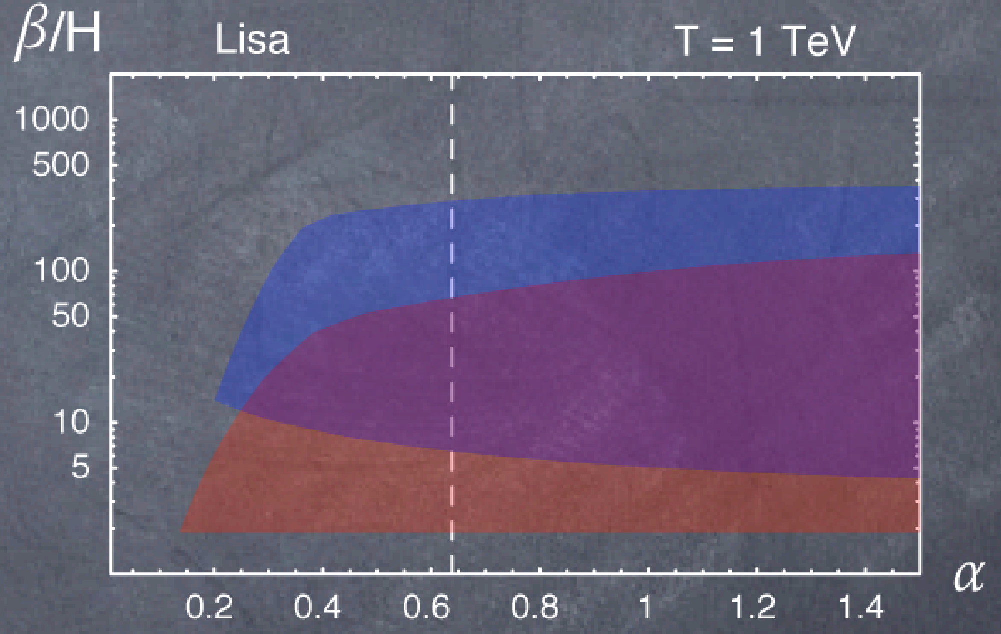
We always have $f_{\text{peak}}^{\text{turb}} \leq f_{\text{peak}}^{\text{coll}}$

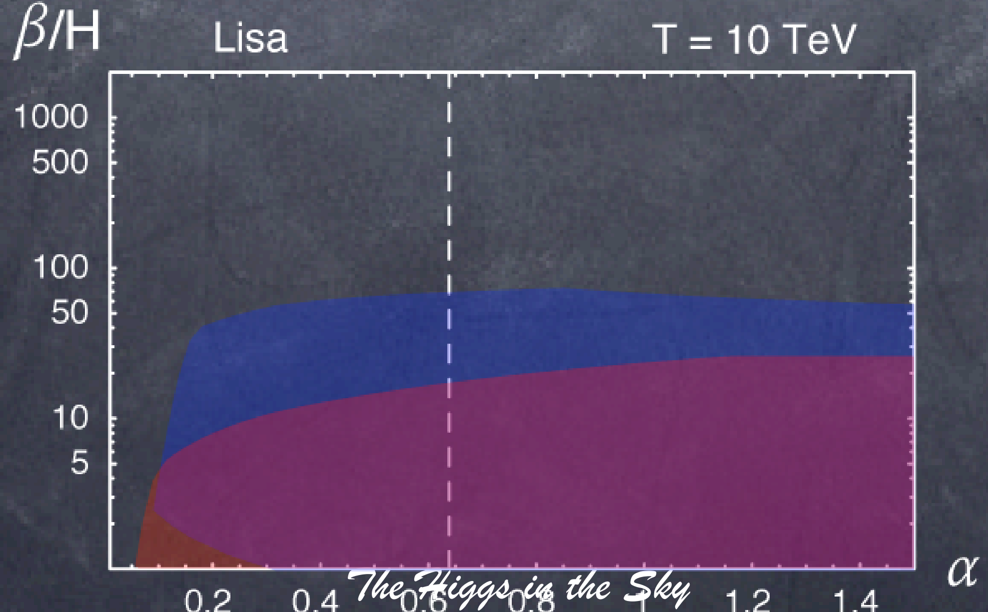
Observability of a 1st order PT at Lisa







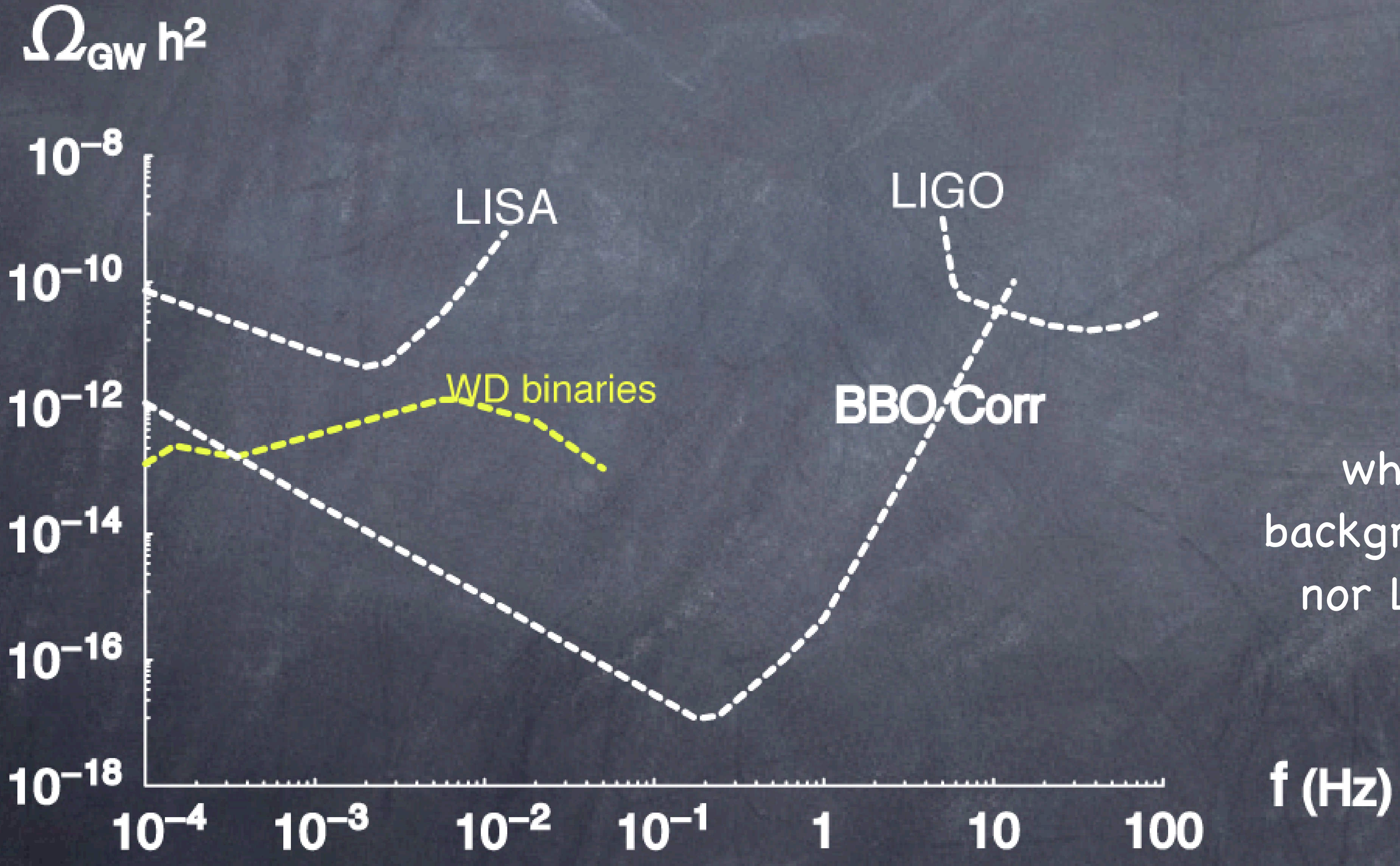




Observability of a 1st order PT at BBO

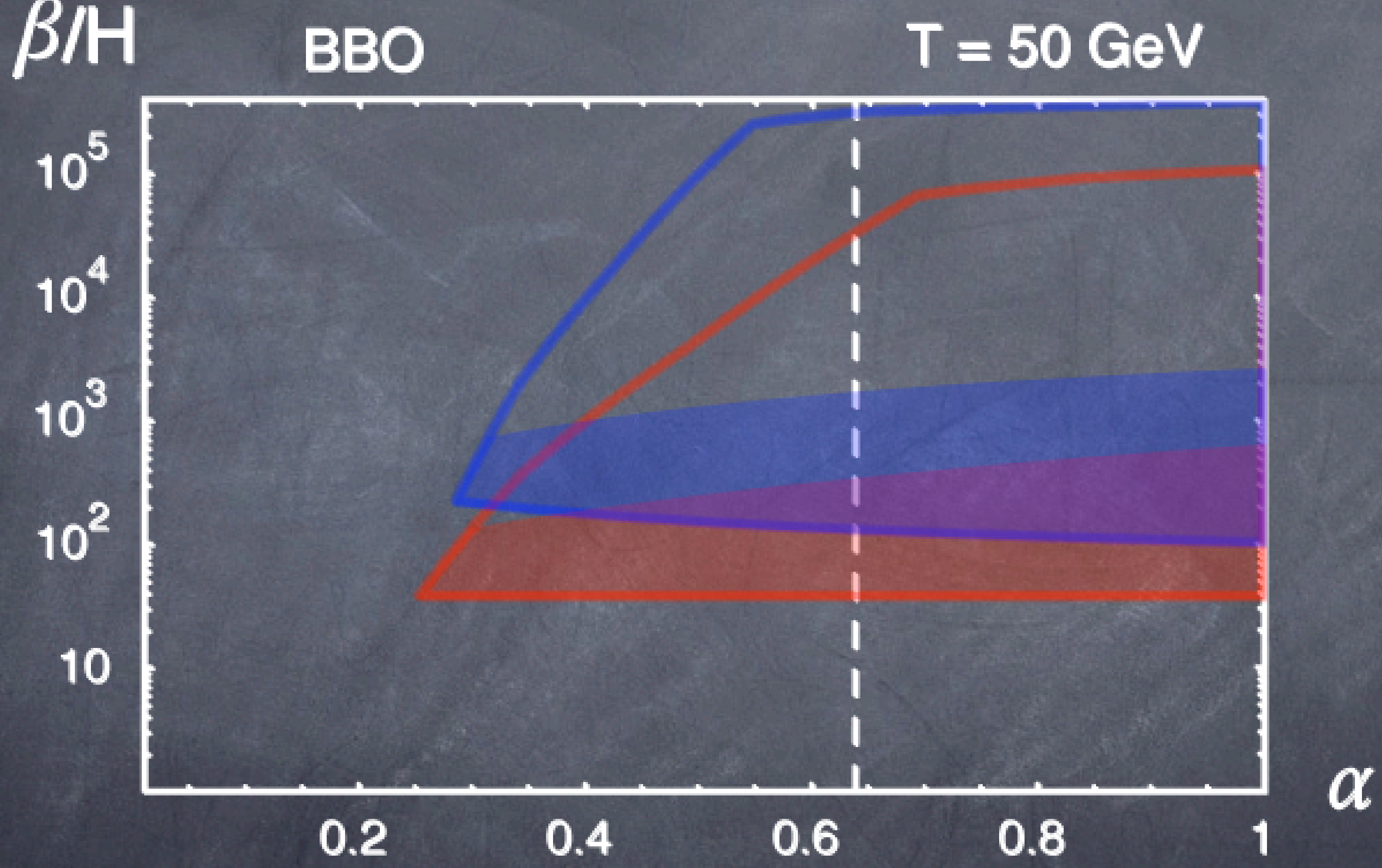
The galactic astrophysical background can be removed since in the galactic plane (sources are removed one by one: lot of work!)

An extra galactic astrophysical background can affect the sensitivity of BBO

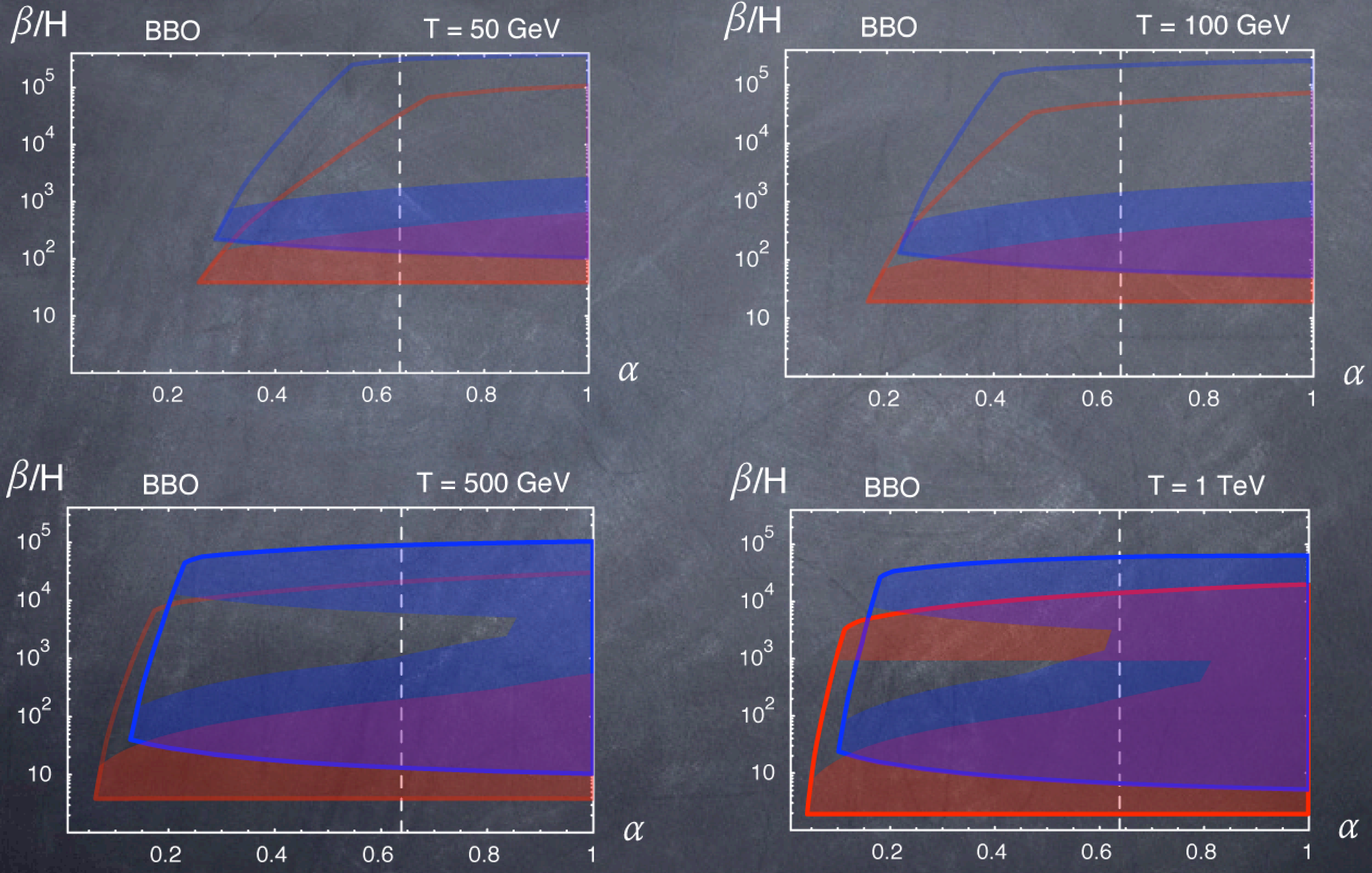


while this extra galactic background doesn't affect LISA nor LIGO, it will affect BBO

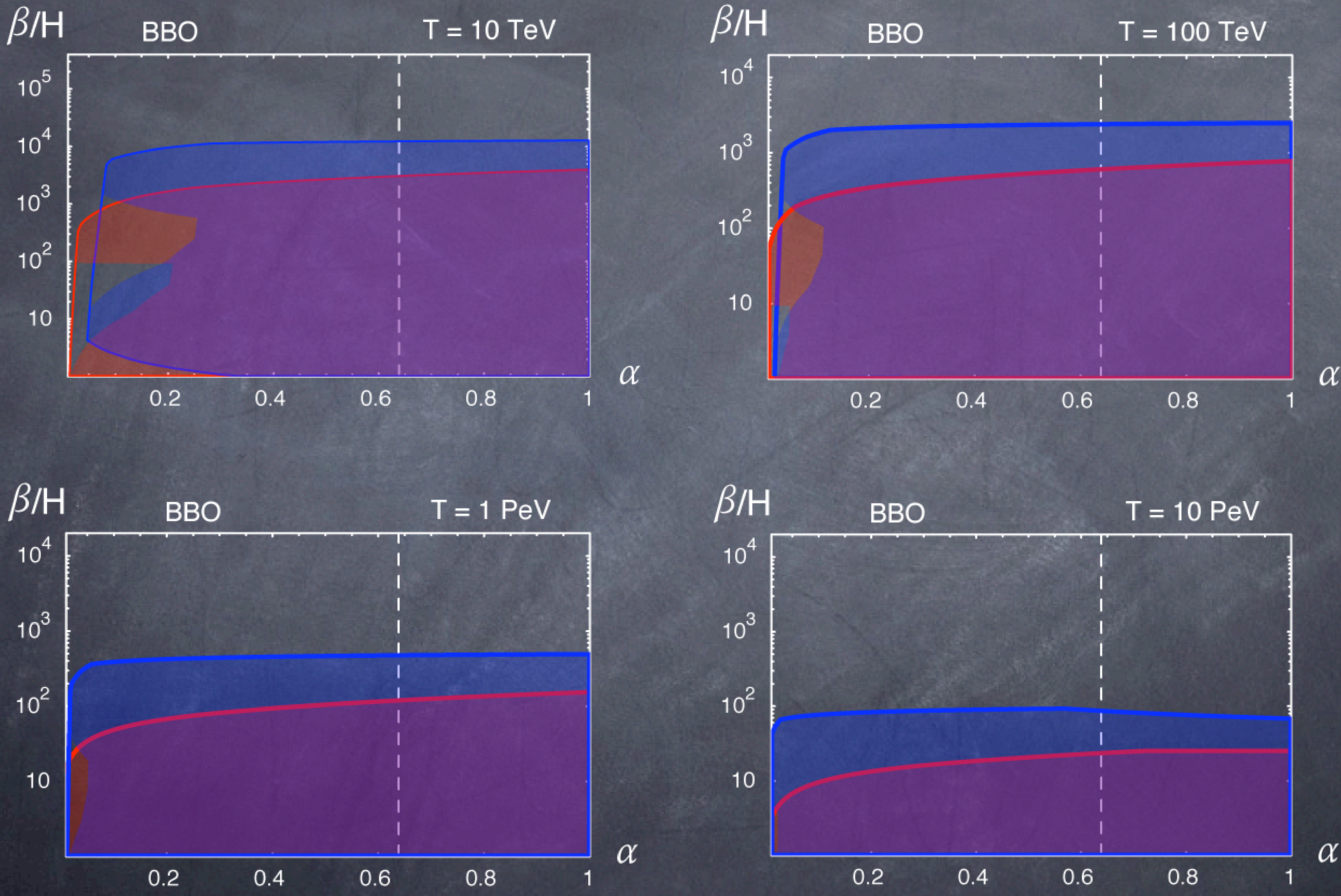
Observability of a 1st order PT at BBO



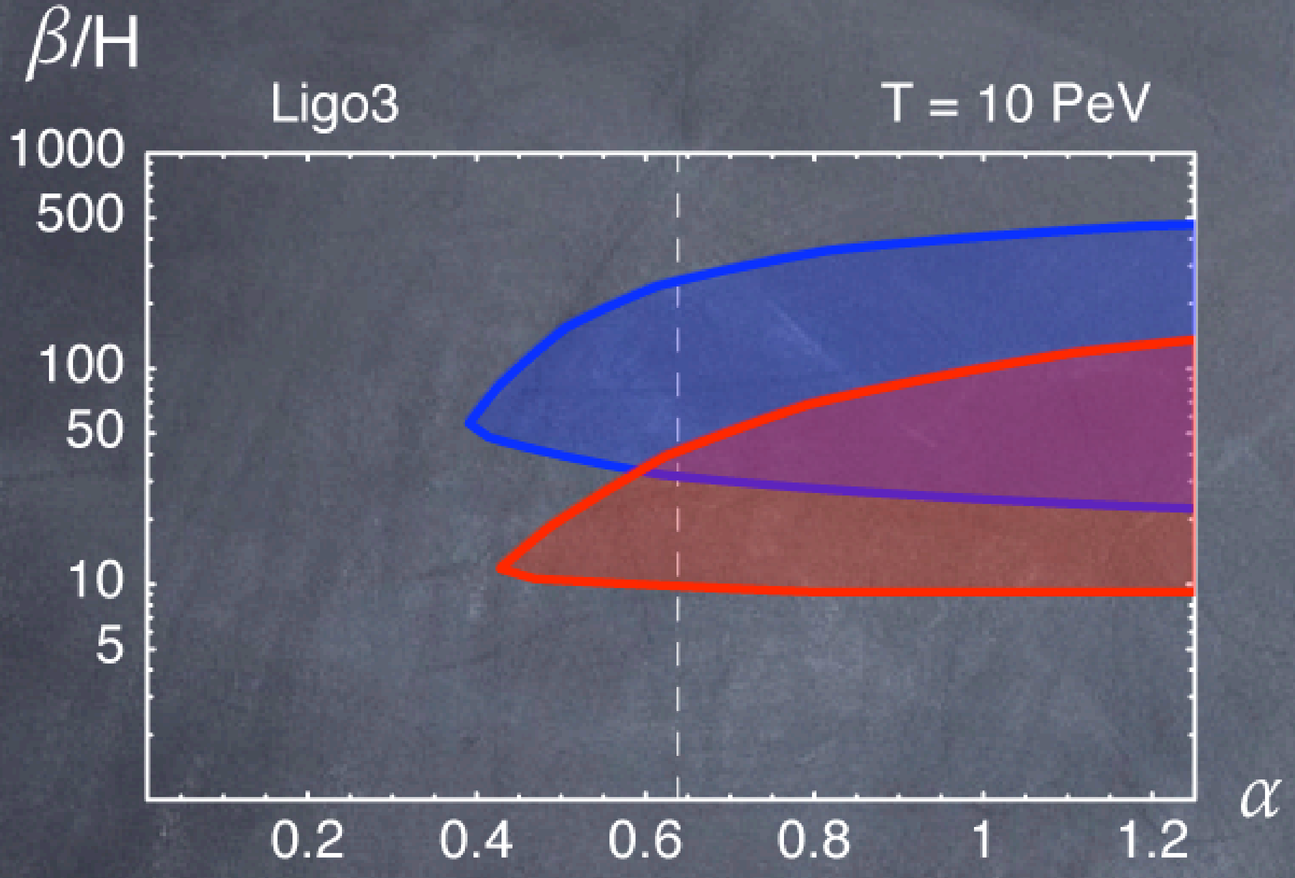
Observability of a 1st order PT at BBO

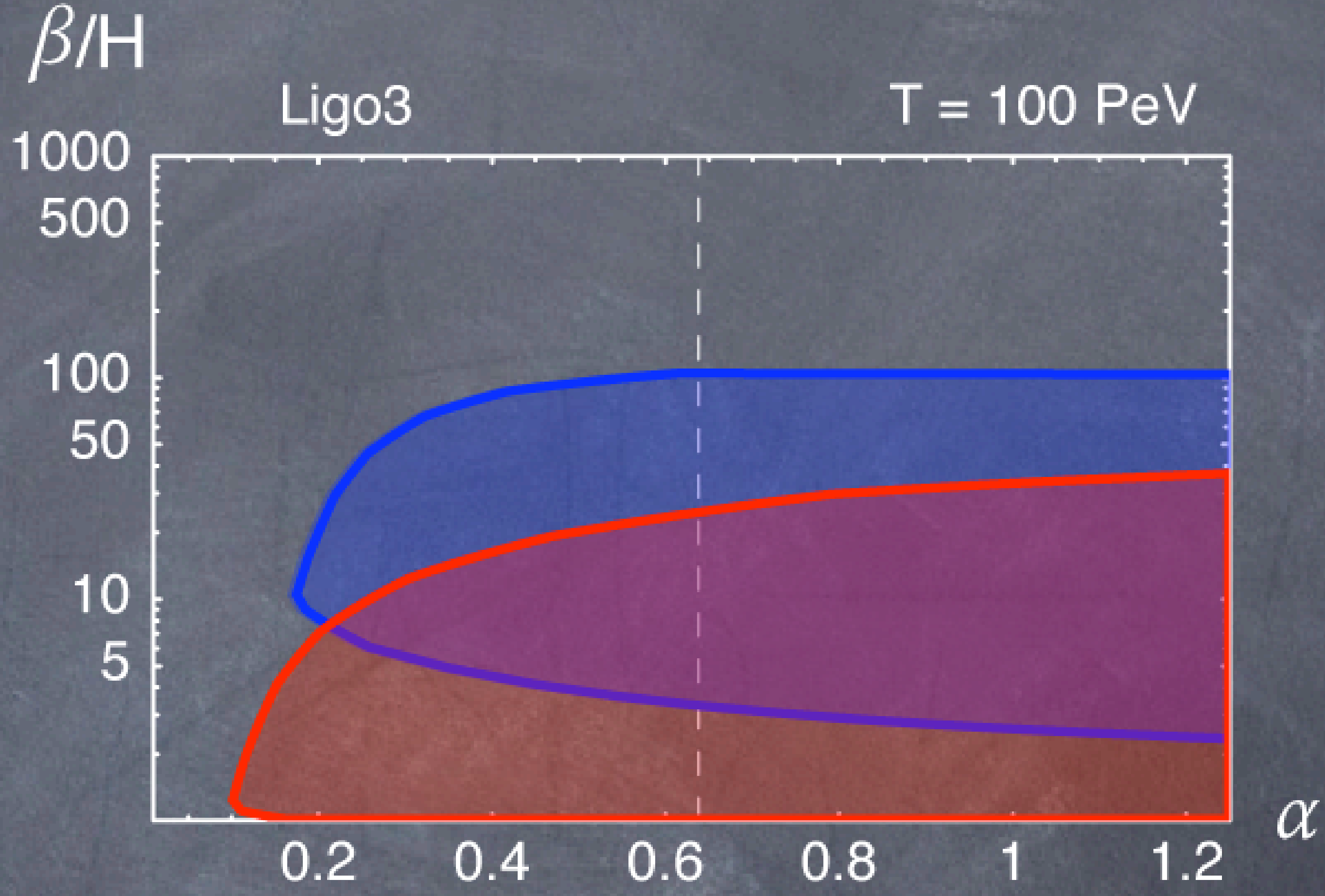


Observability of a 1st order PT at BBO



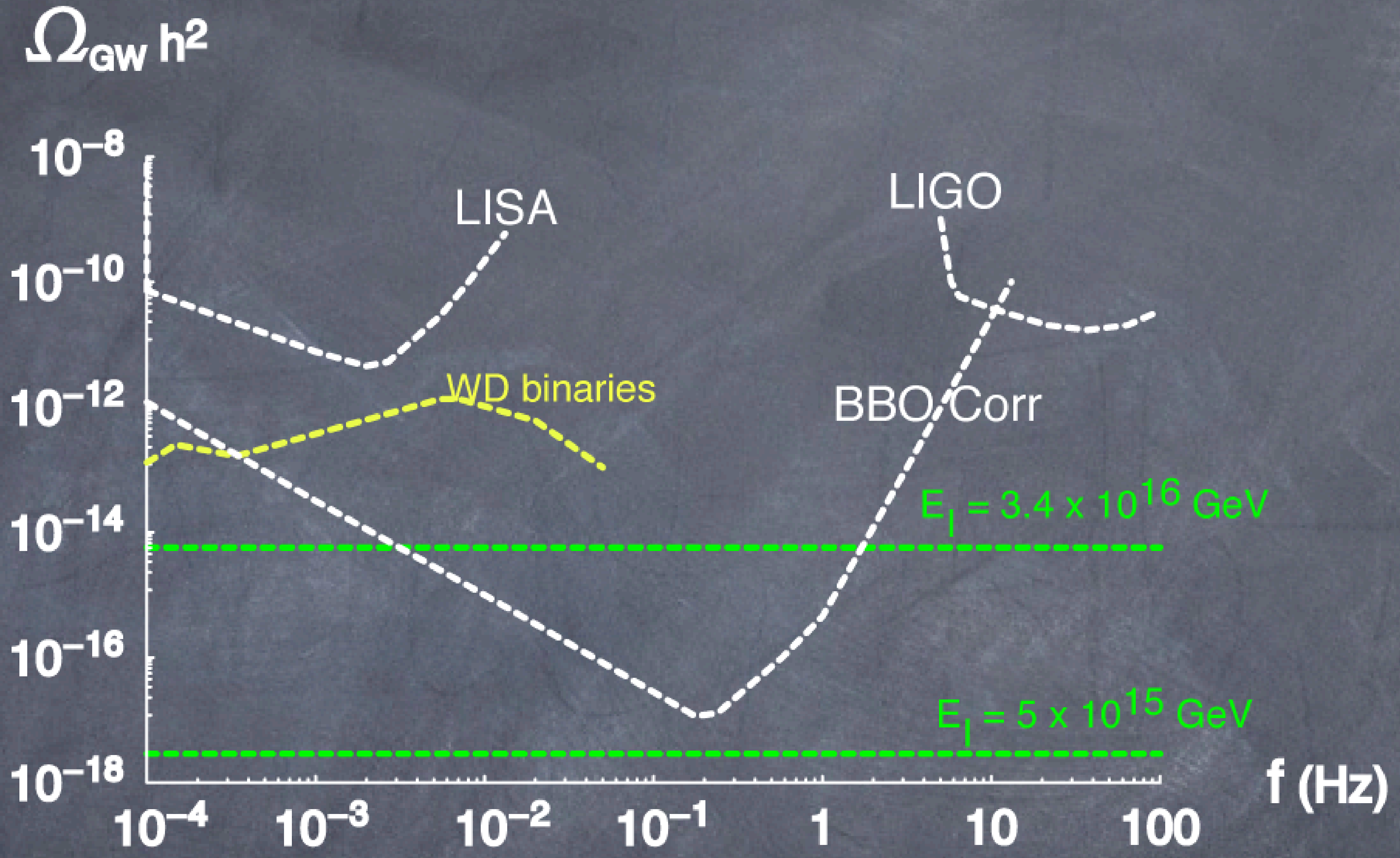
Observability of a 1st order PT at LIGO





A phase transition at T~ 10^7 GeV could be observed at LIGO.

Inflation vs. Phase Transition Signal

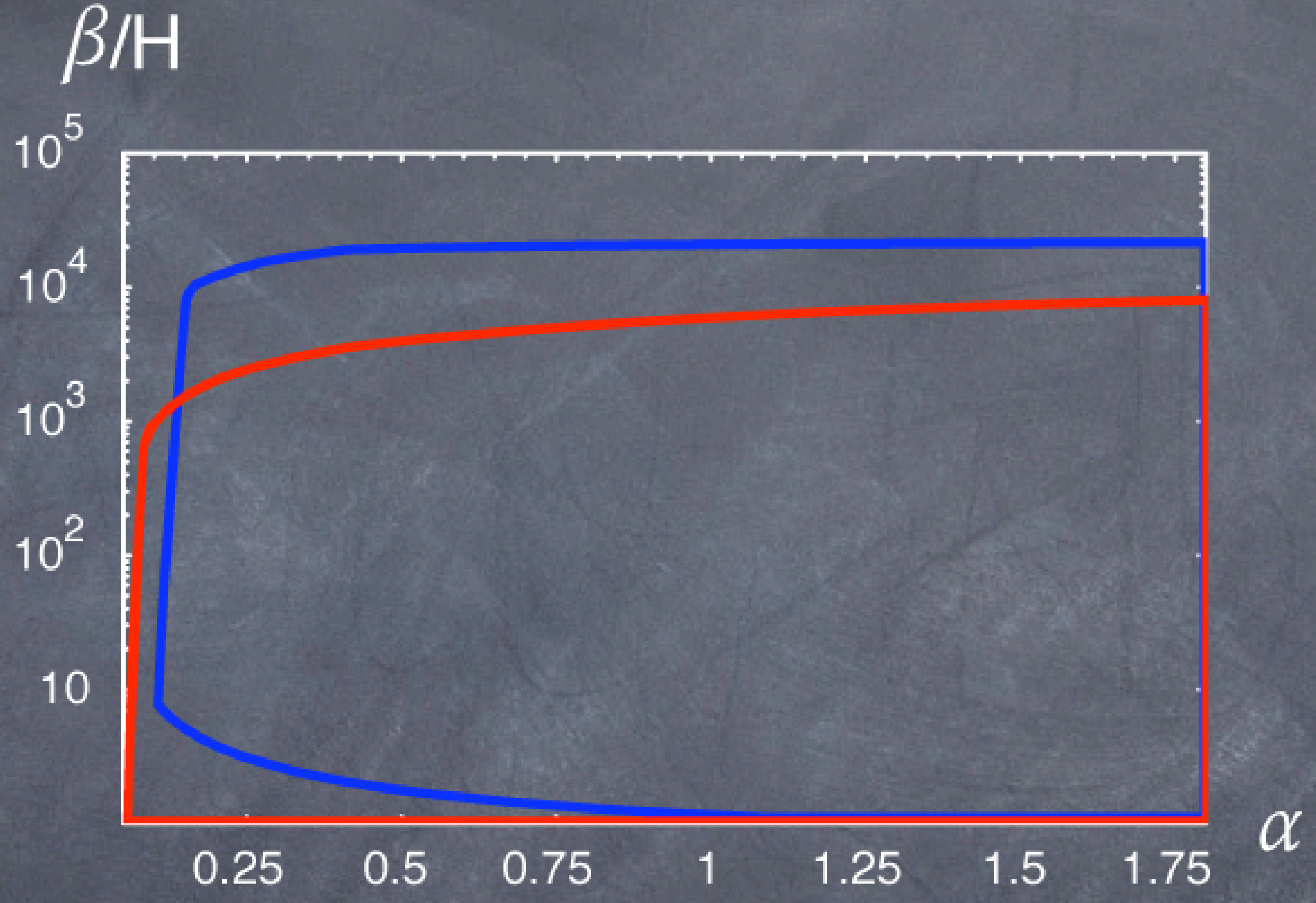


WMAP: B-mode polarization imposes $\Omega_{GW} h^2 \leq 10^{-15} - 10^{-14}$

If the scale of inflation is high enough, its GW signal can be observable at BBO

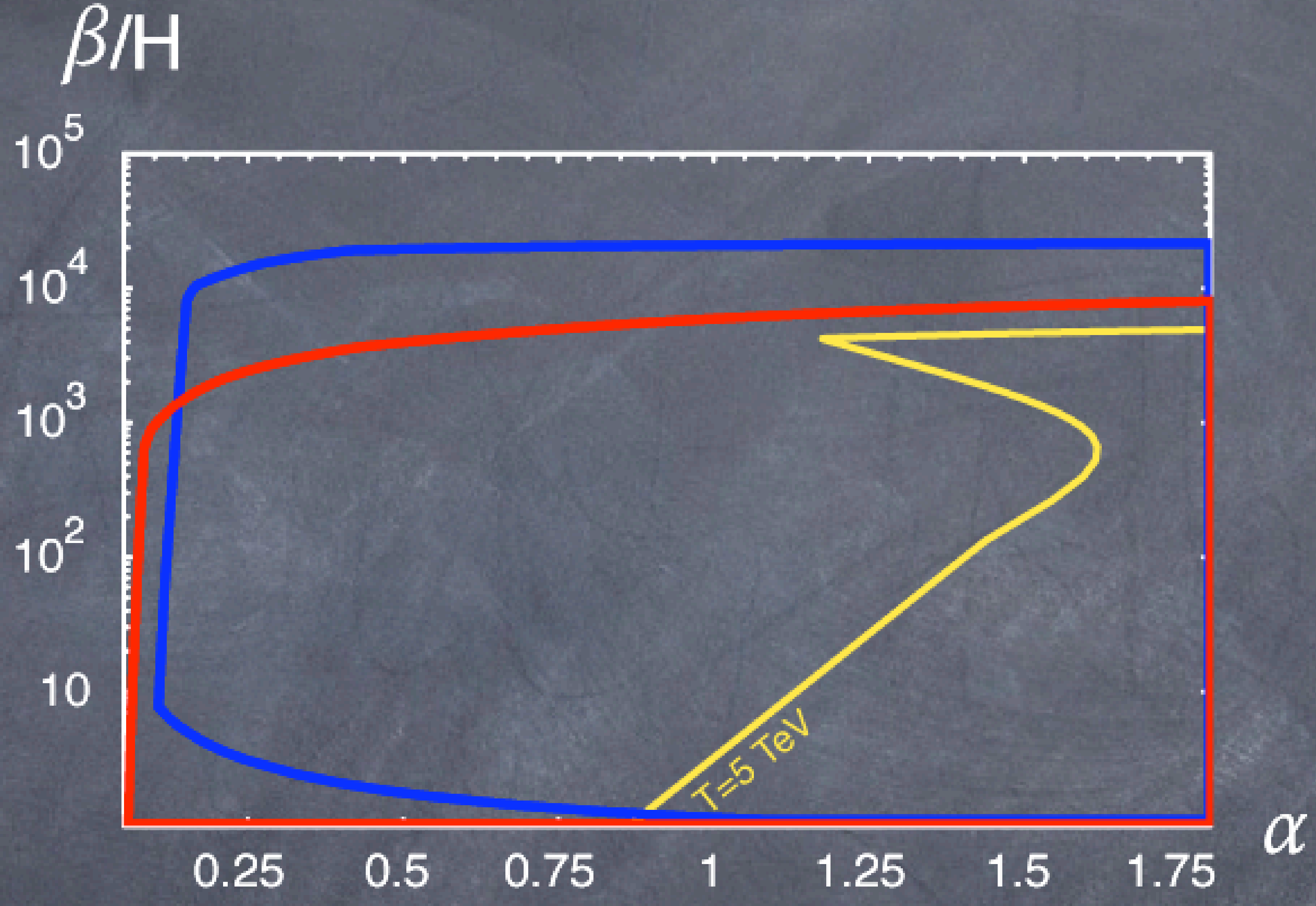
unless it is screened by GW background produced by a 1st order phase transition

Inflation vs. Phase Transition Signal

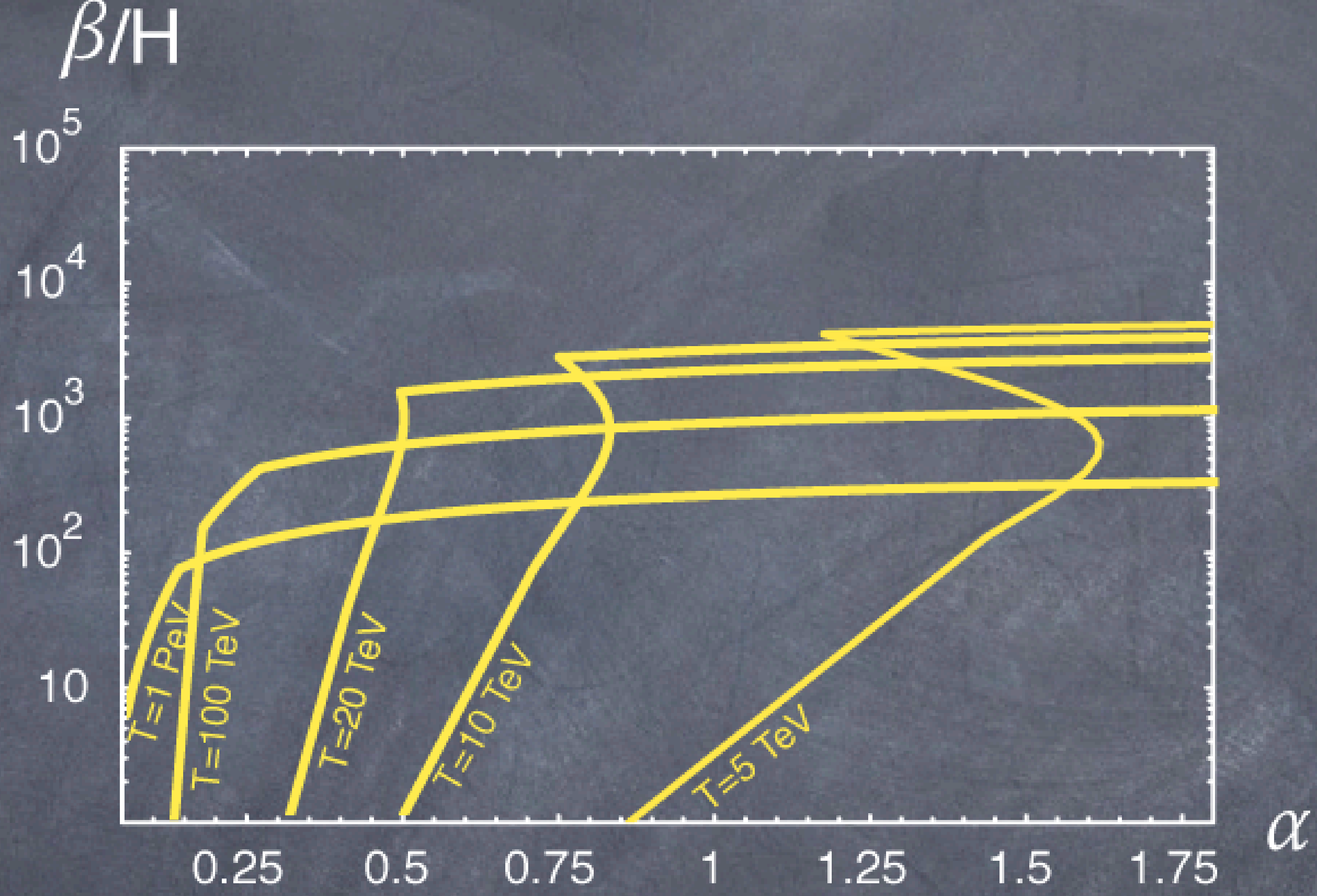


Observability of the peaks at BBO from a PT at T=5 TeV

Inflation vs. Phase Transition Signal



Inflation vs. Phase Transition Signal

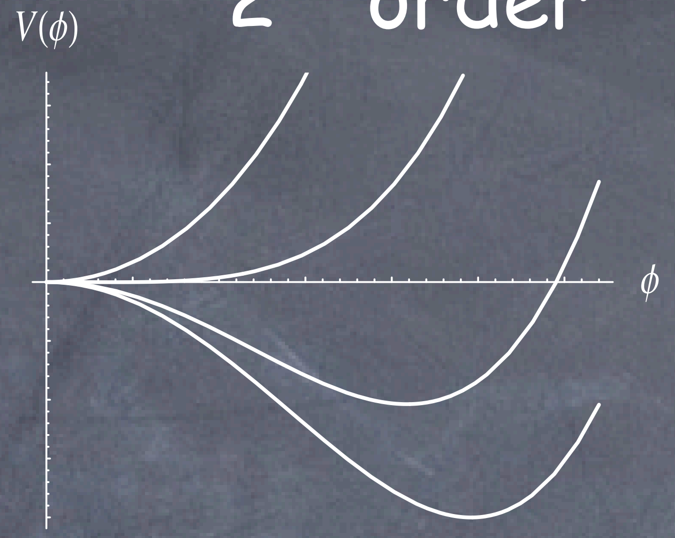


A phase transition up to T = 1 PeV can totally mask a GW signal from inflation at BBO

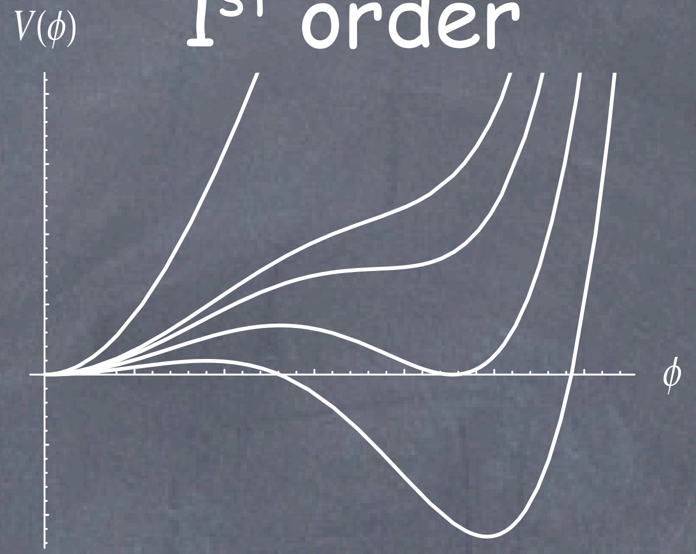
What can we learn from GW in the EW symmetry breaking sector of a particular model?

EW Phase Transition in the Standard Model

2nd order



1st order



In the SM, a 1st order phase transition could occur due to thermally generated cubic Higgs interactions:

$$V(\phi,T) \simeq \frac{1}{2}\left(-\mu_h^2 + cT^2\right)\phi^2 + \frac{\lambda}{4}\phi^4 \boxed{- ET\phi^3} \qquad \boxed{-\frac{T}{12\pi}\sum_{\text{bosons}} m^3(\phi)}$$

In the SM: $\sum_i \simeq \sum_{W,Z}$ $\Rightarrow$ not enough $E = \dfrac{4m_W^3 + 2m_Z^3}{12\pi v_0^3} \sim 6{\cdot}10^{-3}$

$$\frac{\langle\phi(T_c)\rangle}{T_c} = \frac{2\ E\ v_0^2}{\lambda\ v_0^2} = \frac{4\ E\ v_0^2}{m_h^2}$$

$$\frac{\langle\phi(T_c)\rangle}{T_c} \geq 1 \quad \Longleftrightarrow \quad m_h \leq 47\ \text{GeV}$$

A first order EW phase transition from H^6

Grojean, Servant, Wells '04

- does not rely on a thermally generated negative Higgs cubic interaction

- instead, we add a non-renormalizable Φ^6 interaction in the Higgs potential

$$V(\phi) = \mu_h^2|\Phi|^2 - \lambda|\Phi|^4 + \frac{|\Phi|^6}{\Lambda^2}$$

Little Higgs theories

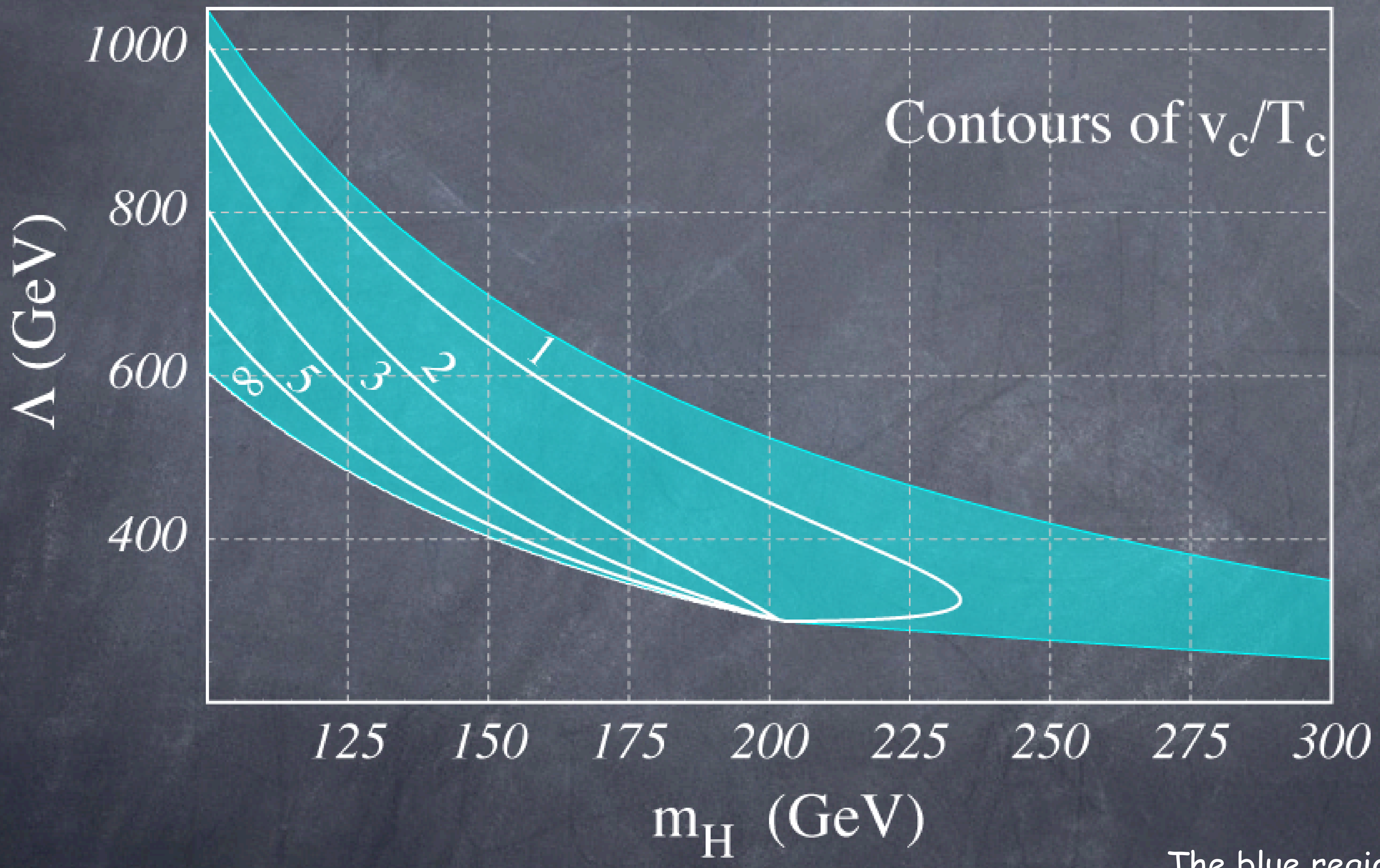
integrating out a singlet coupled to the Higgs

Can induce a strong 1^{st} order phase transition if

$$\Lambda \sim 1 \text{ TeV}$$

$$\langle\phi^2(T_c)\rangle = \frac{3}{2}v_0^2 - \frac{m_h^2\Lambda^2}{2v_0^2} \quad \text{and} \quad T_c^2 = \frac{\Lambda^4 m_h^4 + 2\Lambda^2 m_h^2 v_0^4 - 3v_0^8}{16c\Lambda^2 v_0^4}$$

A strongly 1st order PT at large Higgs mass



The blue region corresponds to a first order phase transition

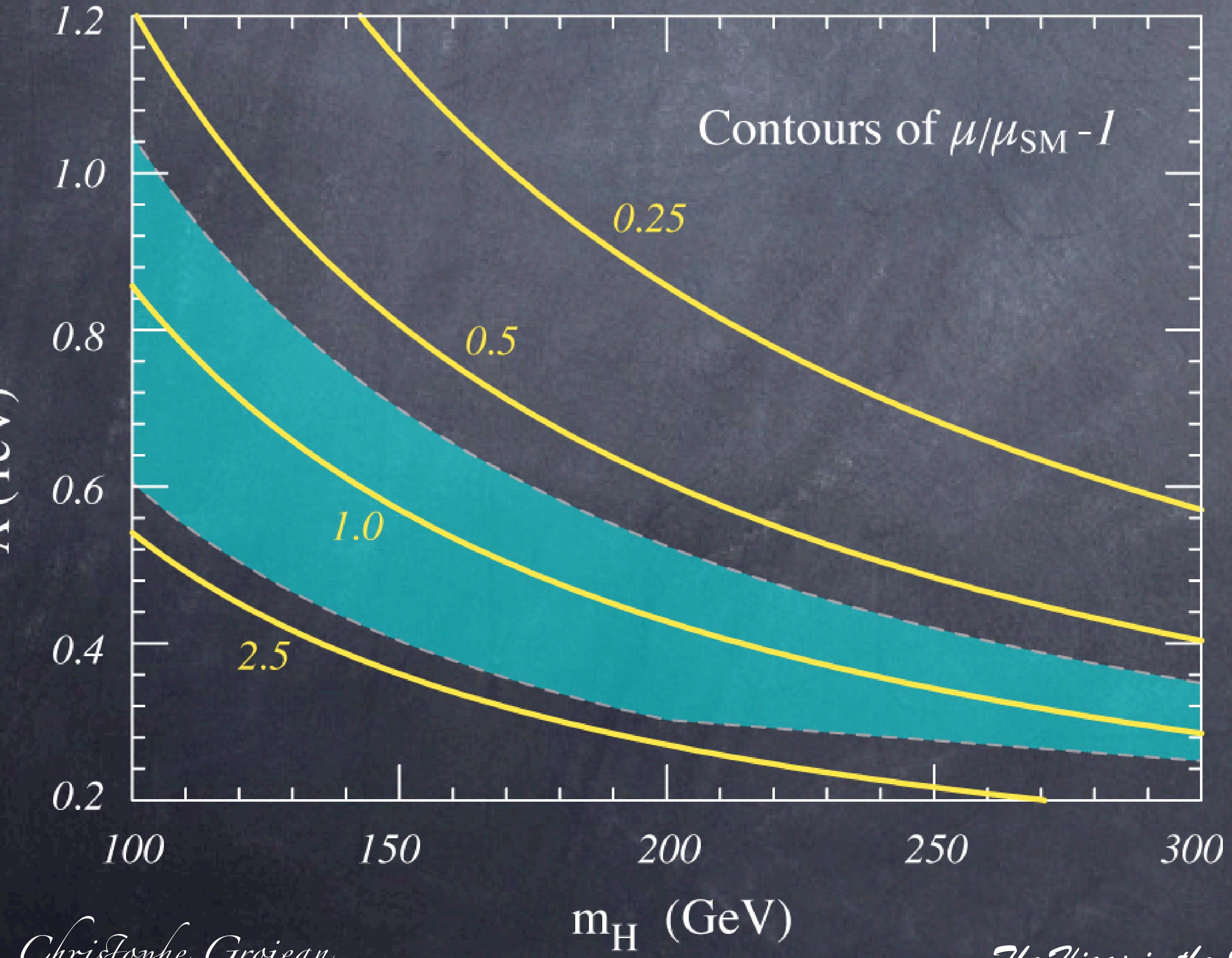
Testing the H^6 interaction @ colliders

The H^6 interaction generates large deviations in the Higgs self-couplings

$$\mathcal{L} = \frac{m_H^2}{2} H^2 + \frac{\mu}{3!} H^3 + \frac{\eta}{4!} H^4 + \ldots$$ where

$$\mu = 3\frac{m_H^2}{v_0} + 6\frac{v_0^3}{\Lambda^2}$$

$$\eta = 3\frac{m_H^2}{v_0^2} + 36\frac{v_0^3}{\Lambda^2}$$

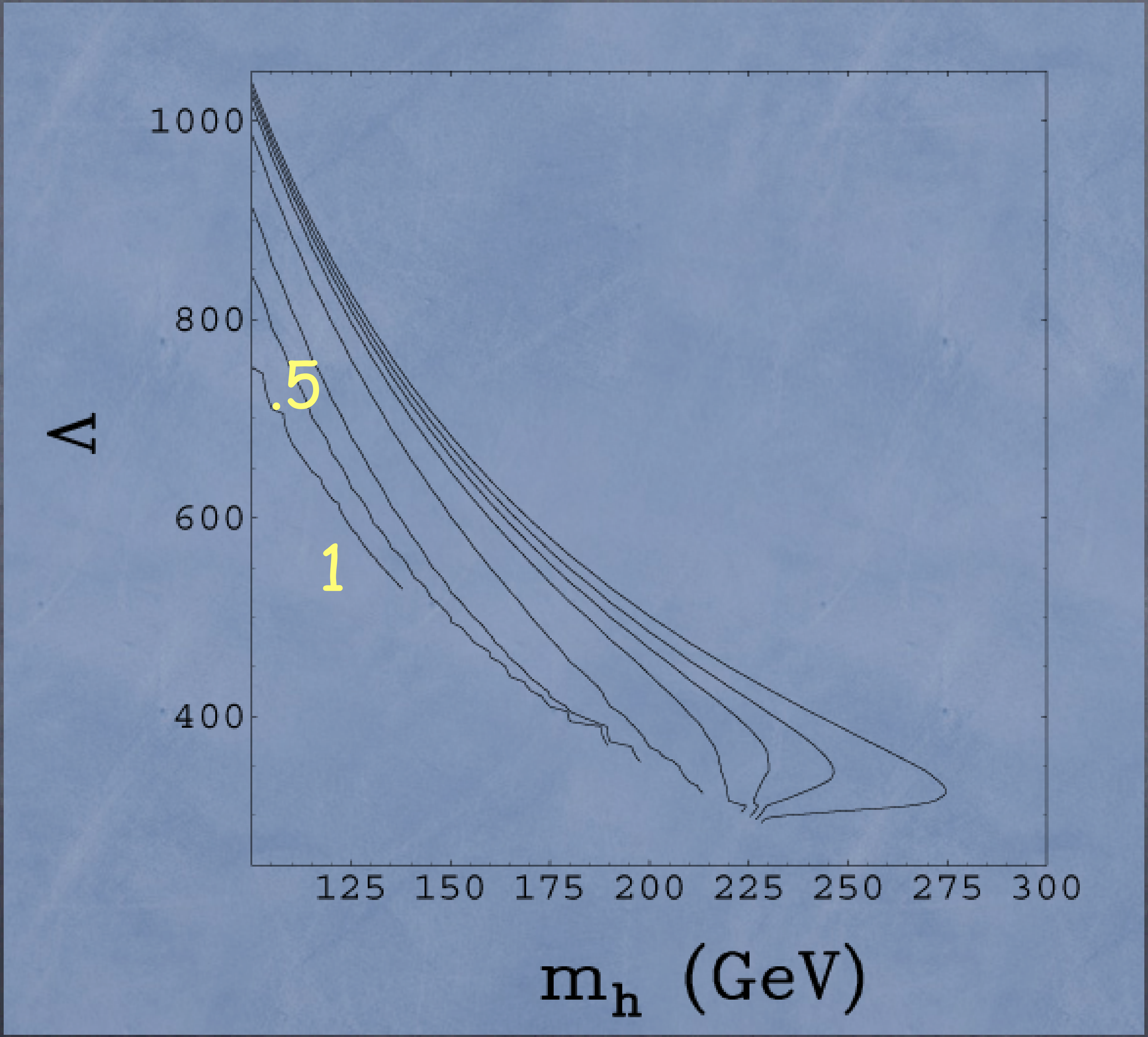


The dotted lines delimit the region for a strong 1rst order phase transition

even deviations of order 1
are difficult to see
at LHC/ILC

Testing the H^6 interaction in the sky

Delaunay, Grojean, Servant, to appear



preliminary

α contour plots

Conclusions

LHC/ILC will tell us about the EWSB sector

GW experiments might also bring interesting and complementary pieces of information

We might well be learning something about the Higgs by looking at the sky