The physical basis and effective self-couplings of Higgs bosons in the CPX scenario

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- Radiatively induced CP violation at the electroweak scale appears naturally in the MSSM Higgs sector. The case of no CPV reqiures an artificial fine tuning of the λ_i phases

- Effective field theory approach
  H.Haber, R.Hempfling, Phys.Rev.D48 (1993) 4280
  Advantages
  - boundary condition at M_{SUSY} respected
  - transparent treatment of the general $\lambda_{1,...7}$ potential for construction of the mass eigenstates in the minimum
  - technically somewhat less complicated (probably) in comparison with diagrammatica
  Disadvantages
  - case of nondegenerate squark masses very difficult
  - limited to external momenta zero

- Experimental reconstruction of the Higgs potential is one of the most important problems for the LHC and NLC

Hermitian two-Higgs-doublet potential in the $-\mu^2\varphi^2 + \lambda\varphi^4$ representation:

$$
\begin{aligned}
U(\Phi_1,\Phi_2) = \quad & -\mu_1^2(\Phi_1^+\Phi_1) - \mu_2^2(\Phi_2^+\Phi_2) \\
& -\mu_{12}^2(\Phi_1^+\Phi_2) - \overset{*}{\mu_{12}^2}(\Phi_2^+\Phi_1) \\
+\frac{\lambda_1}{2}(\Phi_1^+\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^+\Phi_2)^2 & + \lambda_3(\Phi_1^+\Phi_1)(\Phi_2^+\Phi_2) + \lambda_4(\Phi_1^+\Phi_2)(\Phi_2^+\Phi_1) \\
& +\frac{\lambda_5}{2}(\Phi_1^+\Phi_2)(\Phi_1^+\Phi_2) + \frac{\overset{*}{\lambda_5}}{2}(\Phi_2^+\Phi_1)(\Phi_2^+\Phi_1) \\
& +\lambda_6(\Phi_1^+\Phi_1)(\Phi_1^+\Phi_2) + \overset{*}{\lambda_6}(\Phi_1^+\Phi_1)(\Phi_2^+\Phi_1) \\
& +\lambda_7(\Phi_2^+\Phi_2)(\Phi_1^+\Phi_2) + \overset{*}{\lambda_7}(\Phi_2^+\Phi_2)(\Phi_2^+\Phi_1)
\end{aligned}
$$

λ_5, λ_6 λ_7 are complex variables,
no discrete symmetry imposed.

the VEV's

$$
<\varphi_1>= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix} \qquad <\varphi_2>= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 e^{i\theta} \end{pmatrix}
$$

Mass eigenstates h_1, h_2 h_3

($s_\alpha = \mathtt{sin}\alpha$, $c_\beta = \mathtt{cos}\beta$ etc.)

$$\Phi_1 = \begin{pmatrix} -i*(-H^+ s_\beta + G^+ c_\beta) \\ \frac{1}{\sqrt{2}}[v_1 + Hc_\alpha - hs_\alpha + i*(A^0 c_\beta + G^{'} s_\beta)] \end{pmatrix}$$

$$\Phi_2 = e^{i\xi} \begin{pmatrix} -i*(H^+ c_\beta + G^+ s_\beta) \\ \frac{1}{\sqrt{2}}[v_2 e^{i\zeta} + Hs_\alpha + hc_\alpha + i*(-A^0 s_\beta + G^{'} c_\beta)] \end{pmatrix}$$

h, H CP-even bosons, A CP-odd boson, $H^\pm$ charged boson, G Goldstone modes of the CP-conserving limit: $\mathtt{Im}\mu_{12}^2 = 0$, $\mathtt{Im}\lambda_{5,6,7}$ =0, $\xi = \zeta$ =0. With imaginary parts nonzero

$$\begin{pmatrix} h \\ H \\ A \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}$$

In the CP conserving limit the mixing matrix a_{ij}

not only $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, but also $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$, ...

$$U(\Phi_1, \Phi_2) = c_1 hA + c_2 HA + \frac{m_h^2}{2} h^2 + \frac{m_H^2}{2} H^2 + \frac{m_A^2}{2} A^2 + m_{H^\pm}^2 H^+ H^- + \texttt{trilinear and quartic terms in } h, H, A, H^\pm$$

where

$$c_1 = \frac{v^2}{2}(s_\alpha s_\beta - c_\alpha c_\beta)\mathtt{Im}\lambda_5 + v^2\,(s_\alpha c_\beta \mathtt{Im}\lambda_6 - c_\alpha s_\beta \mathtt{Im}\lambda_7)$$

$$c_2 = -\frac{v^2}{2}(s_\alpha c_\beta + c_\alpha s_\beta)\mathtt{Im}\lambda_5 - v^2\,(c_\alpha c_\beta \mathtt{Im}\lambda_6 + s_\alpha s_\beta \mathtt{Im}\lambda_7)$$

To diagonalize $U(\Phi_1, \Phi_2)$ perform orthogonal rotation a_{ij} (i, j =1,2,3) in h, H, A space

$$(h, H, A)\; M^2 \begin{pmatrix} h \\ H \\ A \end{pmatrix} = (h_1, h_2, h_3)\; a^T_{ik}\; M^2_{kl}\; a_{lj} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}$$

with the mass matrix

$$M^2 = \frac{1}{2} \begin{pmatrix} m_h^2 & 0 & c_1 \\ 0 & m_H^2 & c_2 \\ c_1 & c_2 & m_A^2 \end{pmatrix}$$

In the MSSM $\lambda_{5,6,7}$ can be calculated by means of the effective potential method and expressed through the parameters of the squark - Higgs boson sector.

At the one-loop

$$\lambda_5 = -\Delta\lambda_5 = -\frac{3}{96\,\pi^2}\left(h_t^4\left(\frac{\mu A_t}{M_{\rm SUSY}^2}\right)^2 + h_b^4\left(\frac{\mu A_b}{M_{\rm SUSY}^2}\right)^2\right),$$

$$\lambda_6 = -\Delta\lambda_6 = \frac{3}{96\,\pi^2}\left[h_t^4\,\frac{|\mu|^2\mu A_t}{M_{\rm SUSY}^4} - h_b^4\,\frac{\mu A_b}{M_{\rm SUSY}^2}\left(6 - \frac{|A_b|^2}{M_{\rm SUSY}^2}\right) + (h_b^2 A_b - h_t^2 A_t)\,\frac{3\,\mu}{M_{\rm SUSY}^2}\,\frac{g_2^2 + g_1^2}{4}\right],$$

$$\lambda_7 = -\Delta\lambda_7 = \frac{3}{96\,\pi^2}\left[h_b^4\,\frac{|\mu|^2\mu A_b}{M_{\rm SUSY}^4} - h_t^4\,\frac{\mu A_t}{M_{\rm SUSY}^2}\left(6 - \frac{|A_t|^2}{M_{\rm SUSY}^2}\right) + (h_t^2 A_t - h_b^2 A_b)\,\frac{3\,\mu}{M_{\rm SUSY}^2}\,\frac{g_2^2 + g_1^2}{4}\right].$$

(**Tree-level condition at the scale** M_{SUSY}**:**

$$\lambda_1^{SUSY} = \lambda_2^{SUSY} = \frac{g_2^2+g_1^2}{8},\ \lambda_3^{SUSY} = \frac{g_2^2-g_1^2}{4},$$

$$\lambda_4^{SUSY} = -\frac{g_2^2}{2},\quad \lambda_{5,6,7}^{SUSY} = 0.\ )$$

CPX scenario [M.Carena et.al., PL B495 (2000) 155]

Defines a region of MSSM parameter space constrained by

$|\mu| = 4\, M_{SUSY}\,,\; |A_{t,b}| \;=\; 2\, M_{SUSY}$

Moderate $m_{H^\pm} \sim 160 \div 200$ GeV.

Quantum effects of CP-even/CP-odd states mixing depending on $\mathtt{Im}(\mu A_t)/M^2_{SUSY}$ are not small.

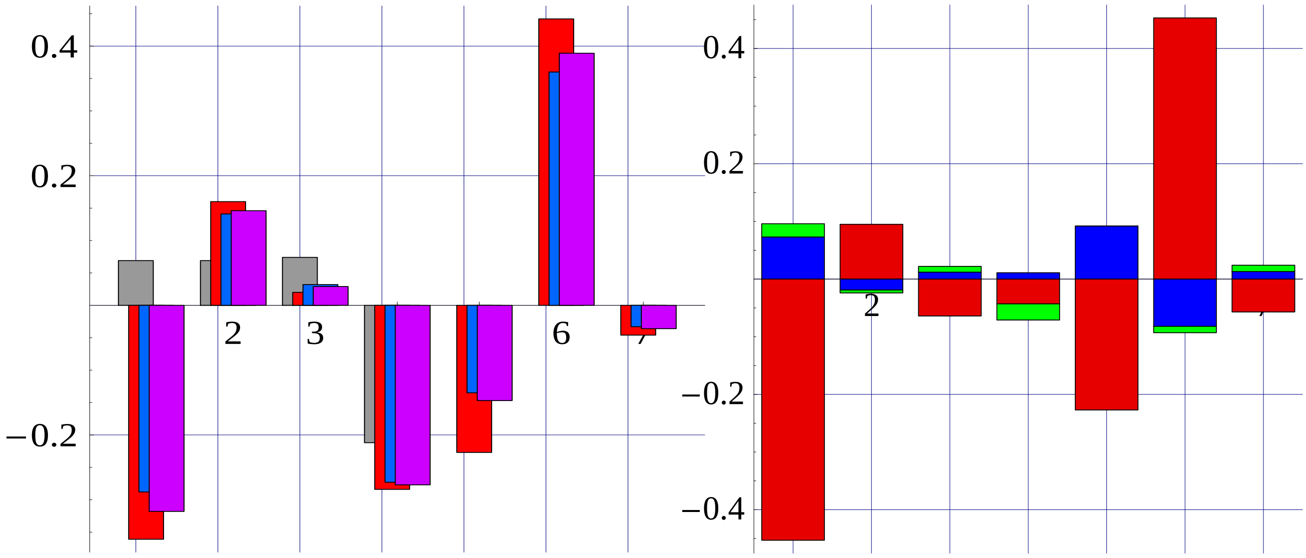
In the following $M_{SUSY} = 500\,$GeV, $|A_t| = |A_b| = A = 1000\,$GeV, $|\mu| = 2000\,$GeV (CPX_{500} scenario).

and

CPsuperH parameter set: $m_Z = 91.19\,$GeV, $m_b = 3\,$GeV, $m_t = 175\,$GeV, $m_W = 79.96\,$GeV, $g_2 = 0.6517$, $g_1 = 0.3573$, $v = 245.4\,$GeV, $G_F = 1.17 \cdot 10^{-5}\,$GeV^{-2}, $\alpha_S(m_t) = 0.1072$, $\sigma = m_t$.

[J.S.Lee et.al., CPC 156 (2004) 283]

$\lambda_{1,\ldots 7}$ and $\Delta\lambda_{1,\ldots 7}$ patterns.
MSSM, CPX_{500} scenario.



$\lambda_{1,\ldots 7}$ $\Delta\lambda_{1,\ldots 7}$

Figure 1: Effective parameters
$\lambda_{1,\ldots 7}$ and $\Delta\lambda_{1,\ldots 7} = \lambda_{1,\ldots 7}^{M_{SUSY}} - \lambda_{1,\ldots 7}$

- tree level$_{M_{SUSY}}$
- one-loop$_{m_{top}}$
- two-loop Yukawa$_{m_{top}175}$
- two-loop Yukawa$_{m_{top}178}$
- one-loop, no D-terms and wfr
- two-loop Yukawa$_{175}$
- D-terms and wfr

The cubic equation for eigenvalues

$$(m_{h_i}^2)^3 + a_2 (m_{h_i}^2)^2 + a_1 m_{h_i}^2 + a_0 = 0$$

Squared masses of the physical states h_1, h_2, h_3 **– the eigenvalues of mass matrix** M^2

$$\begin{aligned}
m_{h1}^2 &= 2\sqrt{(-q)}\cos(\frac{\Theta + 2\pi}{3}) - \frac{a_2}{3} \\
m_{h2}^2 &= 2\sqrt{(-q)}\cos(\frac{\Theta - 2\pi}{3}) - \frac{a_2}{3} \\
m_{h3}^2 &= 2\sqrt{(-q)}\cos(\frac{\Theta}{3}) - \frac{a_2}{3}
\end{aligned}$$

$$\Theta = \arccos \frac{r}{\sqrt{-q^3}}$$

$$r = \tfrac{1}{54}(9a_1 a_2 - 27a_0 - 2a_2^3), \;\; q = \tfrac{1}{9}(3a_1 - a_2^2)$$

$$\begin{aligned}
a_0 &= c_1^2 m_H^2 + c_2^2 m_h^2 - m_h^2 m_H^2 m_A^2, \\
a_1 &= m_h^2 m_H^2 + m_h^2 m_A^2 + m_H^2 m_A^2 - c_1^2 - c_2^2, \\
a_2 &= -m_h^2 - m_H^2 - m_A^2.
\end{aligned}$$

if $c_{1,2} \to 0$ **then** $m_{h_1} \to min(m_h, m_H, m_A)$
and $m_{h_3} \to max(m_h, m_H, m_A)$.

The normalized eigenvectors of the matrix M^2

$a_{ij} = \frac{a'_{ij}}{n_j}$, $\quad n_i = k_i \sqrt{a'^2_{1i} + a'^2_{2i} + a'^2_{3i}}$.

$$
\begin{aligned}
a'_{11} &= ((m_H^2 - m_{h_1}^2)(m_A^2 - m_{h_1}^2) - c_2^2), \\
a'_{21} &= c_1 c_2, \\
a'_{31} &= -c_1(m_H^2 - m_{h_1}^2), \\
a'_{12} &= -c_1 c_2, \\
a'_{22} &= -((m_h^2 - m_{h_2}^2)(m_A^2 - m_{h_2}^2) - c_1^2), \\
a'_{32} &= c_2(m_h^2 - m_{h_2}^2), \\
a'_{13} &= -c_1(m_H^2 - m_{h_3}^2), \\
a'_{23} &= -c_2(m_h^2 - m_{h_3}^2), \\
a'_{33} &= (m_h^2 - m_{h_3}^2)(m_H^2 - m_{h_3}^2)
\end{aligned}
$$

The cubic equation can be rewritten in the form

$$(m_h^2 - m_{h_i}^2)[(m_H^2 - m_{h_i}^2)(m_A^2 - m_{h_i}^2) - c_2^2] - c_1^2(m_H^2 - m_{h_i}^2) = 0.$$

For example at $i = 1$ (first eigenvector)

$$(m_h^2 - m_{h_1}^2)a'_{11} + c_1 a'_{31} = 0$$

so if c_1 =0 then either (I) a'_{11} =0 or (II) $m_{h_1} = m_h$. Case (II) takes place when $\frac{r}{\sqrt{-q^3}}$ =max.value ($\varphi \sim \pi/2$) or min.value ($\varphi \sim \pi$), but $\frac{r}{\sqrt{-q^3}}$ ≠1 or, equivalently Θ ≠0.

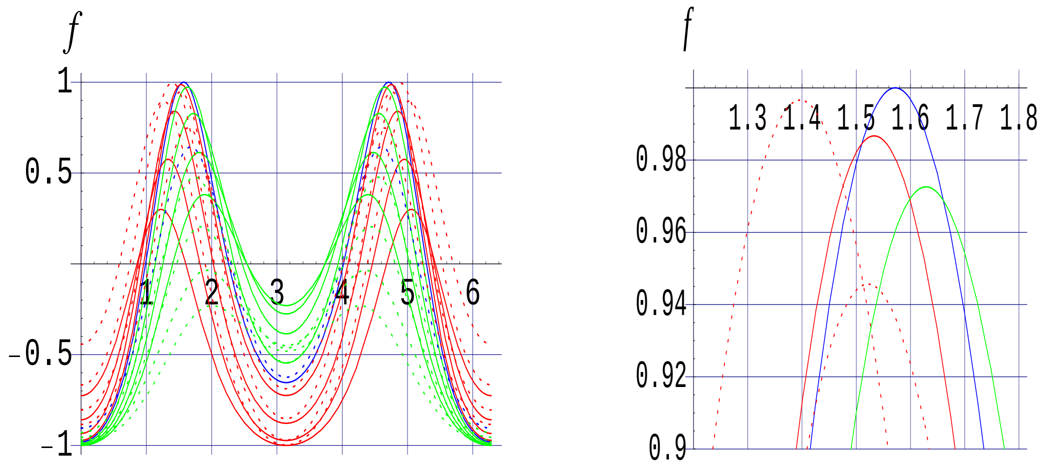


Figure 2: Plots for $f = \frac{r}{\sqrt{(-q^3)}}$ as a function of φ for different of $m_{H^\pm}$ (10 GeV difference for various lines). Blue line – $m_{H^\pm} = 184$ GeV. Dashed – with leading two-loop corrections

Special point, MSSM, CPX$_{500}$ scenario, one-loop:

$$m_{H^\pm}\textbf{=184 GeV.}$$

This change of regime (I)→(II) leads to a discontinuties in $a_{ij}(m_{H^\pm})$.

In the MSSM, CPX class of scenarios

$$c_1 = \frac{v^2}{2}(s_\alpha s_\beta - c_\alpha c_\beta)\mathtt{Im}\lambda_5 + v^2\,(s_\alpha c_\beta \mathtt{Im}\lambda_6 - c_\alpha s_\beta \mathtt{Im}\lambda_7) = 0$$

appears in the vicinity of $\varphi = \pi/2$ as a consequence of the MSSM relation between the radiatively induced phases of $\lambda_{5,6,7}$: $2\,\mathtt{arg}(\lambda_{6,7}) = \mathtt{arg}(\lambda_5)$. In other nonstandard models c_1 =0 may not take place and different structure of discontinuities may appear.

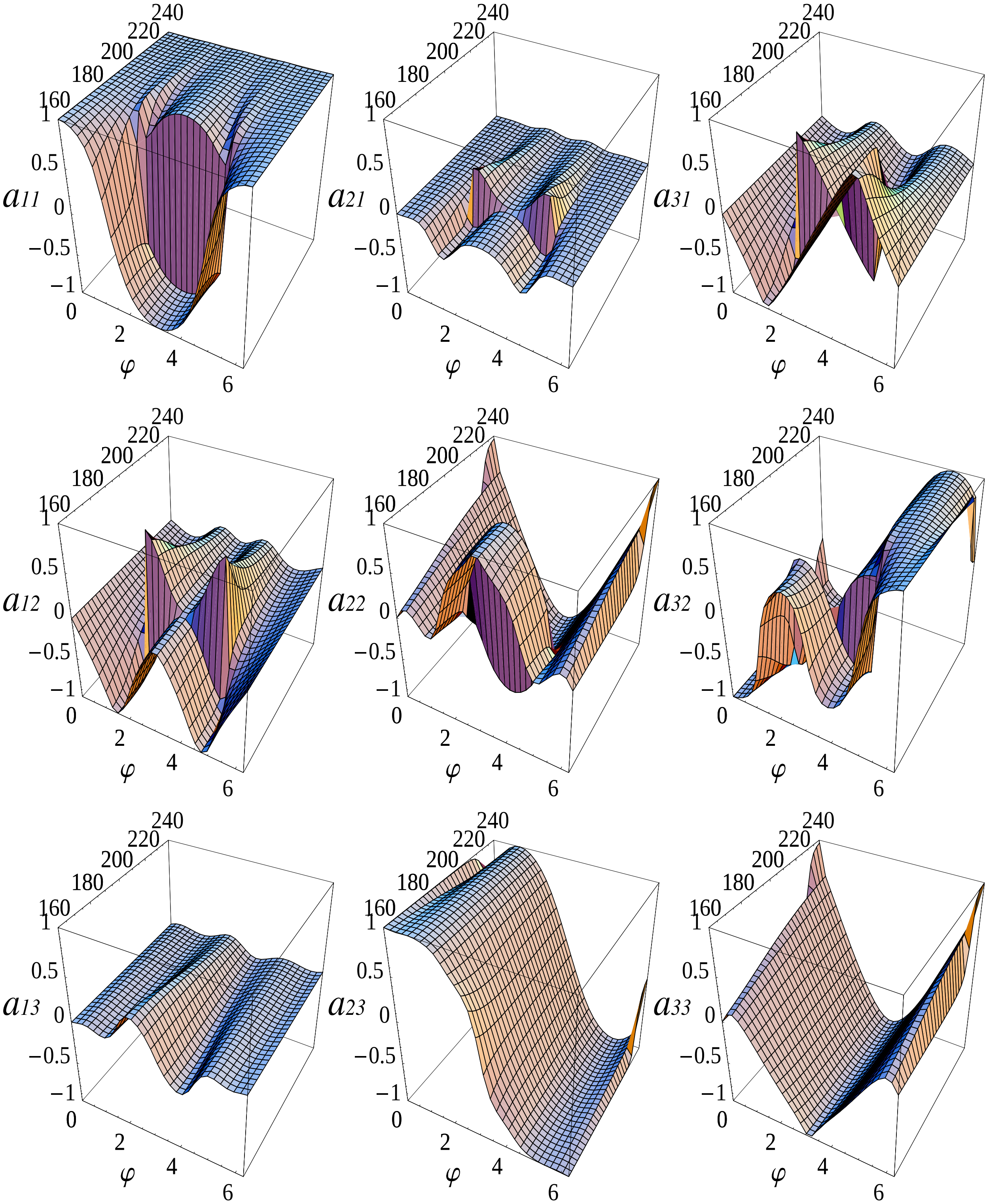


Figure 3: The mixing matrix elements a_{ij} as a two-dimensional functions of the mass $m_{H^\pm}$ (GeV) and the phase φ (rad) at the one-loop approximation for $\lambda_{1,\ldots 7}$, MSSM, CPX$_{500}$

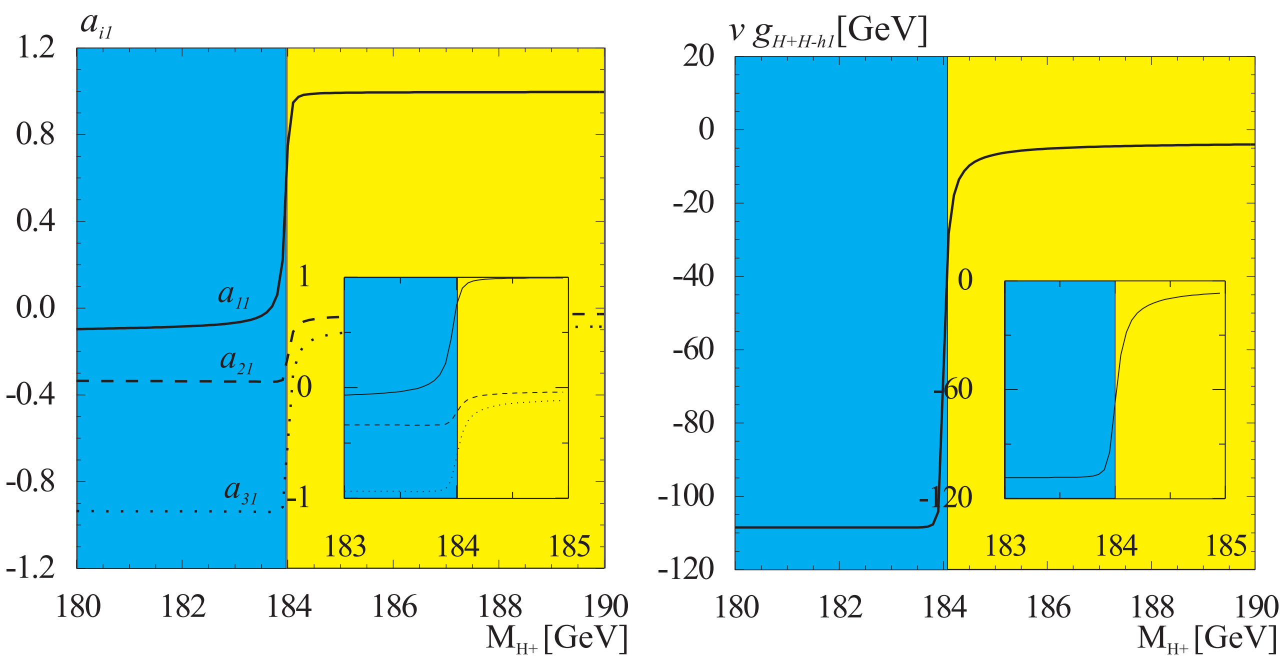


Figure 4: The mixing matrix elements a_{i1} – (a) and the triple Higgs boson interaction vertex $v \cdot g_{H^+ \, H^- \, h_1}$ in GeV – (b) vs the $H^{\pm}$-mass for the one-loop approximation for lambda-couplings of the MSSM two-doublet potential, CPX$_{500}$ scenario

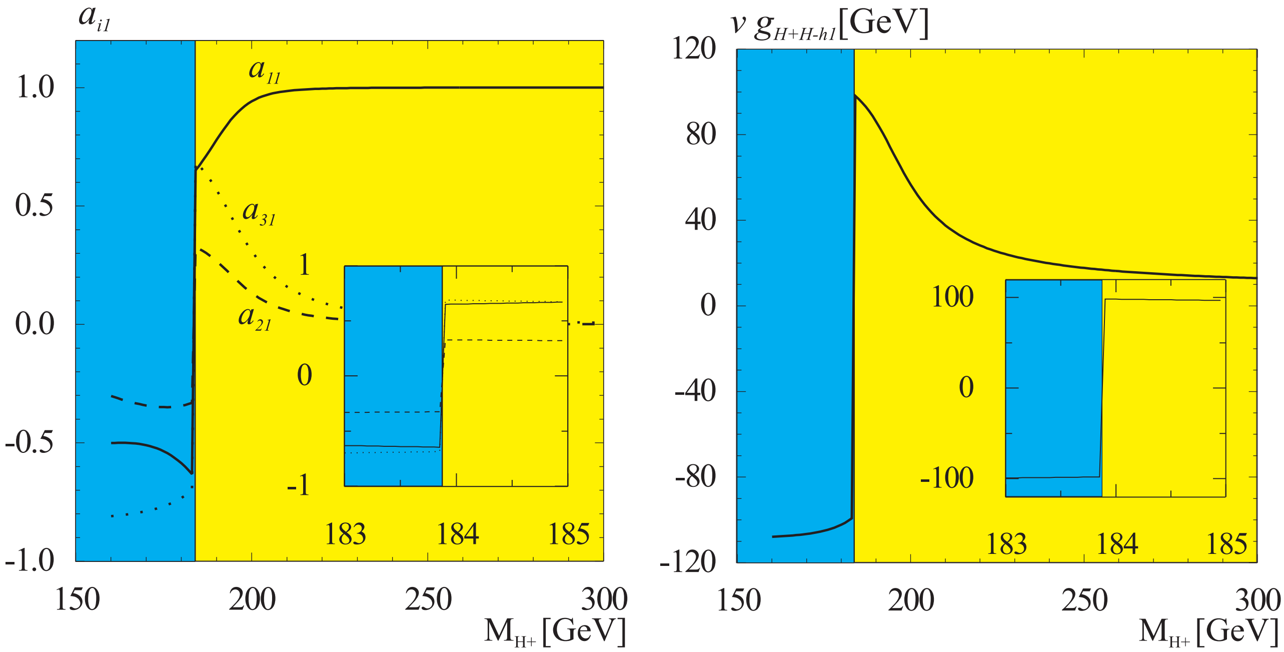


Figure 5: The mixing matrix elements a_{i1} - (a) and the triple Higgs boson interaction vertex $v \cdot g_{H^+H^-h_1}$ - (b) at $\varphi = 7\pi/12$ vs the $H^{\pm}$-mass for the one-loop approximation for lambda-couplings. The discontinuities of a_{i1} at $M_{H\pm}$ =184 GeV are introduced at the phase $\pi/2 < \varphi < 3\pi/2$. Then the eigenvector basis is left-handed at any $\{\varphi, M_H^{\pm}\}$

There are no simple representations for the triple and quartic Higgs boson self-interaction vertices.

$$\mathcal{L}_{3H} = v \sum_{i\geq j\geq k=1}^{3} g_{h_i h_j h_k} \frac{1}{N_S^{ijk}} h_i h_j h_k + v \sum_{i=1}^{3} g_{h_i H^+ H^-} h_i H^+ H^- ,$$

where N_S **are the combinatorial factors and**

$$\begin{aligned} g_{h_i h_j h_k} &= \sum_{\alpha\geq\beta\geq\gamma=1}^{3} \{a_{\alpha i} a_{\beta j} a_{\gamma k}\} \; g_{\alpha\beta\gamma} \\ g_{h_i H^+ H^-} &= - \sum_{\alpha=1}^{3} a_{\alpha i} \, g_{\alpha H^+ H^-} \end{aligned}$$

$$\begin{aligned} \{a_{\alpha i} a_{\beta j} a_{\gamma k}\} \equiv \frac{1}{N_S} \Big(& a_{\alpha i} a_{\beta j} a_{\gamma k} + a_{\alpha i} a_{\beta k} a_{\gamma j} + a_{\alpha j} a_{\beta i} a_{\gamma k} \\ & + a_{\alpha j} a_{\beta k} a_{\gamma i} + a_{\alpha k} a_{\beta i} a_{\gamma j} + a_{\alpha k} a_{\beta j} a_{\gamma i} \Big) , \end{aligned}$$

where $N_S = 6$ **at** $i = j = k$**,** $N_S = 1$ **at** $(i, j, k) = (3, 2, 1)$**, and** $N_S = 2$ **in all other cases. There are discontinuities in** $g_{h_i h_j h_k}$ **because** a_{ij} **products are not always in combinations of "compensating sign".**

In the λ_i basis

$$
\begin{aligned}
g_{1H^+H^-} &= \mathtt{Re}\Delta\lambda_5\, s_\beta c_\beta\, c_{\alpha+\beta} - \mathtt{Re}\Delta\lambda_6\, c_\alpha\, s_\beta^2\, c_\beta + \mathtt{Re}\Delta\lambda_6\, s_\alpha\, s_\beta^3 \\
&\quad - \mathtt{Re}\Delta\lambda_6\, s_\alpha\, s_{2\,\beta}\, c_\beta + \mathtt{Re}\Delta\lambda_7\, c_\beta\, (s_\alpha\, s_\beta\, c_\beta \\
&\quad - c_\alpha\, \left(c_\beta^2 - 2\, s_\beta^2\right)\Big) - 2\, s_\alpha\, s_\beta^2\, c_\beta\, \lambda_1 + 2\, c_\alpha\, s_\beta\, c_\beta^2\, \lambda_2 \\
&\quad - c_\beta^3\, s_\alpha\, \lambda_3 + c_\alpha\, s_\beta^3\, \lambda_3 - c_\beta\, s_\beta\, \lambda_4\, c_{\alpha-\beta} \\
g_{2H^+H^-} &= \mathtt{Re}\Delta\lambda_5\, s_\beta c_\beta\, s_{\alpha+\beta} + 2\, \mathtt{Re}\Delta\lambda_6\, c_\alpha\, s_\beta\, c_\beta^2 - \mathtt{Re}\Delta\lambda_6\, c_\alpha\, s_\beta^3 \\
&\quad - \mathtt{Re}\Delta\lambda_6\, s_\alpha\, s_\beta^2 c_\beta - \mathtt{Re}\Delta\lambda_7\, c_\beta\, (c_\alpha\, s_\beta\, c_\beta \\
&\quad + s_\alpha\, \left(c_\beta^2 - 2\, s_\beta^2\right)\Big) + 2\, c_\alpha\, s_\beta^2\, c_\beta \lambda_1 + 2\, s_\alpha\, s_\beta\, c_\beta^2 \lambda_2 \\
&\quad + c_\alpha\, c_\beta^3 \lambda_3 + s_\alpha\, s_\beta^3 \lambda_3 - c_\beta\, s_\beta\, \lambda_4\, s_{\alpha+\beta} \\
g_{3H^+H^-} &= c_\beta^2\, \mathtt{Im}\Delta\lambda_7 - s_\beta\, c_\beta\, \mathtt{Im}\Delta\lambda_5 + s_\beta^2\, \mathtt{Im}\Delta\lambda_6
\end{aligned}
$$

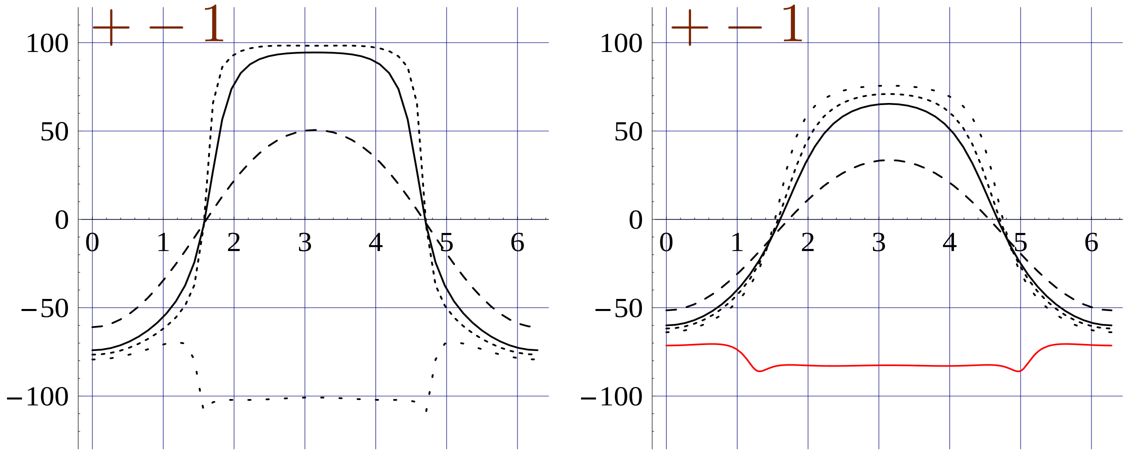


Figure 6: The triple Higgs boson interaction vertex $v \cdot g_{H^+ \, H^- \, h_1}$ (GeV) vs the phase $\mathrm{Arg}(\mu A)$ (the left figure for 1-loop approximation and the right figure with additional leading OCD Yukawa corrections to λ_i) at parameter values of CPX$_{500}$: $M_{SUSY} = 500$ GeV, $\tan\beta$ =5, $A_{t,b}$ =1000 GeV, $\mu = 2000$ GeV. Long dashed line – $m_{H^\pm} = 300$ GeV, solid line – $m_{H^\pm} = 200$ GeV, short dashed line – $m_{H^\pm} = 190$ GeV, rare dotted line – $m_{H^\pm} = 180$ GeV; red line on the right plot – $m_{H^\pm} = 150$ GeV for the intense-coupling regime.

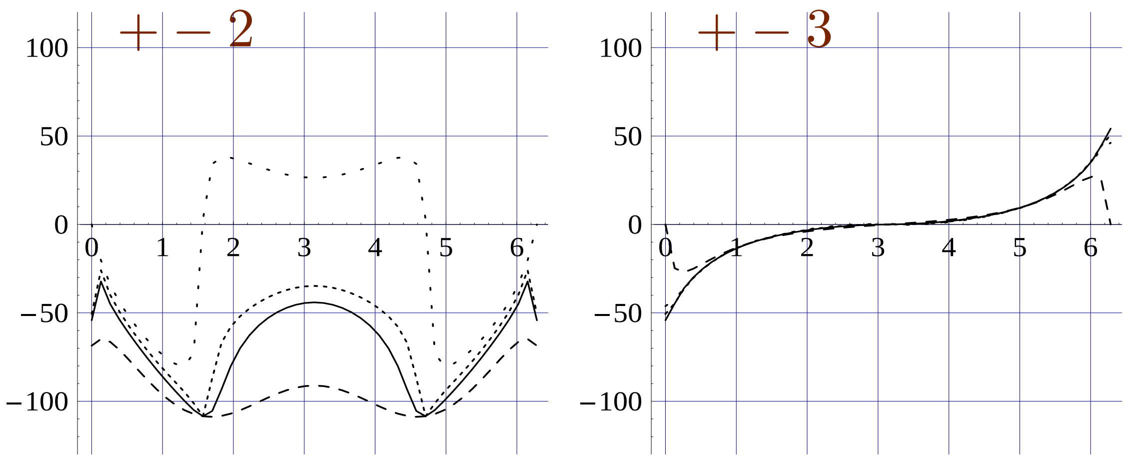


Figure 7: The triple Higgs boson interaction vertices $v \cdot g_{H^+ \, H^- \, h_2}$ (GeV, the left figure) and $v \cdot g_{H^+ \, H^- \, h_3}$ (the right figure) vs the phase $\mathrm{Arg}(\mu A)$ for 1-loop approximation to lambda-couplings at the same values of parameters and notations as fig. 6. Curves on the right plot coincide, except for the case $m_{H^\pm} = 300$ GeV

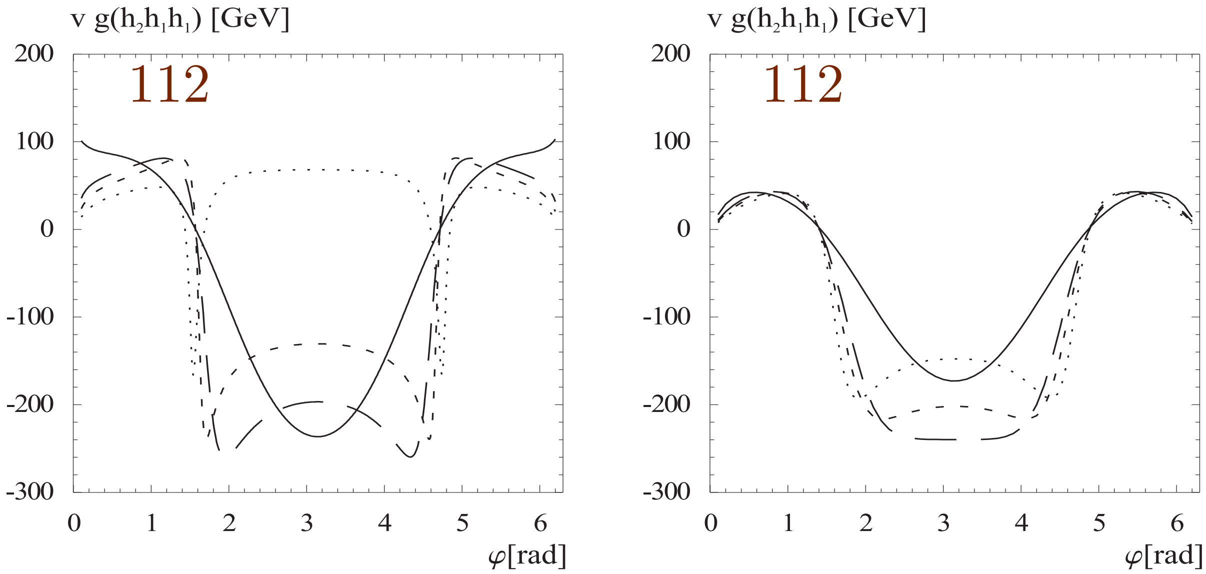


Figure 8: Light Higgs boson interaction vertex $v \cdot g_{h_1 h_1 h_2}$ (GeV) vs the phase $\arg(\mu A)$ at parameter values M_{SUSY} = 500 GeV, $\tan\beta$ = 5, $A_{t,b}$ = 1000 GeV, μ = 2000 GeV. Solid line – $m_{H^\pm}$ = 300 GeV, long dashed line – $m_{H^\pm}$ = 200 GeV, short dashed line – $m_{H^\pm}$ = 190 GeV, dotted line – $m_{H^\pm}$ = 180 GeV. (a) – effective one-loop potential, (b) – leading two-loop corrections included.

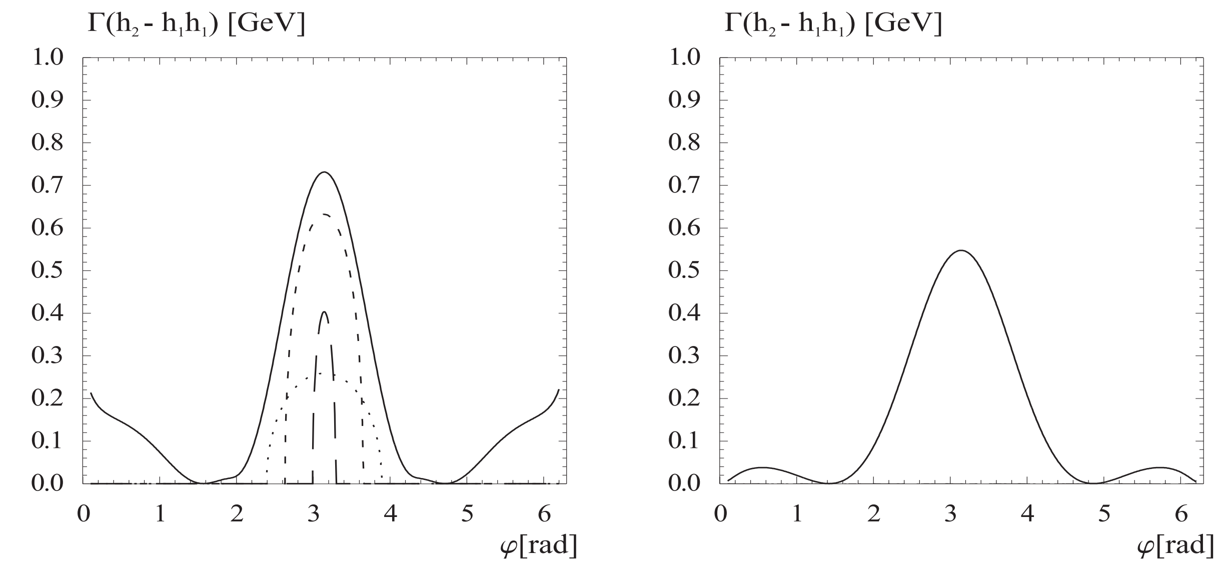


Figure 9: The decay width $h_2 \to h_1 h_1$ (GeV) vs the phase $\mathrm{Arg}(\mu A)$ at parameter values M_{SUSY} = 500 GeV, $\tan\beta$ = 5, $A_{t,b}$ = 1000 GeV, μ = 2000 GeV. Solid line – $m_{H^\pm}$ = 300 GeV, long dashed line – $m_{H^\pm}$ = 200 GeV, short dashed line – $m_{H^\pm}$ = 190 GeV, pointed curve – $m_{H^\pm}$ = 180 GeV. (a) – effective one-loop approach, (b) – with leading two-loop corrections

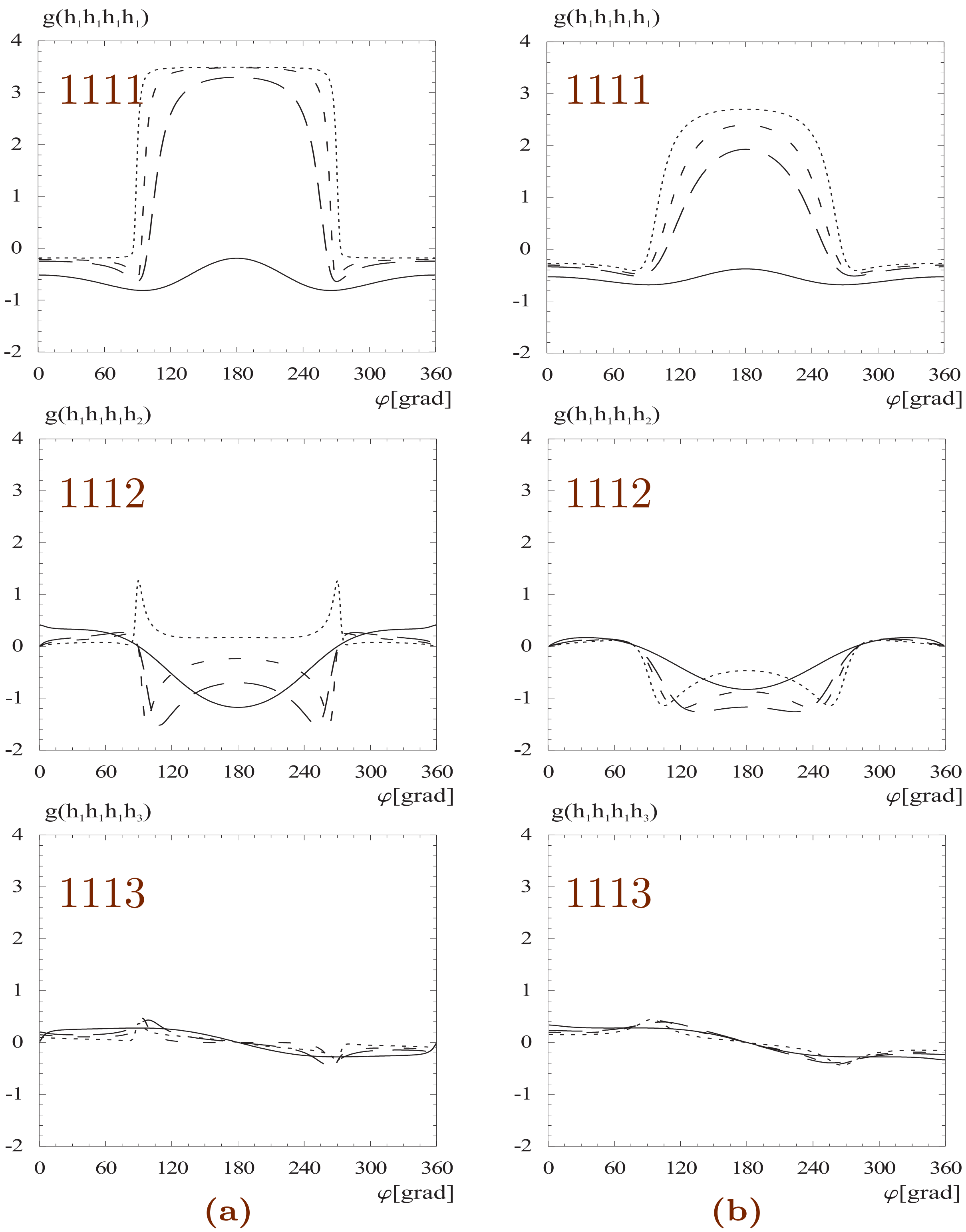


Figure 10: Quartic interaction vertices $g_{h_1 h_1 h_1 h_i}$, i =1,2,3, vs the phase $\mathrm{Arg}(\mu A)$ Solid line – $m_{H^\pm}$ = 300 GeV, long dashed line – $m_{H^\pm}$ = 200 GeV, short dashed line – $m_{H^\pm}$ = 190 GeV, pointed curve – $m_{H^\pm}$ = 180 GeV. (a) – effective one-loop potential, (b) – with leading two-loop corrections included

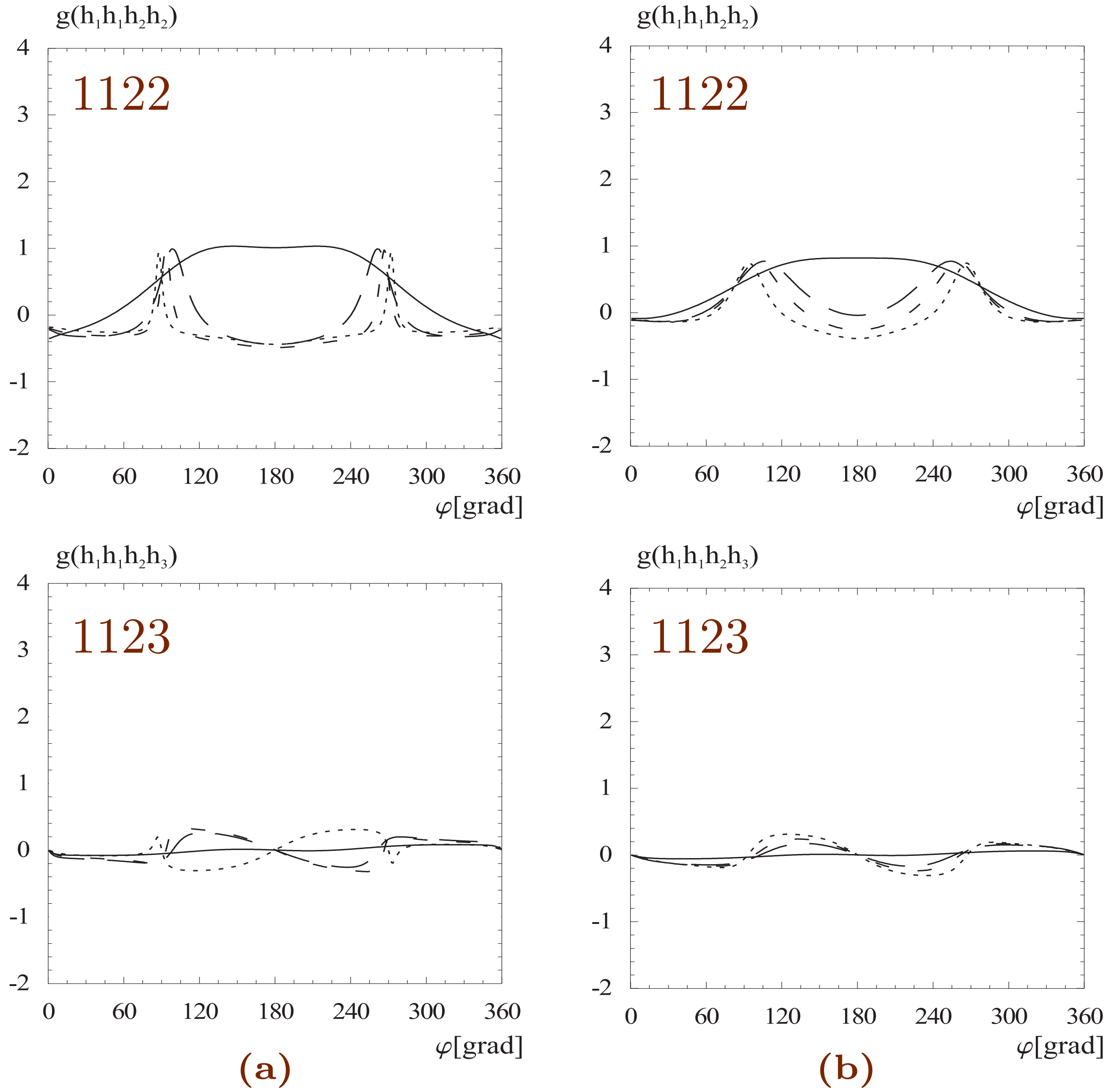


Figure 11: Quartic interaction vertices $g_{h_1 h_1 h_2 h_i}$, i =2,3, vs the phase $\mathrm{Arg}(\mu A)$ Solid line – $m_{H^\pm}$ = 300 GeV, long dashed line – $m_{H^\pm}$ = 200 GeV, short dashed line – $m_{H^\pm}$ = 190 GeV, pointed curve – $m_{H^\pm}$ = 180 GeV. (a) – effective one-loop potential, (b) – with leading two-loop corrections included

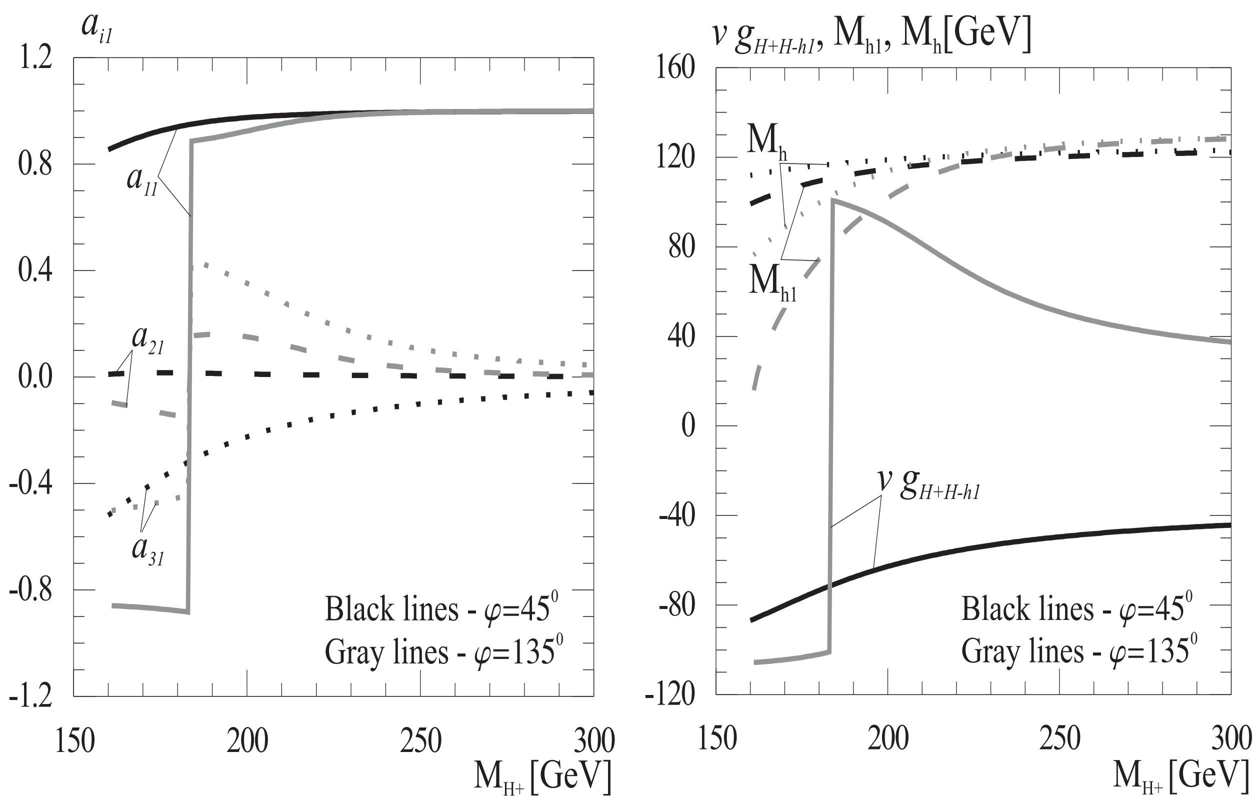


Figure 12: The matrix elements a_{i1} – (a) and the triple Higgs boson interaction vertex $v \cdot g_{H^+\,H^-\,h_1}$ and m_{h_1}, m_h in GeV – (b) vs the $H^{\pm}$-mass for the one-loop approximation for lambda-couplings

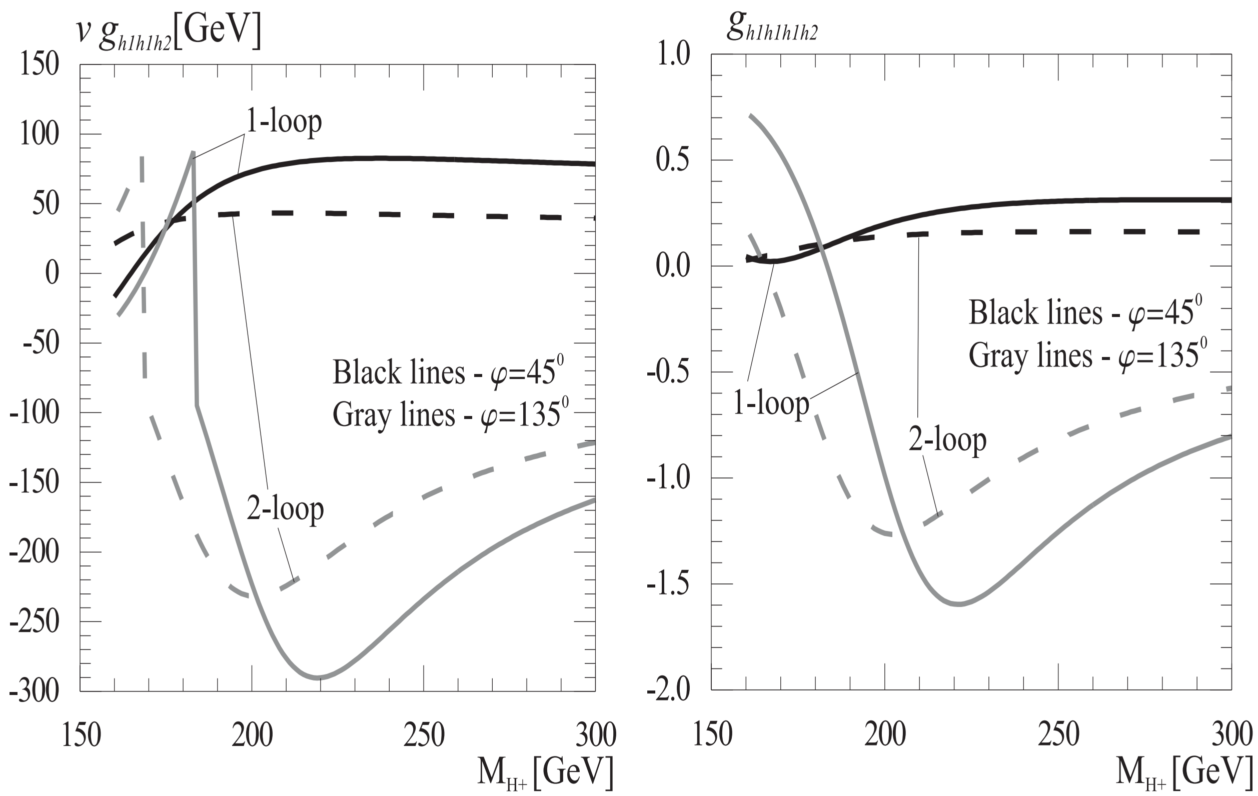


Figure 13: The triple Higgs boson interaction vertex $\cdot g_{h_1h_1h_2}$ – (a) and the quartic Higgs boson interaction vertex $\cdot g_{h_1h_1h_1h_2}$ – (b) vs the $H^{\pm}$-mass for the one-loop and with leading two-loop corrections for lambda-couplings

Summary and work in progress

- In order to keep (1) mass ordering $m_{h_1} \leq m_{h_2} \leq m_{h_3}$ of Higgs bosons, (2) matching of $(h_1,\ h_2,\ h_3)$ states to the $(h,\ H\ A)$ states of the CP-conserving limits and (3) definite (det$\|a_{ij}\|$ =1) orientation of the eigenvector basis at any values of model parameters $m_{H^\pm}$, $\mathtt{tan}\beta$, $\varphi = \mathtt{arg}(\mu A_{t,b})$ a sign prescription for the normalized eigenvectors should be accepted. For example, sign of $\mathbf{e_1}$

$$
k_1 : \quad \begin{cases}
1)\ \text{if } m_H(\varphi=0) > m_A(\varphi=0) \ \textit{and} \\
\quad \begin{cases}
1.1)\ \text{if } m_{h_3}(\varphi=0) \geq m_{H^\pm} \ \textit{and} \\
\quad \begin{cases}
1.1.1)\ \text{if sign}(c_1)\ \text{sign}(\pi-\varphi) = 1 \text{ then } \ k_1 = \text{sign}(m_{h_3} - m_{H^\pm}). \\
1.1.2)\ \text{if sign}(c_1)\ \text{sign}(\pi-\varphi) = -1, \text{ then } \ k_1 = -1.
\end{cases} \\
1.2)\ \text{if } m_{h_3}(\varphi=0) < m_{H^\pm} \quad \text{then } k_1 = 1.
\end{cases} \\
2)\ \text{if } m_H(\varphi=0) \leq m_A(\varphi=0) \qquad \text{then } \ k_1 = 1.
\end{cases}
$$

  The only discontinuity of $a_{ij}(\varphi, m_{H^\pm})$ is then the discontinuity in $m_{H^\pm}$, which takes place at 184 GeV at the one-loop effective λ_i in the MSSM, CPX$_{500}$ scenario. There are discontinuities in the trilinear and quartic couplings.

- Triple and quartic couplings of physical Higgs bosons vary very strongly (by a factor of 5÷10 or more) with $\varphi \in [0, 2\pi]$ and $m_{H^\pm}$ (MSSM, CPX$_{500}$ scenario). Small variations are only at $\varphi \sim 0$ (CP-conserving limit). High sensitivity of self-couplings to radiative corrections is observed. At the phase $\varphi \simeq \pi/2$ (c_1 =0, $m_{H^\pm}$ moderately small, the case of strong mixing) many of them change sign bypassing zero, so in the vicinity of $\varphi \simeq \pi/2$ the physical scalars are extremely weakly self-interacting.

- A discontinuity of the mixing matrix elements a_{ij} leads to a discontinuity in the couplings, where terms odd in a_{ij} ($j = \{1, 2\}$) appear. Such property has no clear physical motivation but could be relevant for systems that evolve in the phase and charged scalar mass, related to the phase transitions in cosmological models. Within the perturbation theory the discontinuities do not show up in the amplitudes (with the sign compensation in the product of matrix elements for each h propagator), however, this could be not the case for the nonperturbative insertions to diagrams.