Quarks-2006, 22 May 2006

Total resummation of leading logarithms of x versus Standard description of the polarized DIS

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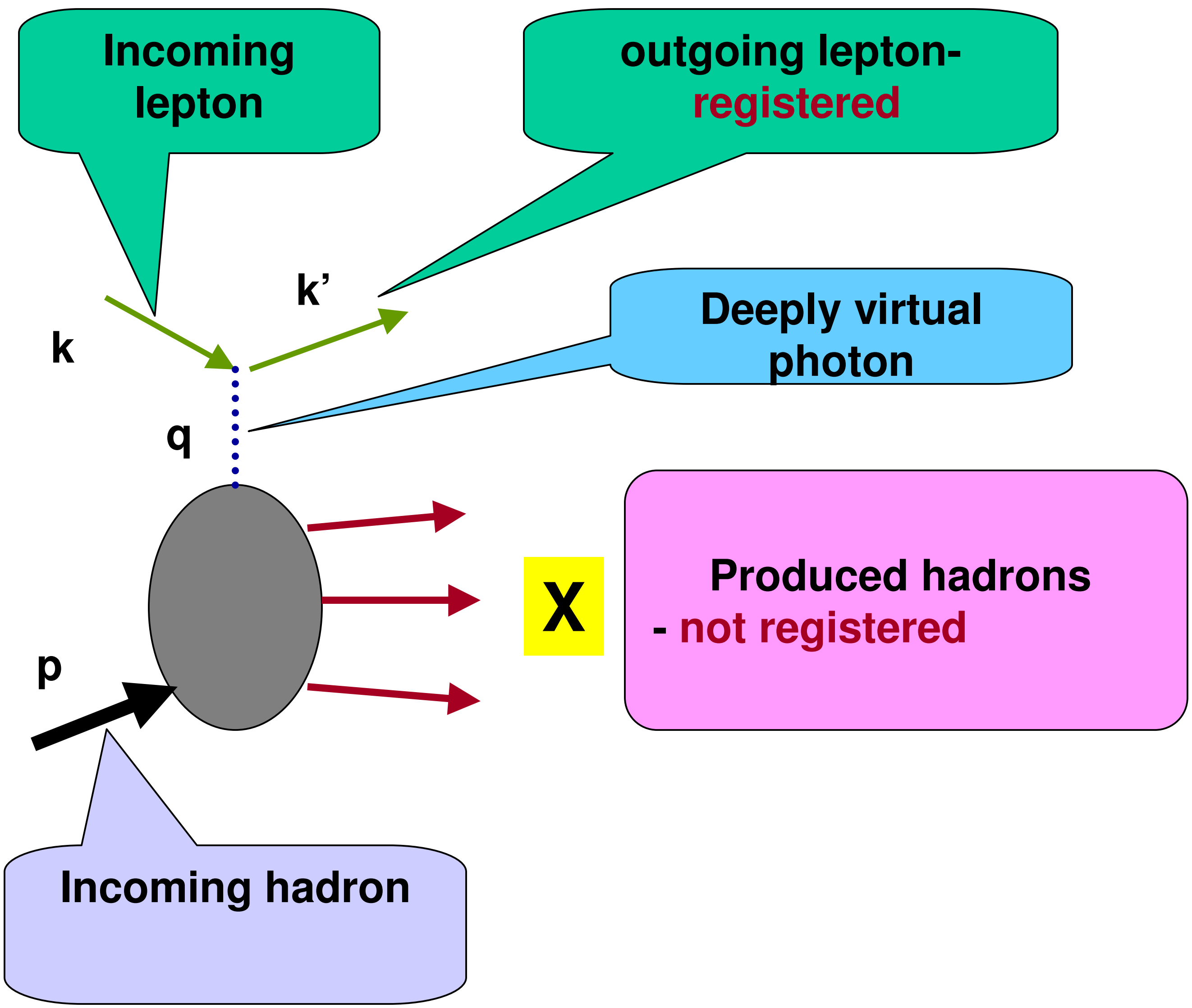
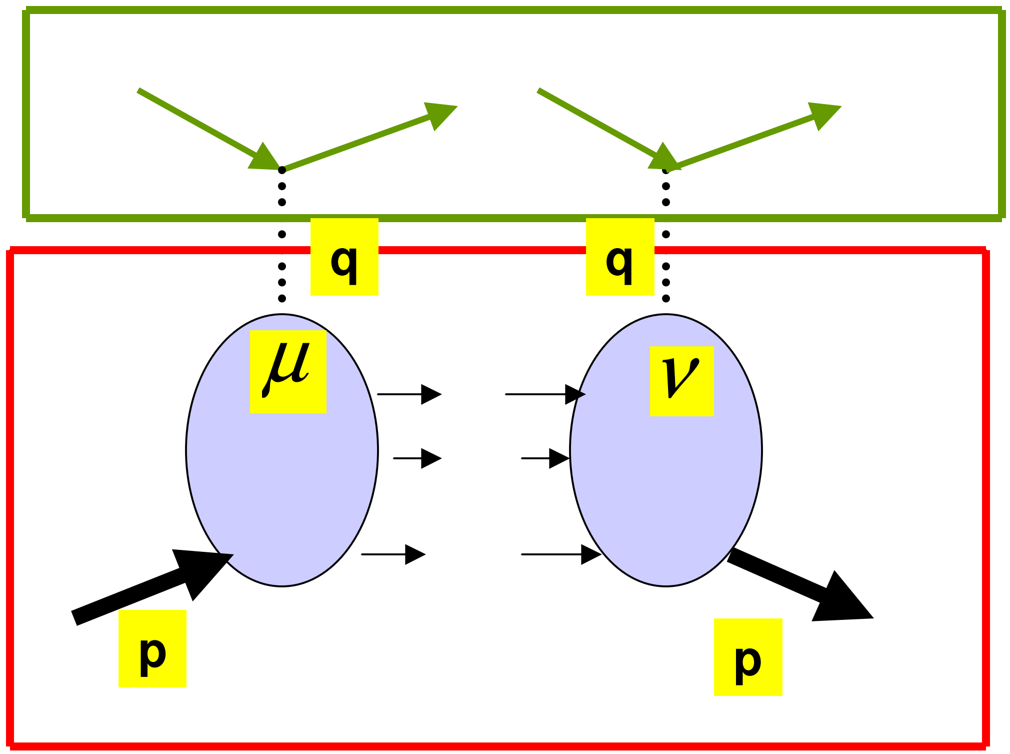
talk based on results obtained in collaboration with

M. Greco and S.I. Troyan

Deep Inelastic e-p Scattering





Leptonic tensor

hadronic tensor $W_{\mu\nu}$

hadronic tensor consists of two terms:

Does not depend on spin

Spin-dependent

$$W_{\mu\nu} = W_{\mu\nu}^{unpolarized}(p,q) + W_{\mu\nu}^{spin}(p,q)$$

symmetric

antisymmetric

The spin-dependent part of $W_{\mu\nu}$ is parameterized by two structure functions:

$$W_{\mu\nu}^{spin} = \frac{m}{pq} i\varepsilon_{\mu\nu\lambda\rho} q_{\lambda} \left[S_{\rho} g_1(x,Q^2) + \left(S_{\rho} - \frac{Sq}{pq} p_{\rho} \right) g_2(x,Q^2) \right]$$

Structure functions

where **m**, **p** and **S** are the hadron mass, momentum and spin;
q is the virtual photon momentum ($Q^2 = -q^2 > 0$). Both of the functions depend on Q^2 and $x = Q^2/2pq$, $0 < x < 1$. When both electron and proton spins are in the **p,q** –plane, g_1 is related to the asymmetry:

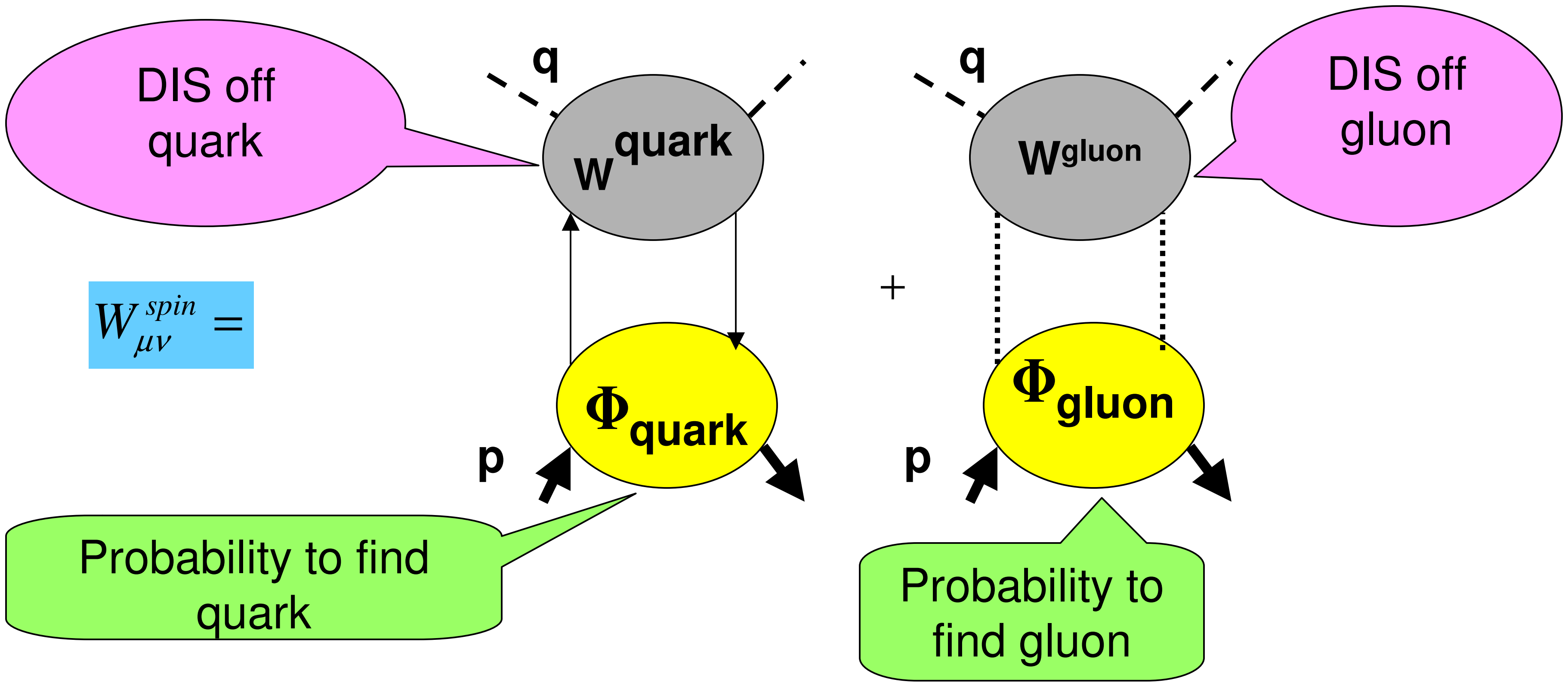
$$g_1 \propto \sigma_{\uparrow\uparrow} - \sigma_{\uparrow\downarrow}$$

Parallel spins

Antiparalel spins

FACTORISATON: $W_{\mu\nu}$ **is a convolution of the the partonic tensor and probabilities to find a polarized parton (quark or gluon) in the hadron :**

$W^{spin}_{\mu\nu} =$



DIS off quark and gluon can be studied with perturbative QCD, with calculating involved Feynman graphs.

Probabilities, Φ_{quark} and Φ_{gluon} involve non-perturbaive QCD. There is no a regular analytic way to calculate them. Usually they are defined from experimental data at large **x** and small **Q^2**, they are called the initial quark and gluon densities and are denoted δ**q** and δ**g** .

So, the conventional form of the hadronic tensor is:

$$W_{\mu\nu} = W_{\mu\nu}^{quark} \otimes \delta q + W_{\mu\nu}^{gluon} \otimes \delta g$$

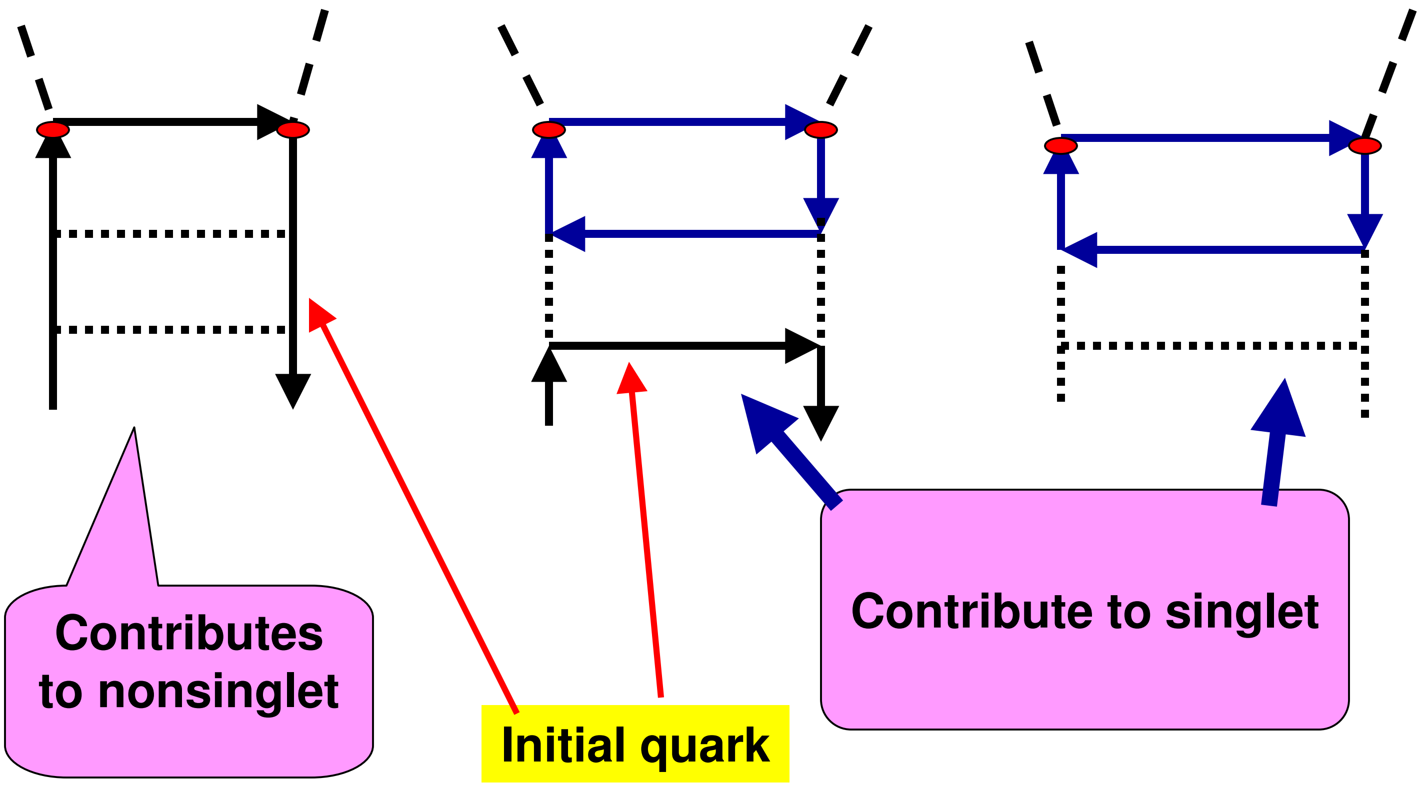
Initial quark distribution

Initial gluon distribution

DIS off the quark

DIS off the gluon

Piece of terminology



Each structure function has both the non-singlet and singlet components:
$g_1 = g_1^{NS} + g_1^{S}$

The Standard Approach includes the DGLAP Evolution Equations and the Standard Fits for initial paron densities

Evolution Equations: Altarelli-Parisi, Gribov-Lipatov, Dokshitzer

In particular, non-singlet g_1:

$$g_1^{NS}(x,Q^2) = (e_q^2/2)C_{NS}(x/y) \otimes \Delta q^{NS}(y,Q^2)$$

Coefficient function

Evolved quark distribution

where

$$\frac{d\Delta q^{NS}}{d\ln Q^2} = \frac{\alpha_s}{2\pi} P_{qq} \otimes \Delta q^{NS}$$

Splitting function

Expression for the singlet g_1 is similar, though more involved. It includes

coefficient function $C_q,\ C_g$ and splitting functions $P_{qq},\ P_{qg},\ P_{gq}, P_{gg}$

in order to make the quark and gluon **evolved distributions** Δq **and** Δg

Applying the Mellin transform, obtain expression for g_1^{NS} in a simpler form :

Initial quark density

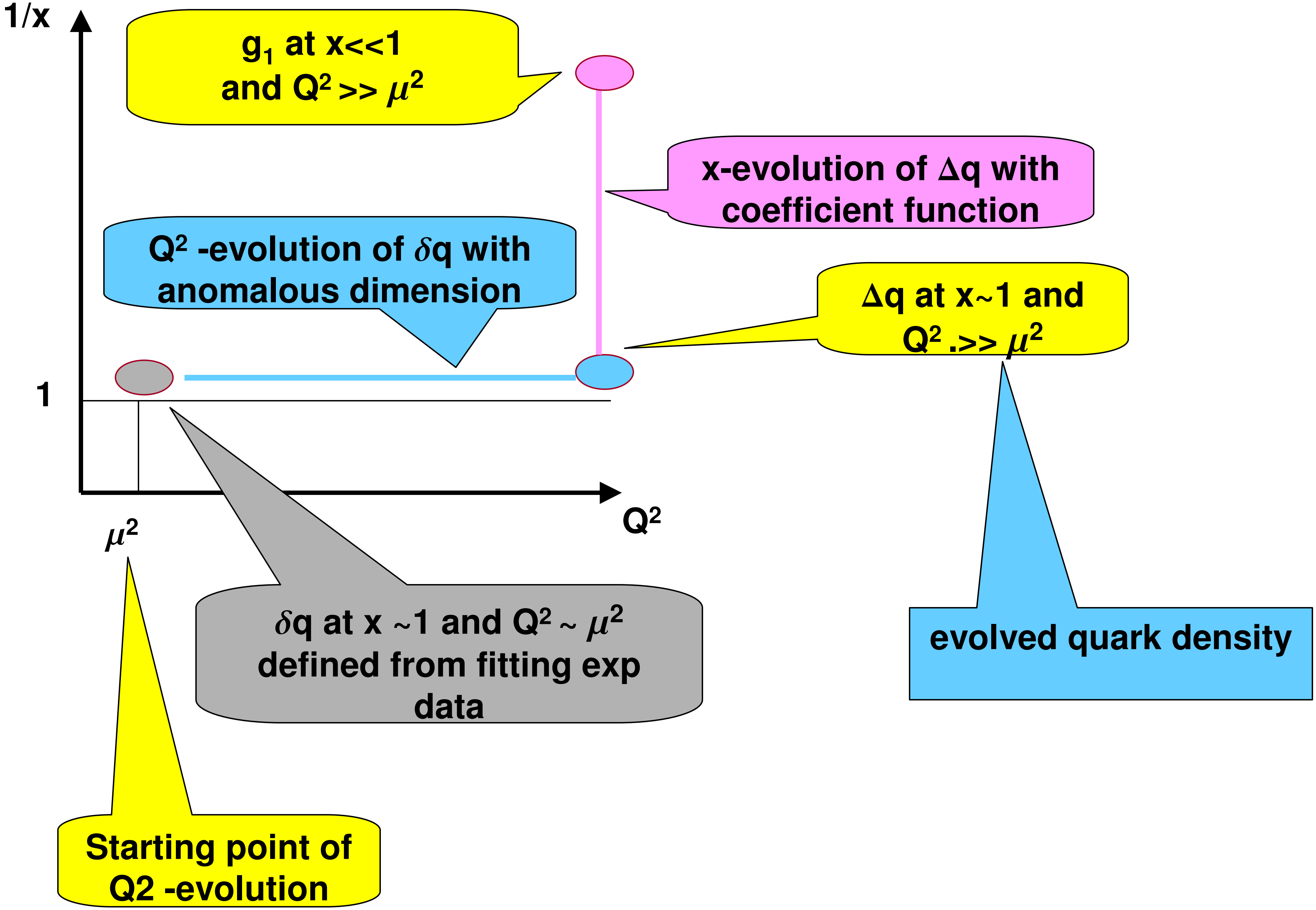
$$g_1^{NS}(x,Q^2) = (e_q^2/2) \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} \left(\frac{1}{x}\right)^{\omega} C(\omega)\delta q(\omega) \exp\left[\int_{\mu^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \gamma(\omega, \alpha_s(k_\perp^2))\right]$$

Coefficient function

Anomalous dimension

Non-Pert QCD

Pert QCD


1/x
g_1 at x<<1
and $Q^2 >> \mu^2$
x-evolution of Δq with
coefficient function
Q^2 -evolution of δq with
anomalous dimension
Δq at x~1 and
Q^2 .>> μ^2
1
Q^2
μ^2
δq at x ~1 and $Q^2 \sim \mu^2$
defined from fitting exp
data
evolved quark density
Starting point of
Q2 -evolution

In DGLAP, coefficient functions and anomalous dimensions are known with LO and NLO accuracy, often at integer $\omega = n$

LO

NLO

$$C(\omega) = 1 + (\alpha_s(Q^2)/2\pi)\, C^{(1)}(\omega) + \ldots$$

$$\gamma(\omega) = (\alpha_s(Q^2)/4\pi)\, \gamma^{(0)}(\omega) + (\alpha_s(Q^2)/2\pi)^2 \gamma^{(1)}(\omega) + \ldots$$

LO

NLO

One can say that DGLAP includes both Science and Art :

SCIENCE

LO splitting functions	**Ahmed-Ross, Altarelli-Parisi, Sasaki,**
NLO splitting functions	**Floratos, Ross, Sachradja, Gonzale- Arroyo, Lopes, Yandurain, Kounnas, Lacaze, Curci, Furmanski, Petronzio, Zijlstra, Mertig, van Neerven, Vogelsang**
Coefficient functions $C^{(1)}_k$, $C^{(2)}_k$	**Bardeen, Buras, Muta, Duke, Altarelli, Kodaira, Efremov, Anselmino, Leader, Zijlstra, van Neerven**

ART

= the art of composing the fits for initial parton densities

Altarelli-Ball-Forte-Ridolfi, Blumlein- Botcher, Leader- Sidorov-Stamenov, Hirai et al

There are different fits for initial parton densities. For example,

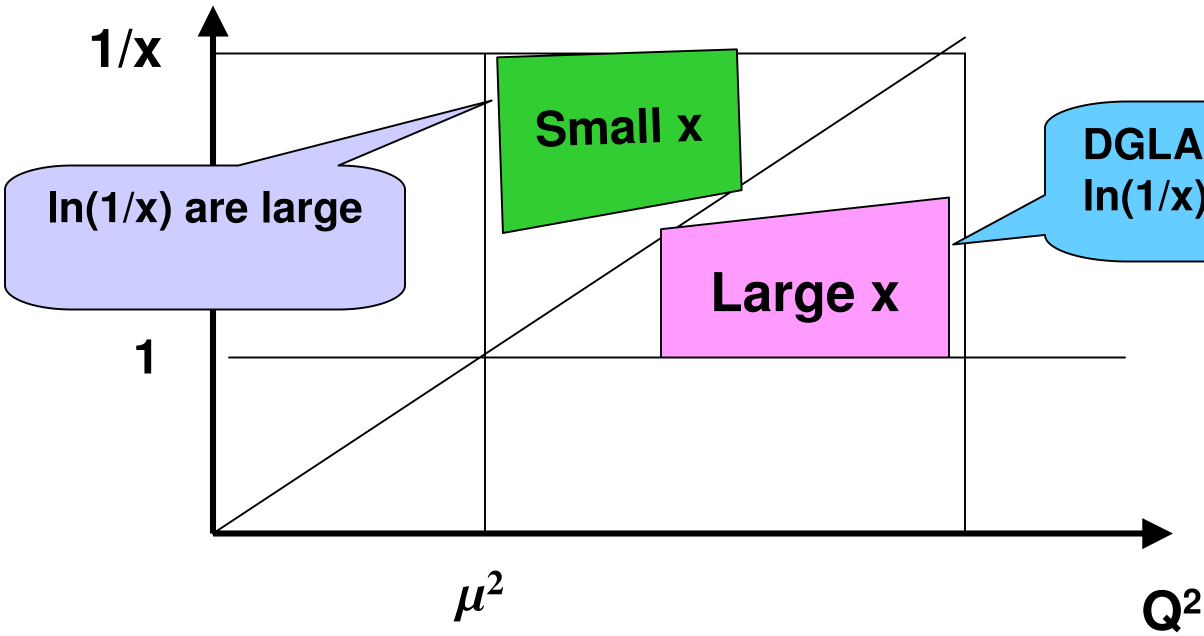
$$\delta q = N x^{-\alpha} [(1-x)^{\beta} (1+\gamma x^{\delta})]$$

$$\delta q = N [\ln^{\alpha}(1/x) + \gamma x \ln^{\beta}(1/x)]$$

Altarelli-Ball-Forte-Ridolfi,

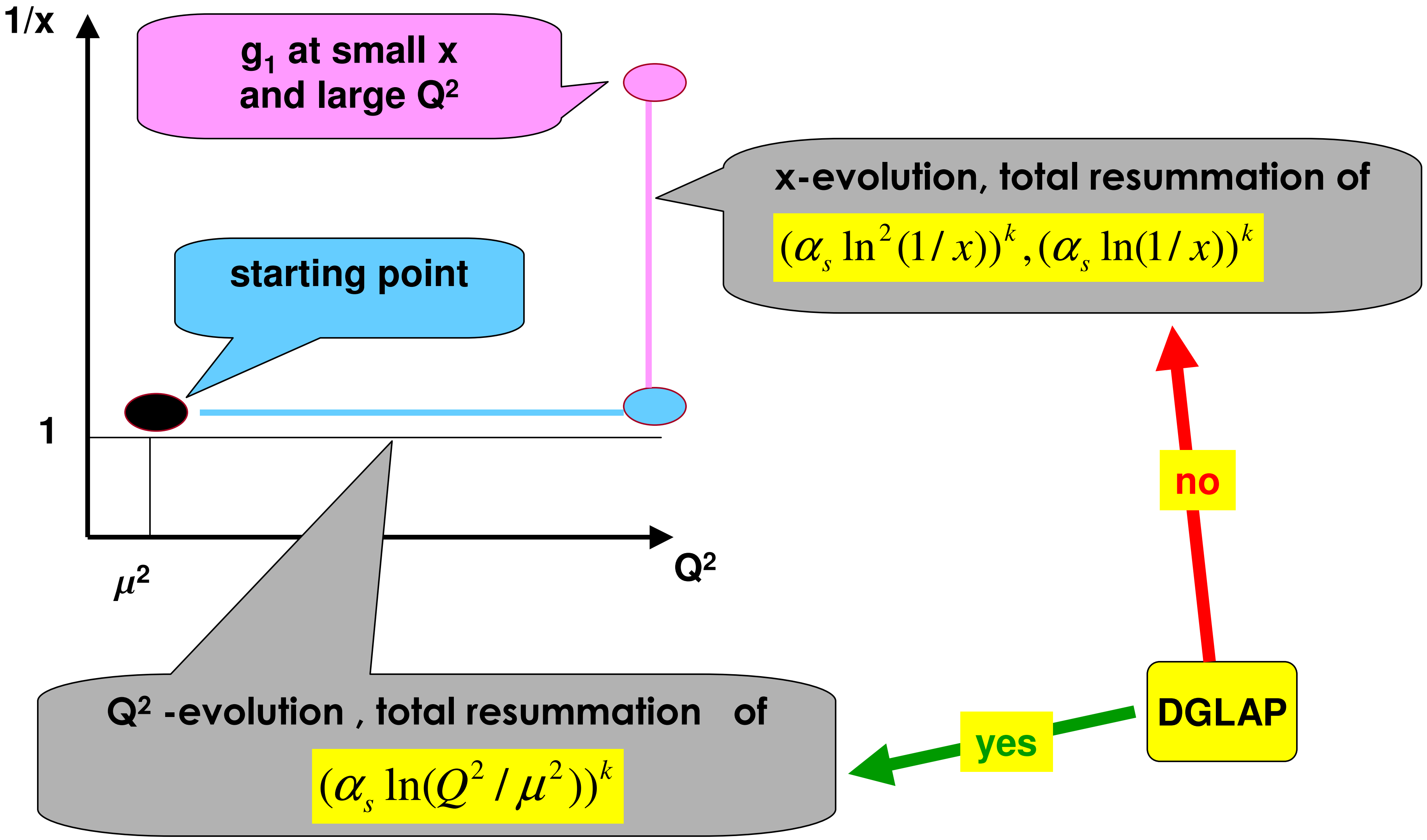
Parameters $N, \alpha, \beta, \gamma, \delta$ should be fixed from experiment

This combination of Science and Art works well at large and small x, though strictly speaking, DGLAP is not supposed to work at the small- x region:

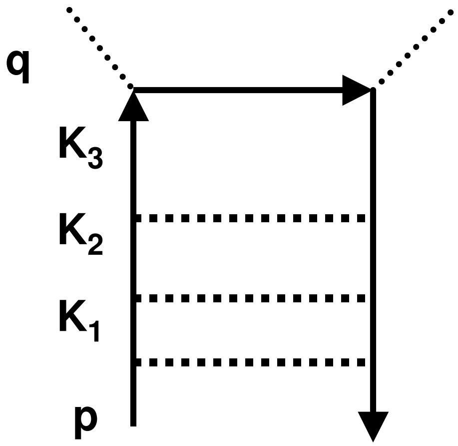


DGLAP accounts for $\ln(Q^2)$ to all orders in α_s and neglects $(\alpha_s \ln^2(1/x))^k, (\alpha_s \ln(1/x))^k$ **with k>2**

However, these contributions become leading at small x and should be accounted for to all orders in the QCD coupling.



DGLAP cannot do total resummation of logs of x because of the DGLAP-ordering – KEYSTONE of DGLAP



DGLAP –ordering:

$$\mu^2 < k_{1\perp}^2 < k_{2\perp}^2 < k_{3\perp}^2 < Q^2$$

good approximation for large x when logs of x can be neglected. At x << 1 the ordering has to be lifted

DGLAP small-x asymptotics of g_1 is well-known:

$$g_1^{DGLAP} \sim \exp\left[\ln(1/x)\ln\ln\left(Q^2/\Lambda_{QCD}^2\right)\right]^{1/2}$$

when the initial parton densities are not singular functions of x

When the DGLAP –ordering is lifted and all double logarithms of x are accounted for, the asymptotics is different:

$$g_1^{DL} \sim (1/x)^{\Delta}\left(Q^2/\mu^2\right)^{\Delta/2}$$

intercept

Bartels- Ermolaev-
Manaenkov-Ryskin

Obviously $g_1^{DL} \gg g_1^{DGLAP}$ when $x \rightarrow 0$

Intercepts of g_1 in Double-Logarithmic Approximation:

non- singlet intercept

$$\Delta_{NS} = (8\alpha_s/3\pi)^{1/2},$$

singlet intercept

$$\Delta_S = 3.45\,(3\alpha_s/2\pi)^{1/2}$$

The weakest point of this approach: the QCD coupling α_s is fixed at an unknown scale.

On the contrary, DGLAP equations have always operated with running α_s

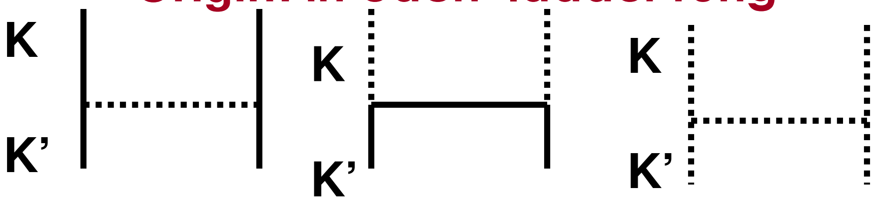
$$\alpha_s = \alpha_s(Q^2)$$

DGLAP-
parameterization

Arguments in favor of the Q^2- parameterization:

Bassetto-Ciafaloni-Marchesini - Veneziano, Dokshitzer-Shirkov

Origin: in each ladder rung



$$\alpha_s = \alpha_s(k_\perp^2)$$

DGLAP-parameterization

However, such a parameterization is good for large x only. At small **x** :

Ermolaev-Greco-Troyan

$$\alpha_s = \alpha_s((k-k')^2) \approx \alpha_s((k_\perp^2 + k_\perp'^2)/x) \approx \alpha_s(k_\perp^2)$$

time-like argument

Participates in the Mellin transform

When DGLAP-ordering is used and x ~1

DGLAP -parametrization

$$\alpha_s = \alpha_s(Q^2)$$

space-like argument,

no Mellin transform

Obviously, this parameterization and the DGLAP one converge when x is large but differ a lot at small x

So, for studying g_1 in the small-x region, it is necessary to do:

1. Total resummation of logs of x
2. New parameterization of the QCD coupling

The basic idea: the formula $\alpha_s(k^2) = 1/(b \ln(k^2/\Lambda^2))$

valid when $k^2 >> \Lambda^2$

$\Downarrow$

it is necessary to introduce an infrared cut-off for k^2

It is convenient to introduce the cut-off in the transverse space:

$k_\perp^2 > \mu^2$

Lipatov

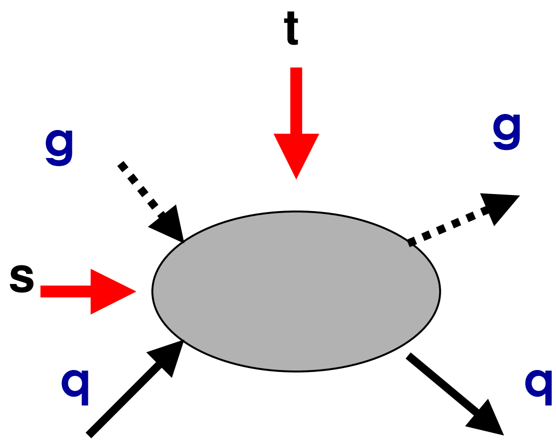
As value of the cut-off is not fixed, one can evolve the structure functions with respect to μ → the name of the method:

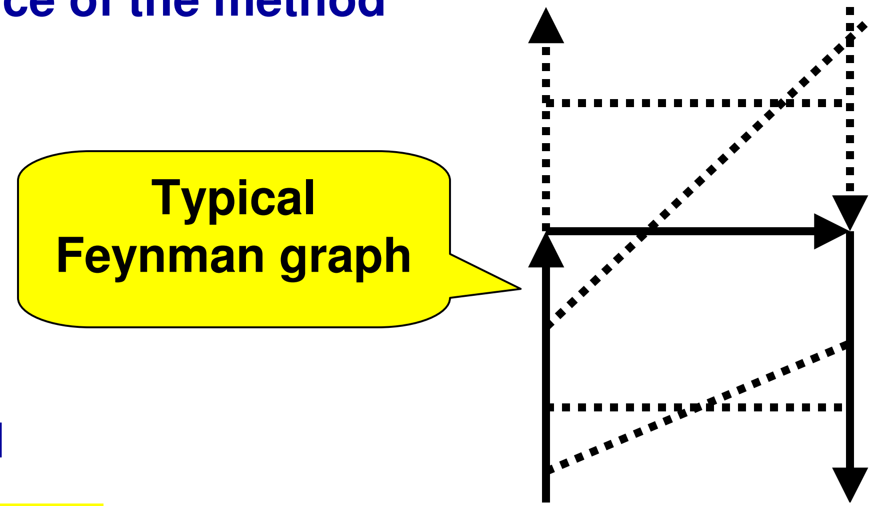
Infra-Red Evolution Equations (IREE)

Highlights of the history of the method

- ☆ Analyses of two-particle cuts in Regge kinematics — Gribov
- ☆ Factorization of photons with small transverse momenta — Gribov
- ☆ Infrared cut-off in the transverse momentum space — Lipatov
- ☆ Quark-quark scattering amplitudes — Kirschner-Lipatov
- ☆ Generalization of Gribov bremsstrahlung theorem to QCD , inelastic quark form factors — Ermolaev-Fadin -Lipatov
- ☆ QCD inelastic processes in Regge kinematics — Ermolaev-Lipatov
- ☆ Applications to Polarized Deep-Inelastic scattering — Bartels-Ermolaev -Manaenkov-Ryskin- Greco-Troyan

Essence of the method





Introduce IR cut-off: $k_\perp > \mu$ for all virtual particle momenta.

DL and SL contributions arrive from the integration region where $k_\perp < k_\parallel$

the cut-off works both in the longitudinal and in the transverse space

DL contributions come from the region where all transverse momenta are widely different one can factorize the phase space into a set of separable sub-regions, in each region some virtual particle has minimal $k_\perp$
Let us call such a particle the softest one.

DL and SL contributions of softest particles can be factorized

Expression for the non-singlet g_1 at large Q^2: $Q^2 >> 1$ GeV^2

Initial quark density

$$g_1^{NS} = \frac{e_q^2}{2}\int \frac{d\omega}{2\pi i}\left(\frac{1}{x}\right)^{\omega}\left(\frac{\omega}{\omega - H(\omega)}\right)\delta q(\omega)\left(\frac{Q^2}{\mu^2}\right)^{H(\omega)}$$

Coefficient function

Anomalous dimension

Compare our non-singlet anomalous dimension to the LO DGLAP one:

expand C and H into series in $1/\omega$

$$H = \frac{A(\omega)C_F}{2\pi}\left[\frac{1}{\omega} + \frac{1}{2}\right] + \ldots$$

coincide, save the treatment of α_s

$$\gamma_{NS}^{\text{LO DGLAP}} = \frac{\alpha_s(Q^2)C_F}{2\pi}\left[\frac{1}{n(n+1)} + \frac{3}{2} - S_2(n)\right] \approx \frac{\alpha_s(Q^2)C_F}{2\pi}\left[\frac{1}{n} + \frac{1}{2} + O(n)\right]$$

when n < 1

where

$$S_k(n) = \sum_{j=1}^{n}\frac{1}{j^k}$$

small/large x $\Longleftrightarrow$ **small/large n**

Compare our coefficient function and the NLO DGLAP one

$$C = \frac{\omega}{\omega - H(\omega)} = 1 + \frac{A(\omega) C_F}{2\pi}\left[\frac{1}{\omega^2} + \frac{1}{2\omega}\right] + ...$$

coincide, save the treatment of α_s

LO

NLO

$$C_{NS}^{\mathrm{DGLAP}} = 1 + \frac{\alpha_s(Q^2) C_F}{2\pi}\left[\frac{1}{n^2} + \frac{1}{2n} + \frac{1}{2n+1} - \frac{9}{2} + \left(\frac{3}{2} - \frac{1}{n(1+n)}\right) S_1(n) + S_1^2(n) - S_2(n)\right]$$

when n < 1

$$\approx \frac{\alpha_s(Q^2) C_F}{2\pi}\left[\frac{1}{n^2} + \frac{1}{2n} + O(n)\right]$$

Expression for the singlet g_1 at large Q^2:

$$g_1^S = \frac{< e_q^2 >}{2} \int \frac{d\omega}{2\pi i} \left(\frac{1}{x}\right)^{\omega}$$

$$\left[\left(C_q^{(+)} \delta q + C_q^{(+)} \delta g\right) \left(\frac{Q^2}{\mu^2}\right)^{\Omega^{(+)}} + \left(C_q^{(-)} \delta q + C_q^{(-)} \delta g\right) \left(\frac{Q^2}{\mu^2}\right)^{\Omega^{(-)}} \right]$$

$$\Omega^{(+)} > \Omega^{(-)}$$

Large Q^2 means

$$Q^2 > \mu^2; \;\; \mu \approx 5 \text{ GeV}$$

Small –x symptotics of g_1: when $x \to 0$, the saddle-point method leads to

$$g_1^{NS} \sim \frac{e_q^2}{2} (1/x)^{\Delta_{NS}} (Q^2/\mu^2)^{\Delta_{NS}/2} \delta q$$

Nonsinglet intercept $\Delta_{NS} = 0.42$

At large x, g_1^{NS} and g_1^{S} are positive

$\delta q > 0$ → $g_1^{NS} > 0$ In the whole range of x at any Q^2

Asymptotics of the singlet g_1 are more involved

$$g_1^S \sim \frac{\langle e_q^2 \rangle}{2} S(\Delta_S) \, (1/x)^{\Delta_S} \left(Q^2/\mu^2\right)^{\Delta_S/2}$$

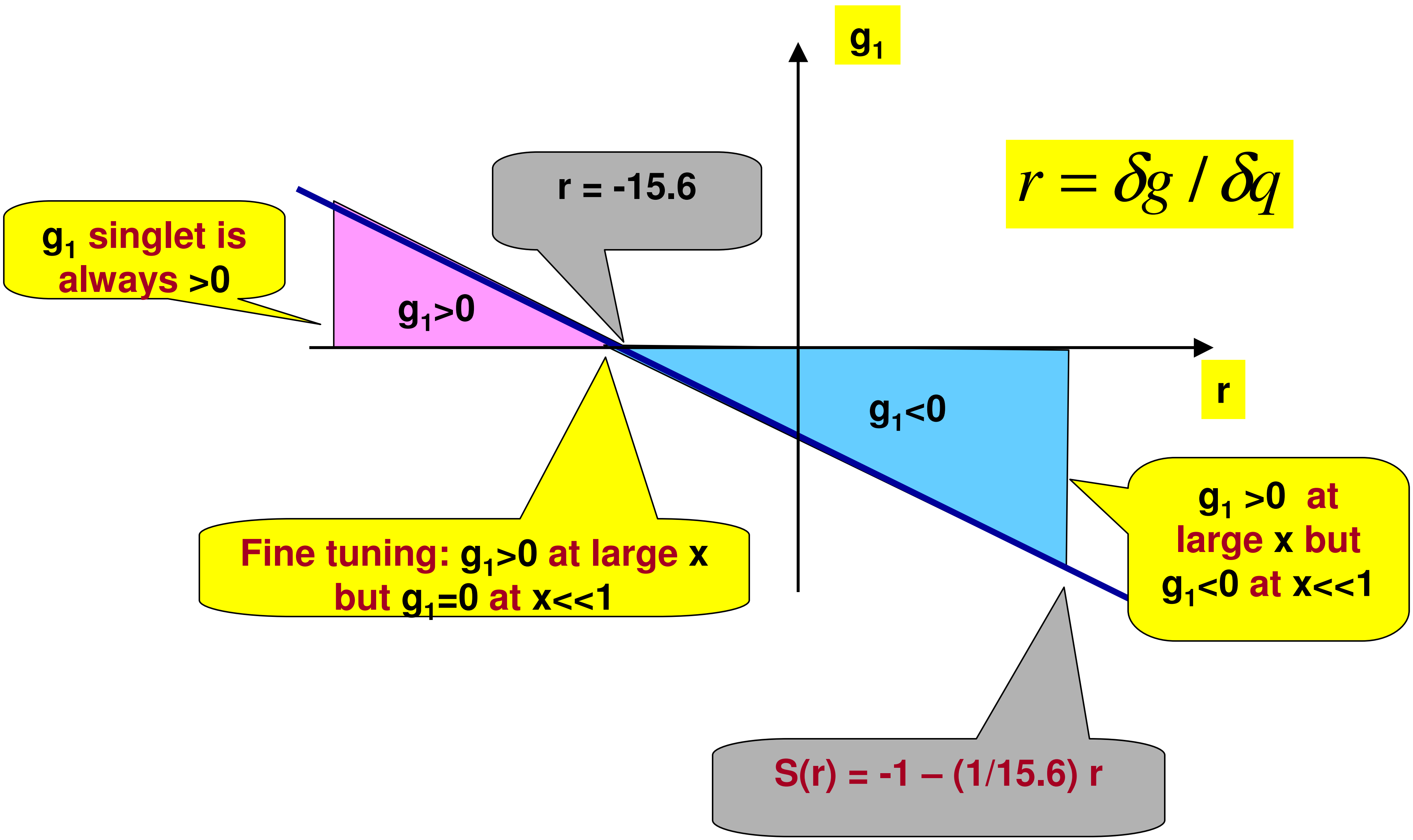
With intercept $\Delta_S = 0.86$

and $S(\Delta_S) = -1.2 \; \delta q - 0.08 \; \delta g$

Interplay between the quark and gluon densities can lead to different sign of g_1 singlet at $x \ll 1$

Warning: asymptotic expressions $g_1 \sim (1/x)^{\Delta}$ are reliable at $x < 10\text{-}5$

At large x, g_1 singlet is positive . When x--> 0, the sign of asymtpotics of the singlet g_1 depends on the ratio between the initial parton densities



Values of the intercepts perfectly agree with results of several groups who fitted experimental data.

non-singlet intercept — Soffer-Teryaev, Kataev-Sidorov-Parente, Kotikov-Lipatov-Parente-Peshekhonov-Krivokhijine-Zotov,

singlet intercept — Kochelev-Lipka-Vento-Novak-Vinnikov

Anatomy of the singlet intercept

A. Graphs with gluons only: $\Delta_S = 1.1$ → violates unitarity → similar to LO BFKL

B. All graphs $\Delta_S = 0.86$ → No violation of unitarity

Comparison of our results to DGLAP at finite x –no asymptotic formulae used

Comparison depends on the assumed shape of initial parton densities.

The simplest option: use the bare quark input

$$\delta q(x) = \delta(\mathrm{x}) \quad \Longleftrightarrow \quad \delta q(\omega) = 1$$

in x- space

in Mellin space

Numerical comparison shows that the impact of the total resummation of logs of x becomes quite sizable at x = 0.05 approx.

Hence, DGLAP should have Failed at x < 0.05.
However, it does not take place.

In order to understand what could be the reason for success of DGLAP at small **x**, let us consider in more detail

standard fits for initial parton densities.

$$\delta q(x) = N\, x^{-\alpha}\left[(1+\gamma x^{\delta})(1-x)^{\beta}\right]$$

Altarelli-Ball-Forte-Ridolfi

normalization

singular factor

regular factors

parameters $\alpha \approx 0.58,\ \beta \approx 2.7, \gamma \approx 34.3,\ \delta \approx 0.75$

are fixed from fitting experimental data at large x

In the Mellin space this fit is

$$\delta q(\omega) = N[(\omega-\alpha)^{-1} + \sum_{k=1}^{\infty} c_k((\omega+k-\alpha)^{-1} + \gamma(\omega+k+1-\alpha)^{-1})]$$

Leading pole
α**=0.58 >0**

Non-leading poles
-k +α**<0**

the small-x DGLAP asymptotics of g_1 is (inessential factors dropped)

$$g_1^{DGLAP} \sim (1/\mathrm{x})^{\alpha}$$

phenomenology

Comparison it to our asymptotics

$$g_1 \sim (1/x)^{\Delta_{NS}}$$

calculations

shows that the singular factor in the DGLAP fit mimics the total resummation of ln(1/x) . However, the value α = 0.58 sizably differs from our non-singlet intercept =0.4

Although our and DGLAP asymptotics lead to the x- behavior of Regge type, they predict different intercepts for the x- dependence and different Q^2 -dependence:

our calculations

$$g_1 \sim (1/x)^{\Delta} \left(Q^2/\mu^2\right)^{\Delta/2}$$

whereas DGLAP predicts the steeper x-behavior and
the flatter Q^2 -behavior:

DGLAP

$$g_1^{DGLAP} \sim (1/\mathrm{x})^{\alpha} (\ln Q^2)^{\gamma(\alpha)}$$

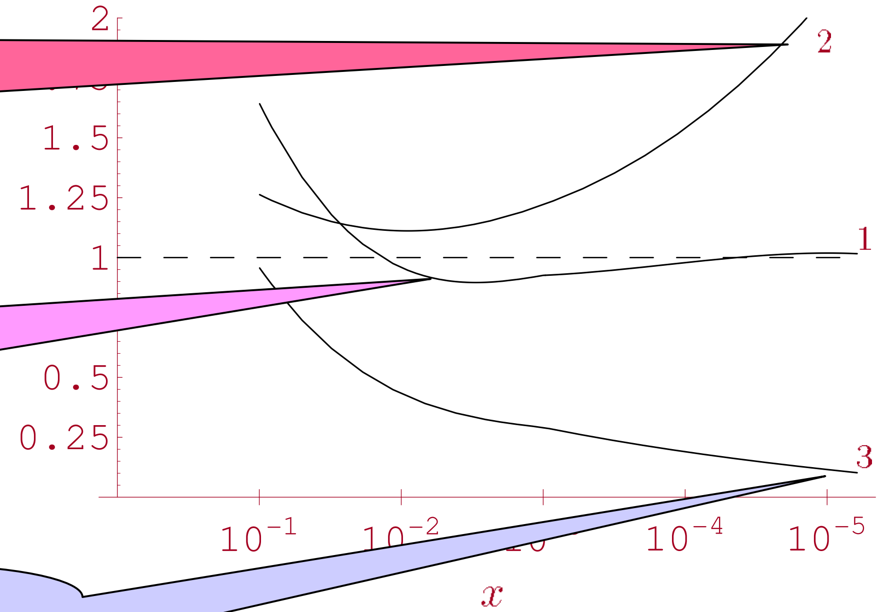
x-asymptotics was checked with extrapolating available exp data to x→ 0.
It agrees with our values of Δ
Contradicts DGLAP

our and the DGLAP Q^2 –asymptotics have not been checked yet.

Common opinion: the total resummation is not relevant at available x
Actually: the resummation has always been accounted for through the standard fits, however without realizing it

Numerical comparison of DGLAP with our approach at small but finite x, using the same DGLAP fit for initial quark density.

$R = g_1^{\,our}/g_1^{\,DGLAP}$



x

Only regular factors in $g_1^{\,our}$ and $g_1^{\,DGLAP}$

Regular term in $g_1^{\,our}$ vs regular + singular in $g_1^{\,DGLAP}$

Whole fit in $g_1^{\,our}$ and $g_1^{\,DGLAP}$: regular + singular

Common opinion: fits for δq are singular but defined and large x,
then convoluting them with coefficient functions weakens the singularity

$$C(x,y)\otimes\delta q(y)=\Delta q(x)$$

initial

x-evolved

Obviously, it is not true:
They both are singular equally

Structure of DGLAP fit once again:

$$\delta q(x)=N\,\mathrm{x}^{-\alpha}[(1+\gamma \mathrm{x}^{\delta})(1-x)^{\beta}]$$

Can be dropped when ln(x) are resummed

x-dependence is weak at x<<1 and can be dropped

Therefore at x << 1

$$\delta q(x)\approx \mathrm{N}(1+\mathrm{ax})$$

Common opinion: DGLAP fits are related to the structure of hadrons, so they mimic effects of Non-Perturbative QCD, using many phenomenological parameters fixed from experiment .

Actually, singular factors in the fits mimic effects of Perturbative QCD and can be dropped when logarithms of x are resummed

Non-Perturbative QCD effects are accumulated in the regular parts of DGLAP fits. Obviously, impact of Non-Pert QCD is not strong in the region of small x. In this region, the fits approximately = overall factors N and a term linear in x

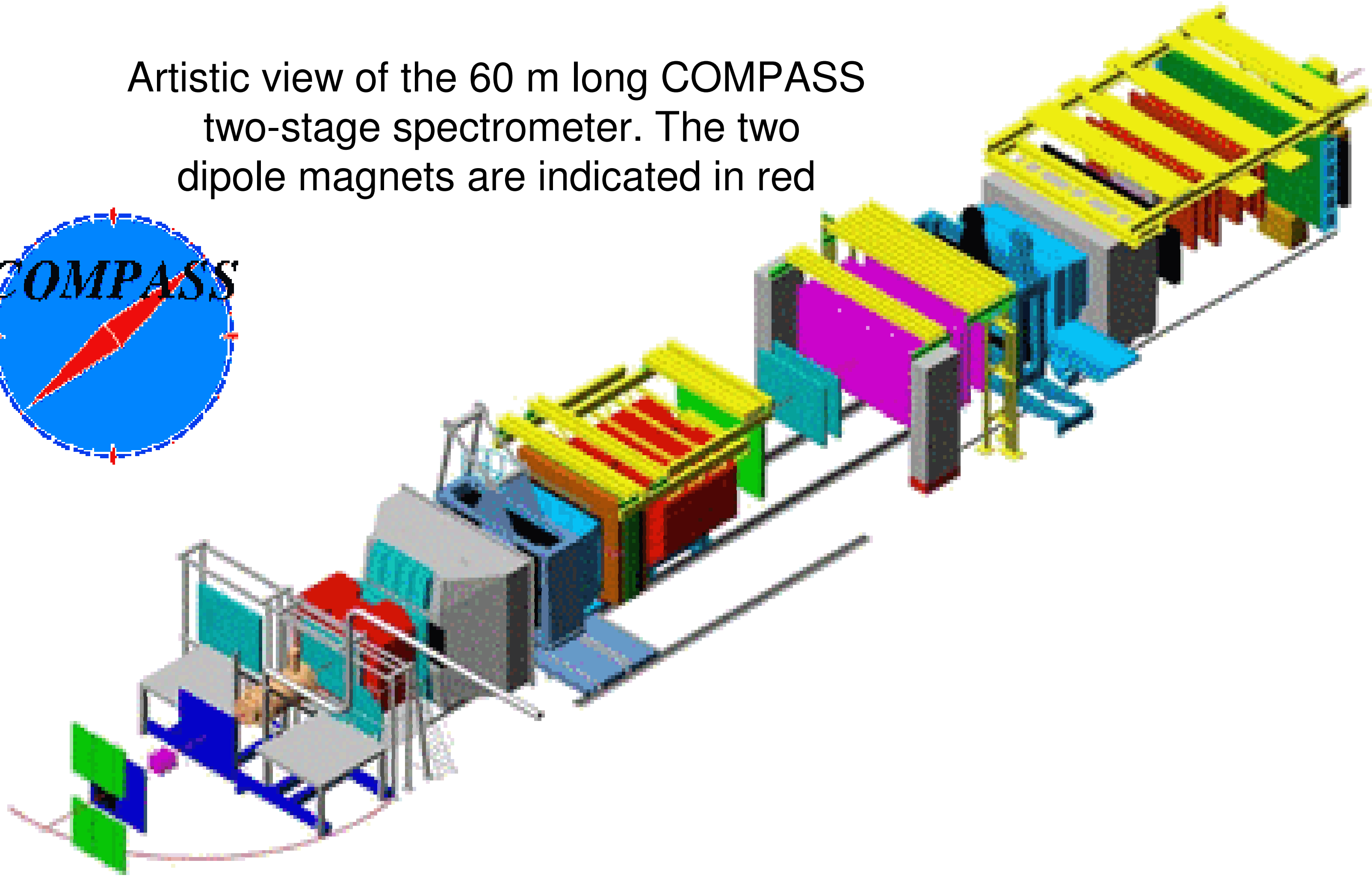
Taken from wwwcompass.cern.ch

COMPASS is a high-energy physics experiment at the Super Proton Synchrotron (SPS) at CERN in Geneva, Switzerland. The purpose of this experiment is the study of hadron structure and hadron spectroscopy with high intensity muon and hadron beams.
On February 1997 the experiment was approved conditionally by CERN and the final Memorandum of Understanding was signed in September 1998. The spectrometer was installed in 1999 - 2000 and was commissioned during a technical run in 2001. Data taking started in summer 2002 and continued until fall 2004. After one year shutdown in 2005, COMPASS will resume data taking in 2006.
Nearly 240 physicists from 11 countries and 28 institutions work in COMPASS

Taken from wwwcompass.cern.ch

COMPASS

COmmon Muon Proton Apparatus for Structure and Spectroscopy



Artistic view of the 60 m long COMPASS two-stage spectrometer. The two dipole magnets are indicated in red

COMPASS operates with small Q^2 ($Q^2 < 10^{-1}$ GeV^2) and small x

In order to generalize our results to the region of small Q^2, one should remember that $\ln(Q^2/\mu^2)$ is the result of the integration $\int_{\mu^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2}$

However it is valid for large Q^2 only. For arbitrary Q^2 :

$$\int_{\mu^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \to \int_{0}^{Q^2} \frac{dk_\perp^2}{k_\perp^2 + \mu^2} = \ln\left(\frac{Q^2+\mu^2}{\mu^2}\right)$$

Introduced as a "mass" of virtual quarks and gluons to regulate infrared singularities

It leads to new expressions: **Small Q^2 non-singlet g_1**

$$z = \frac{\mu^2}{2pq} > x = \frac{Q^2}{2pq}$$

$$g_1^{NS} = \frac{e_q^2}{2} \int \frac{d\omega}{2\pi i} \left(\frac{1}{z+x} \right)^{\omega} \left(\frac{\omega}{\omega - H(\omega)} \right) \delta q(\omega) \left(\frac{\mu^2 + Q^2}{\mu^2} \right)^{H(\omega)}$$

weak x -dependence

Anomalous dimension

weak Q^2 -dependence

Coefficient function

Initial quark density

Small Q^2 Singlet g_1

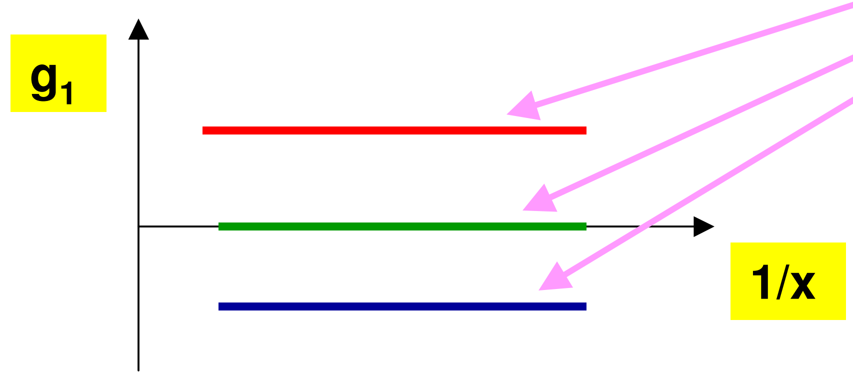
$$z = \frac{\mu^2}{2pq}, \quad x = \frac{Q^2}{2pq}$$

$$g_1^S = \frac{\langle e_q^2 \rangle}{2} \int \frac{d\omega}{2\pi i} \left(\frac{1}{z+x} \right)^{\omega} \left[C_q \delta q + C_g \delta g \right]$$

$$C_g = C_g^{(+)} \left(\frac{\mu^2 + Q^2}{\mu^2} \right)^{\Omega^{(+)}} + C_g^{(-)} \left(\frac{\mu^2 + Q^2}{\mu^2} \right)^{\Omega^{(-)}}$$

$$C_q = C_q^{(+)} \left(\frac{\mu^2 + Q^2}{\mu^2} \right)^{\Omega^{(+)}} + C_q^{(-)} \left(\frac{\mu^2 + Q^2}{\mu^2} \right)^{\Omega^{(-)}}$$

when $Q^2 \ll \mu^2$ both x- and Q^2- dependences are flat, even for $x \ll 1$.



Location of the line is determined by the z-dependence

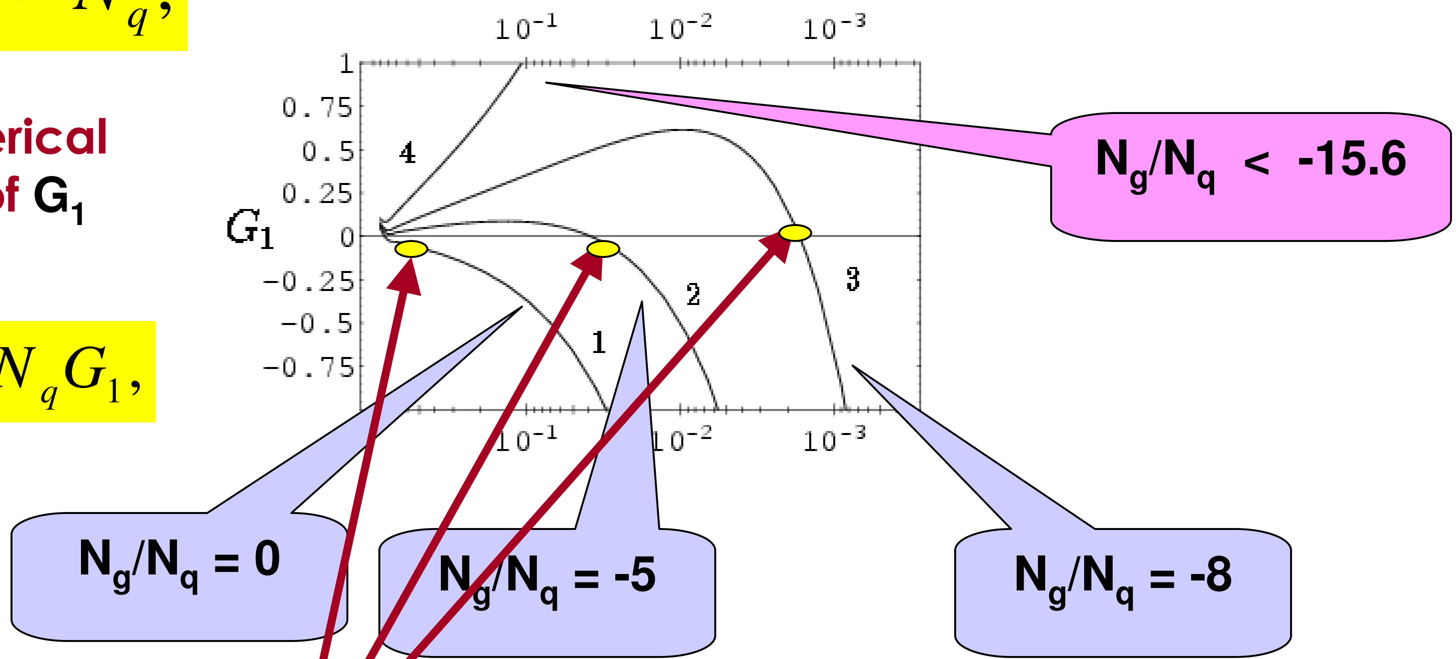
$$g_1(z) = \left(\frac{e_q^2}{2}\right)\int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i}\left(\frac{1}{z}\right)^{\omega}\left[C_q(\omega)\delta q + C_g(\omega)\delta g\right]$$

Approximating

$$\delta q \approx N_q, \delta q \approx N_q,$$

perform numerical calculations of G_1

$$g_1 = (e_q^2/2) N_q G_1,$$



Position of the turning point is sensitive to N_g/N_q , so the experimental detection of it will allow to estimate Ng/Nq

Conclusion

Standard Approach (SA) includes the DGLAP evolution equations and fits for initial parton densities.

DGLAP was originally developed for operating at the region where both x and Q^2 are large.

In order to extend DGLAP to the region of small x, phenomenological singular factors x^{-a} have to be incorporated into the fits

These factors mimic the total resummation of leading logaritms of x and lead to the Regge behavior of g_1 at small x. Without such factors, SA would become unreliable at $x < 0.05$. When the resummation is taken into account, the fits can be strongly simplified. In other words, the impact of Non-Perturbative QCD at $x \ll 1$ is not large compared to Pert QCD.

The region of small Q^2 is also beyond the reach of SA. We predict that g_1 at small Q^2 is almost independent of x, even at $x \ll 1$. Instead, it depends on 2pq only. At a certain relation between the initial quark and gluon densities, g1 can be pretty close to zero in the range of 2pq investigated now experimentally by COMPASS.