3,5,7... -quark wave functions of ordinary baryons and the width of the exotic Θ^+

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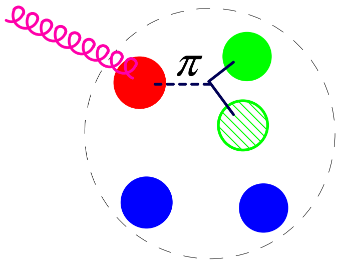
1. Quantum field theory is a necessity
2. Relativistic Mean Field Approximation to baryons
3. Baryon wave functions
4. Θ^+ width

Based on hep-ph/0406043, and D.D. and V. Petrov, hep-ph/0505201



Uncertainty principle at work: When one attempts to measure the quark position in the nucleon to an accuracy better than the pion Compton wave length of $1\,\mathrm{fm}$ one produces a pion, i.e. a new $Q\bar{Q}$ pair. Hence, the quantum-mechanical description of baryons with a fixed number of quarks, is senseless.

The statement "nucleons are made of three quarks" has a limited accuracy.
For some observables the accuracy is very poor, for example,

"Spin crisis": 0.3 ± 0.1 of the nucleon spin is carried by the three valence quarks

Nucleon σ-term: only $\frac{1}{4}$ of it is carried by the three valence quarks,

$$\sigma = \frac{m_u + m_d}{2} < N|\bar{u}u + \bar{d}d|N > \quad = \quad 67 \pm 6\,\mathrm{MeV},$$

$$\text{From valence quarks}: \frac{4\,\mathrm{MeV} + 7\,\mathrm{MeV}}{2} \times (\leq 3) \quad \leq \quad 17.5\,\mathrm{MeV}.$$

Both paradoxes are explained by additional $\bar{Q}Q$ pairs in baryons.

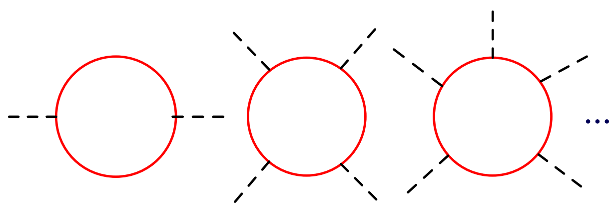
One needs means to describe baryons as $QQQ + QQQQ\bar{Q} + QQQQQ\bar{Q}\bar{Q} + \ldots$ states at low virtuality. Such means are provided by the **chiral theory**.

As a result of the **spontaneous chiral symmetry breaking** nearly massless u, d, s quarks obtain a dynamical mass $M(p)$ and hence necessarily have to interact strongly (!) with the pseudoscalar fields:

$$\mathcal{L}_{\rm eff} = \bar{q}\,\left[i\partial\!\!\!/ \, - M\exp(i\,\gamma_5\,\pi^A\lambda^A/F_\pi)\right]\,q, \qquad \pi^A = \pi, K, \eta, \qquad g_{\pi qq}(0) = \frac{M(0)}{F_\pi} \simeq 4$$

Models with massive quarks and (confining) gluon interaction, $\bar{q}\,[i\partial\!\!\!/ + A\!\!\!/ - M]\,q$, contradict chiral symmetry, as they violate invariance under

$$q \to \exp(i\gamma_5\alpha^A\lambda^A)q, \qquad \bar{q} \to \bar{q}\exp(i\gamma_5\alpha^A\lambda^A), \qquad e^{i\pi} \to e^{i\alpha}e^{i\pi}e^{i\alpha}.$$



Pseudoscalar mesons are themselves bound states of constituent quarks: they propagate and interact via virtual quark-antiquark pairs. The sum of all diagrams with any number of external legs is called the effective chiral lagrangian.

Integrating out quarks one obtains the **effective chiral action** [D.D. and Eides (1983), Dhar, Shankar and Wadia (1985)]

$$S[\pi(x)] \;=\; \int d^4x\,\left\{\frac{F_\pi^2}{4}{\rm Tr}\,(\partial_\mu U^\dagger\partial_\mu U) + \frac{N_c}{192\pi^2}{\rm Tr}\,[\partial_\mu U^\dagger\partial_\nu U]^2 + \text{infinite series}\right\}$$

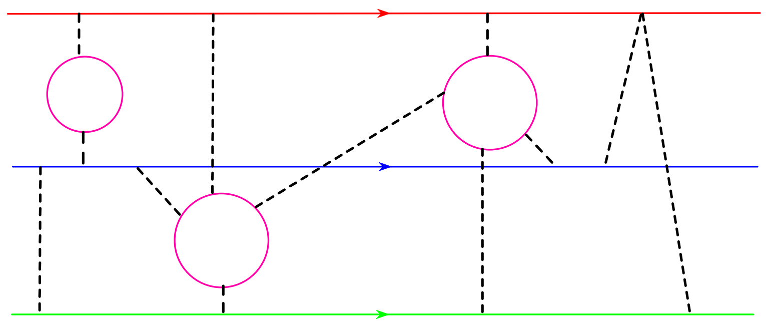
$$+ \quad \frac{iN_c}{240\pi^2}\int d^5x\, \epsilon^{\alpha\beta\gamma\delta\epsilon}\, \mathrm{Tr}\,(U^\dagger\partial_\alpha U\partial_\beta U^\dagger\partial_\gamma U\partial_\delta U^\dagger\partial_\epsilon U) + \ldots,$$

$$U \quad = \quad \exp(i\pi^A\lambda^A).$$

In contrast, the Skyrme model

- neglects all higher derivative terms [it is like replacing e^{-x} by $1-x$]
- adds the Wess–Zumino term by hand

No wonder the agreement of the Skyrme model with data is only qualitative. One has to use the **full** chiral lagrangian to describe baryons.

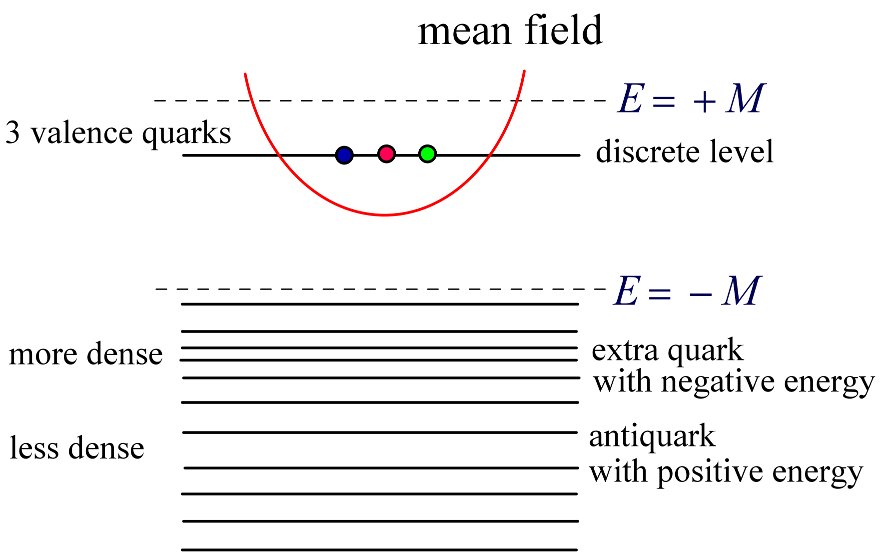


Quarks in the nucleon (solid lines), interacting via pion fields (dashed lines).

If the number of colours N_c is large, the summation of all such diagrams becomes equivalent to finding the **mean chiral field**.

Relativistic Mean Field Approximation **or** the Chiral Quark Soliton Model

[DD and Petrov + Pobylitsa (1986)]



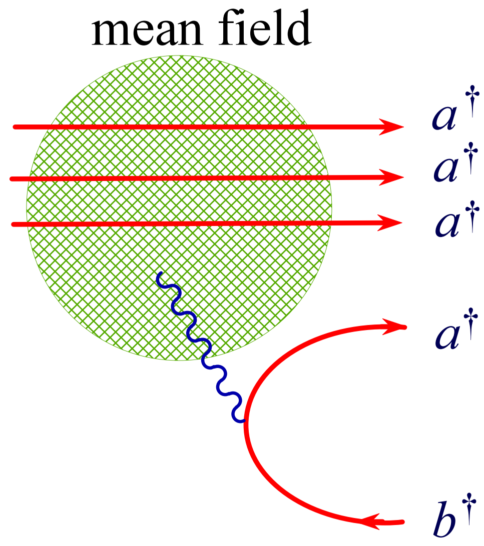


Figure 1: A schematic view of baryons in the Relativistic Mean Field Approximation. There are three "valence" quarks at a discrete energy level created by the mean field, and the negative-energy Dirac continuum distorted by the mean field, as compared to the free one.

Figure 2: Equivalent view of baryons, where the polarized Dirac sea is presented as $Q\bar{Q}$ pairs. Their wave function is given by the quark Green function in the background mean field, at equal times [Petrov and Polyakov (2003)].

$$\text{Baryon mass } = N_c\ (E_{\text{lev}}[\pi(x)] + E_{\text{sea}}[\pi(x)]) \Longleftarrow \text{minimize it in } \pi(x)\,!$$

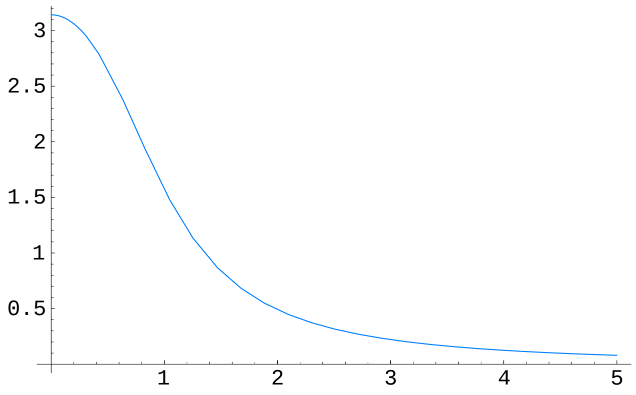


Figure 3: The space profile of the self-consistent chiral field $|\pi|(r)$ in light baryons. One unit on the horizontal axis is $r_0 = 0.8/M = 0.46\,\mathrm{fm}$.

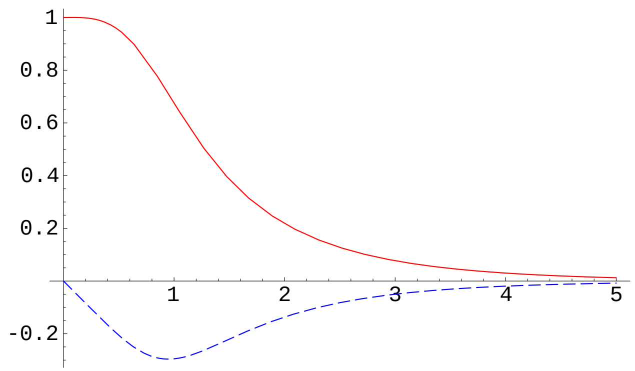


Figure 4: Bound-state quark upper s-wave component $h(r)$ (red) and the lower p-wave component $j(r)$ (blue) in light baryons.

The only input $M(0) = 345\,\mathrm{MeV} \Longrightarrow E_{\rm lev} = 200\,\mathrm{MeV}$. The three "valence" quarks are rather tightly bound and relativistic!

In the Relativistic Mean Field Approximation **all** properties of **all** baryons from the $\left(\mathbf{8}, \frac{1}{2}^+\right)$, $\left(\mathbf{10}, \frac{3}{2}^+\right)$ and $\left(\overline{\mathbf{10}}, \frac{1}{2}^+\right)$ multiplets follow from the shape of the self-consistent pion field, including masses, magnetic moments, formfactors, parton distributions at low virtuality, etc.

Error sources:

- Precise form of $M(p)$ is presently not known – only certain integrals relating $M(p)$ to F_π, $<\bar{q}q>$
- Corrections from fluctuations about the mean field $\sim 1/N_c$, actually $\sim 1/(2\pi N_c) \approx 6\%$
- Residual color Coulomb interactions, estimated as small by DD, Jaenicke and Polyakov (1992)

On the whole, all baryon properties computed so far are within 15% from the data, with no adjusted or fitting parameters!

Baryon wave functions

The Dirac sea is presented by the coherent exponent of the quark $a^\dagger$ and antiquark $b^\dagger$ creation operators:

$$\text{coherent exponent} = \exp\left(\int (d\mathbf{p})(d\mathbf{p}')\, a^\dagger(\mathbf{p})\, W(\mathbf{p},\mathbf{p}')\, b^\dagger(\mathbf{p}')\right)|0>,$$

where $W(\mathbf{p_1},\mathbf{p_2})$ is the (calculated) quark Green function at equal times in the background chiral field.

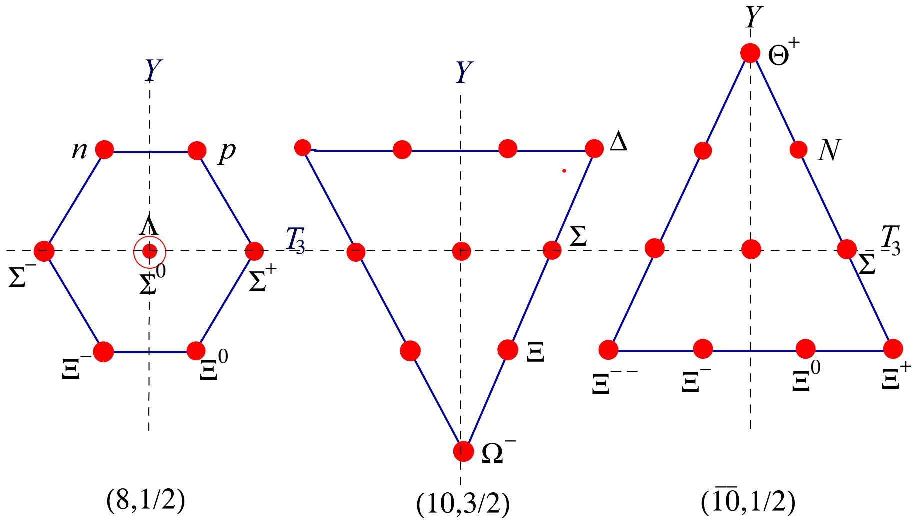
Baryon wave function of N_c "valence" quarks and arbitrary number of $Q\bar{Q}$ pairs:

$$\Psi^B[a^\dagger, b^\dagger] = \prod_{\text{color}=1}^{N_c} \int (d\mathbf{p})\, F(\mathbf{p})\, a^\dagger(\mathbf{p})$$
$$\cdot \quad \exp\left(\int (d\mathbf{p})(d\mathbf{p}')\, a^\dagger(\mathbf{p})\; W(\mathbf{p}, \mathbf{p}')\, b^\dagger(\mathbf{p}') \right) |0> \; .$$

The mean chiral field is degenerate in overall rotations in ordinary and flavor spaces; hence one has to **project** it to a given flavor and spin baryon state:

$$\Psi^B_k = \int dR\, B^*_k(R)\, \epsilon^{\alpha_1\alpha_2\alpha_3} \prod_{n=1}^{3} \int (d\mathbf{p_n})\, R^{f_n}_{j_n}\, F^{j_n\sigma_n}(\mathbf{p_n})\, a^\dagger_{\alpha_n f_n \sigma_n}(\mathbf{p_n})$$
$$\cdot \exp\left(\int (d\mathbf{p})(d\mathbf{p}')\, a^\dagger_{\alpha f \sigma}(\mathbf{p})\, R^f_j\, W^{j\sigma}_{j'\sigma'}(\mathbf{p}, \mathbf{p}')\, R^{\dagger\, j'}_{f'}\, b^{\dagger\, \alpha f' \sigma'}(\mathbf{p}') \right) |0> \; .$$

This is the generating functional **for all** $qqq,\; qqqq\bar{q},\; qqqqq\bar{q}\bar{q}$ quark wave functions in the $\left(\mathbf{8}, \frac{1}{2}^+\right)$, $\left(\mathbf{10}, \frac{3}{2}^+\right)$ and $\left(\overline{\mathbf{10}}, \frac{1}{2}^+\right)$ baryons in the Relativistic Mean Field Approximation. R^f_j is an $SU(3)$ matrix parameterized by 8 "Euler angles". $B^*_k(R)$ is a given baryon's rotational wave function depending on those angles.



Examples of the baryons' (conjugate) rotational wave functions $B^*(R)$:

$$\text{proton, spin projection } k: \qquad p_k^*(R) = \sqrt{8}\,\epsilon_{kl} R_1^{\dagger\,l} R_3^3,$$

$$\text{neutron, spin projection } k: \qquad n_k^*(R) = \sqrt{8}\,\epsilon_{kl} R_2^{\dagger\,l} R_3^3,$$

$$\Delta^{++}, \text{ spin projection } +\frac{3}{2}: \qquad \Delta_{\uparrow\uparrow}^{++\,*}(R) = \sqrt{10}\, R_1^{\dagger\,2} R_1^{\dagger\,2} R_1^{\dagger\,2},$$

$$\Theta^+, \text{ spin projection } k: \qquad \Theta_k^*(R) = \sqrt{30}\, R_3^3 R_3^3 R_k^3, \qquad \left(\times \left(R_3^3\right)^{N_c-3}\right)$$

Baryon wave functions in terms of quarks

If the coherent exponent with $Q\bar{Q}$ pairs is ignored, one gets the 3-quark Fock component of the octet and decuplet baryons. It depends on the quark "coordinates" $\mathbf{r}, \alpha,\ f,\ \sigma$ and on the baryon spin projection k.

For example, the neutron $3Q$ wave function is

$$(|n>_k)^{f_1 f_2 f_3, \sigma_1 \sigma_2 \sigma_3}(\mathbf{r_1}, \mathbf{r_2}, \mathbf{r_3}) = \epsilon^{f_1 f_2}\, \epsilon^{\sigma_1 \sigma_2}\, \delta_2^{f_3}\, \delta_k^{\sigma_3}\, h(r_1) h(r_2) h(r_3)$$
$$+ \text{permutations of } 1,2,3 \qquad (\otimes \quad \epsilon^{\alpha_1 \alpha_2 \alpha_3})\,.$$

It is better known in the form

$$|n\uparrow> \;=\; 2\, d\uparrow(r_1) d\uparrow(r_2) u\downarrow(r_3) - d\uparrow(r_1) u\uparrow(r_2) d\downarrow(r_3) - u\uparrow(r_1) d\downarrow(r_2) d\uparrow(r_3)$$
$$+ \quad \text{permutations of } r_1, r_2, r_3,$$

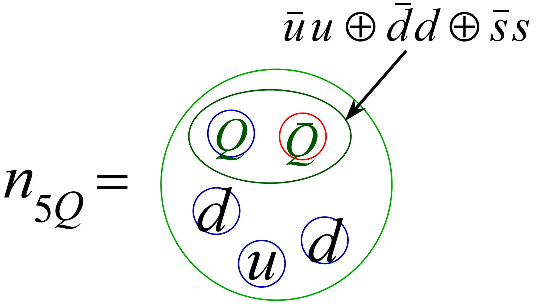
which is the well-known non-relativistic $SU(6)$ wave function of the nucleon!

There are relativistic corrections to the $SU(6)$-symmetric formulae, arising from i) lower p-wave component of the discrete level, ii) additional $Q\bar{Q}$ pairs. Both effects are not small.

The $5Q$ component of a baryon is obtained when one expands the coherent exponent to the first, linear order and then projects it onto the concrete baryon in question. For example, the $5Q$ component of the **neutron** has the wave function

$$
\begin{aligned}
&(|n>_k)^{f_1 f_2 f_3 f_4, \sigma_1 \sigma_2 \sigma_3 \sigma_4}_{f_5, \sigma_5}(\mathbf{r_1}, \mathbf{r_2}, \mathbf{r_3}, \mathbf{r_4}, \mathbf{r_5}) = h(r_1) h(r_2) h(r_3)\, W^{j_4 \sigma_4}_{j_5 \sigma_5}(\mathbf{r_4}, \mathbf{r_5}) \\
\cdot \quad & \left\{ \epsilon^{f_1 f_2} \epsilon_{j_1 j_2} \left[\delta^{f_3}_2 \delta^{f_4}_{f_5} \left(4 \delta^{j_5}_{j_4} \delta^{k'}_{j_3} - \delta^{j_5}_{j_3} \delta^{k'}_{j_4} \right) + \delta^{f_4}_2 \delta^{f_3}_{f_5} \left(4 \delta^{j_5}_{j_3} \delta^{k'}_{j_4} - \delta^{j_5}_{j_4} \delta^{k'}_{j_3} \right) \right] \right. \\
+ \quad & \left. \epsilon^{f_1 f_4} \epsilon_{j_1 j_4} \left[\delta^{f_2}_2 \delta^{f_3}_{f_5} \left(4 \delta^{j_5}_{j_3} \delta^{k'}_{j_2} - \delta^{j_5}_{j_2} \delta^{k'}_{j_3} \right) + \delta^{f_3}_2 \delta^{f_2}_{f_5} \left(4 \delta^{j_5}_{j_2} \delta^{k'}_{j_3} - \delta^{j_5}_{j_3} \delta^{k'}_{j_2} \right) \right] \right\} \epsilon_{k'k} \epsilon^{j_1 \sigma_1} \epsilon^{j_2 \sigma_2} \epsilon^{j_3 \sigma_3} \\
+ \quad & \text{permutations of } (1, 2, 3)
\end{aligned}
$$

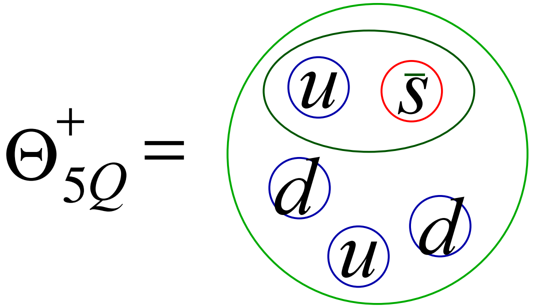
Indices 1-3 refer to quarks at the discrete level, 4 refers to the quark in the additional pair, and 5 refers to the antiquark in the pair. $\delta^{f_3}_{f_5} \sim s\bar{s} + u\bar{u} + d\bar{d}$.



Additional $Q\bar{Q}$ pair is a combination of partial waves corresponding to the scalar, pseudoscalar, vector and axial mesons. The partial waves depend separately on the $Q, \bar{Q}$ coordinates $\mathbf{r_{4,5}}$, see `hep-ph/0408219`, `hep-ph/0505201`.

Exotic baryons from the $\left(\overline{10}, \frac{1}{2}^+\right)$

Projecting the three quarks from the discreet level on the Θ^+ rotational function gives an **identical zero**, in accordance with the fact that the Θ^+ cannot be made of 3 quarks.



The spatial structure (but not the spin-flavor one) of the leading $5Q$ component of the Θ^+ is similar to that of the non-leading $5Q$ component of the nucleon.

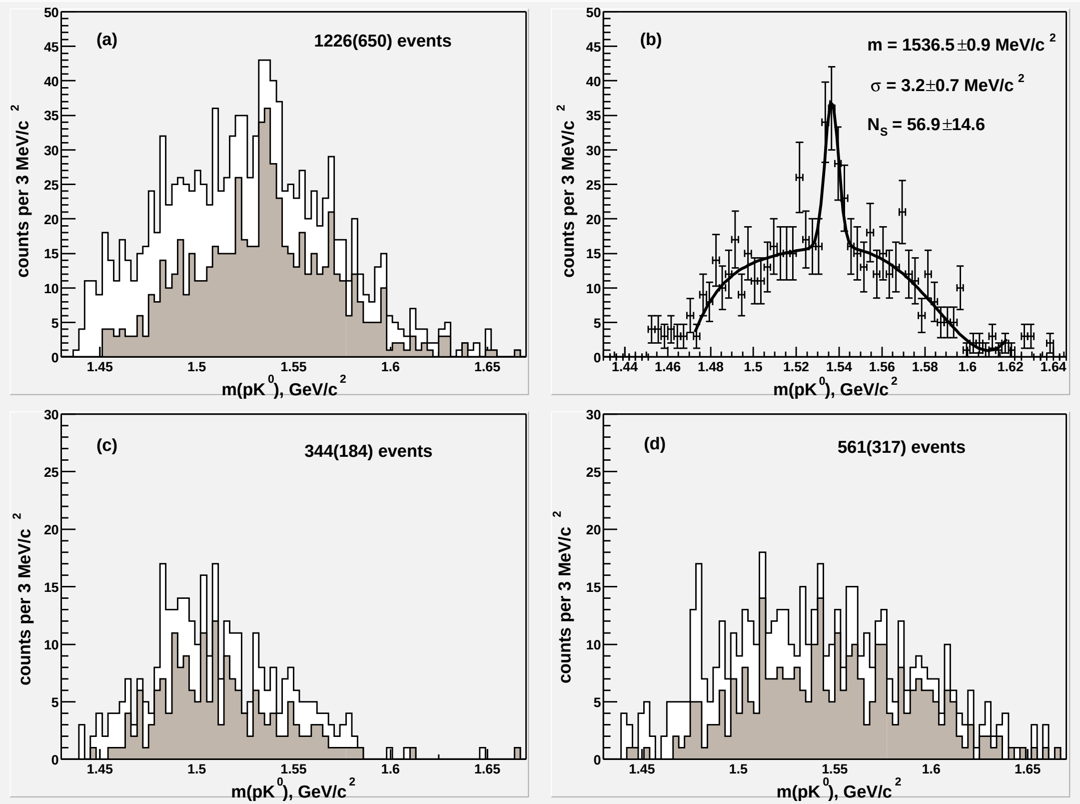
The non-zero projection is achieved when one expands the coherent exponent at least to the linear order. One gets the $5Q$ component of the Θ wave function:

$$|\Theta_k^+ >^{f_1 f_2 f_3 f_4, \sigma_1 \sigma_2 \sigma_3 \sigma_4}_{f_5, \sigma_5} (\mathbf{r_1} \ldots \mathbf{r_5}) = \epsilon^{f_1 f_2} \epsilon^{f_3 f_4} \delta^3_{f_5} \, \epsilon^{\sigma_1 \sigma_2}$$

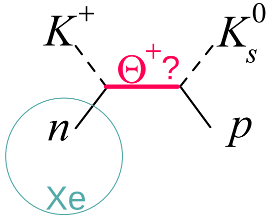
$$\cdot \, h(r_1) h(r_2) h(r_3) \, W^{\sigma_3 \sigma_4}_{k \, \sigma_5}(\mathbf{r_4}, \mathbf{r_5}) + \text{permutations of } 1, 2, 3$$

$$\Theta^+ = uud d\bar{s}.$$

New analysis of the **formation** reaction $K^+n(\mathrm{Xe}) \to K^0p$ at ITEP
[A. Dolgolenko et al. (2006), hep-ex/0603017]



Mass distribution in the range of the incident K^+ momenta where the Θ^+ resonance **can be** formed due to Fermi motion.



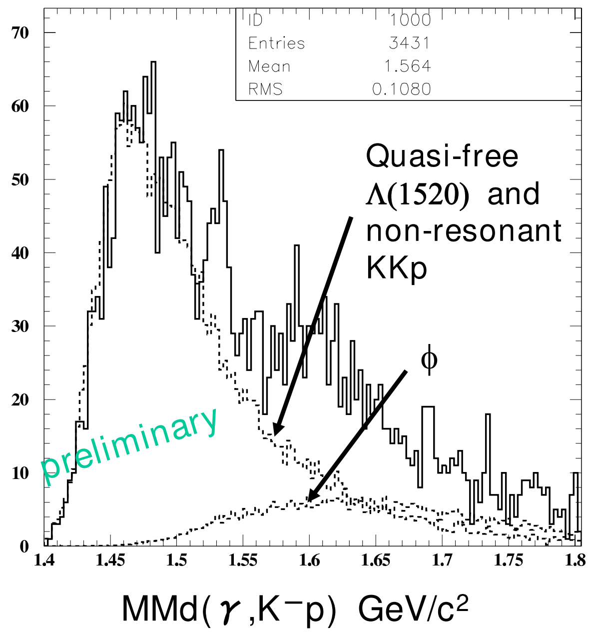
Mass distributions for the range of incident momenta where Θ^+ **cannot be** formed or is suppressed kinematically.

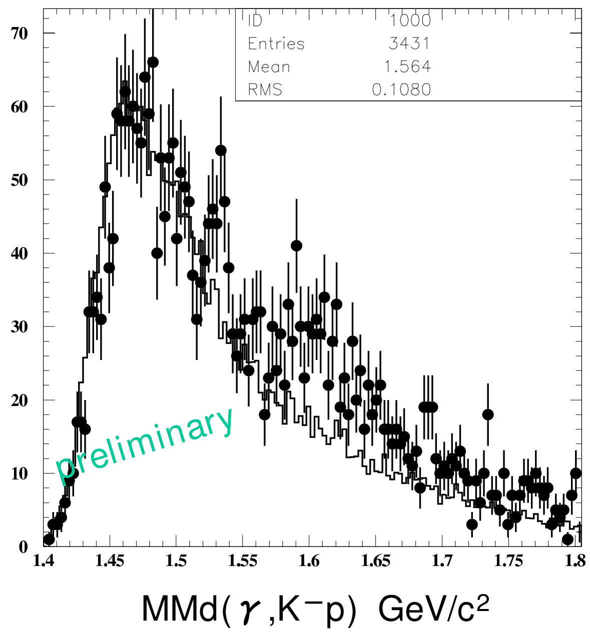
This is the **only direct formation experiment.** BELLE (R. Mizuk) came with an upper limit for Θ^+ production, which does not contradict a very small Θ^+ width obtained by Dolgolenko et al.,

$$\Gamma_\Theta = 0.36 \pm 0.11\,\mathrm{MeV}\,!$$

A *quasi*-formation experiment where a quasi-free K^+ is scattering on a quasi-free neutron inside a deutron: T. Nakano et al. at SPring-8

K⁻p missing mass for Λ(1520)





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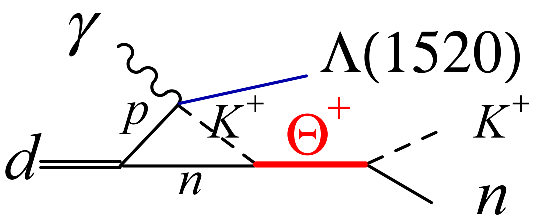
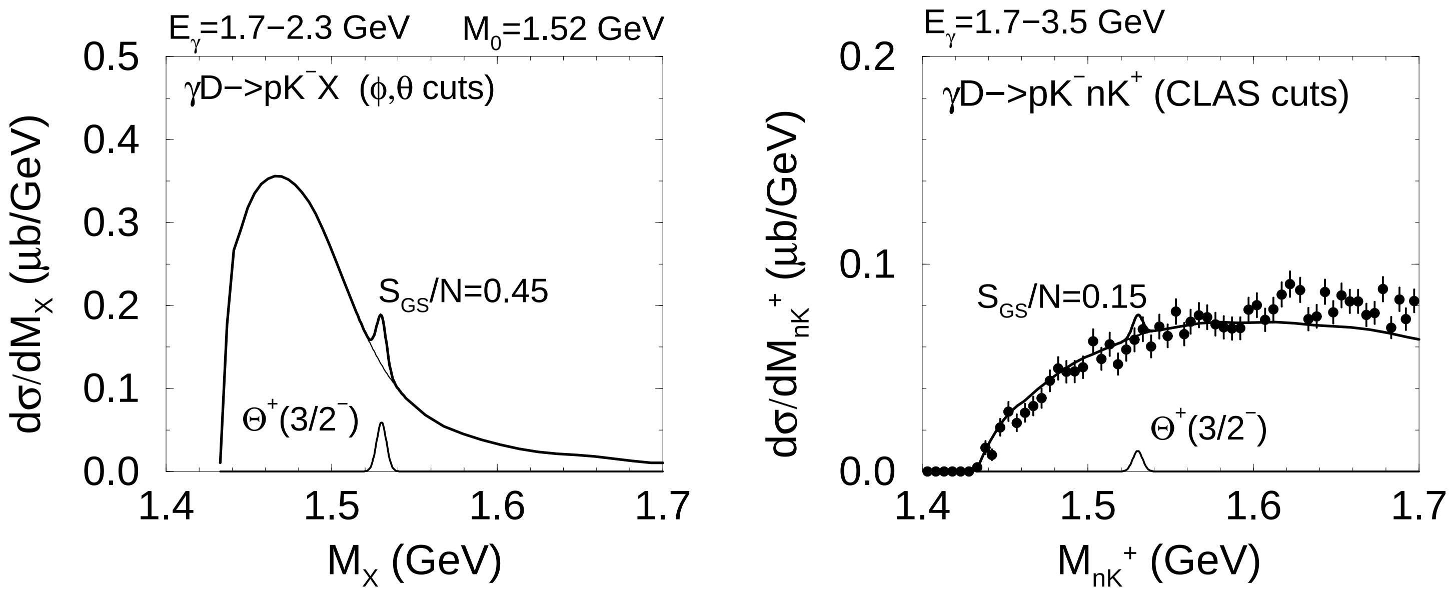


Why Nakano et al. see the Θ^+ peak whereas CLAS@ JLab does not?

In both cases it is the γd reaction but the kinematics and the detector acceptance are different. A. Titov (Dubna, 2006) made a calculation specific for both experiments:



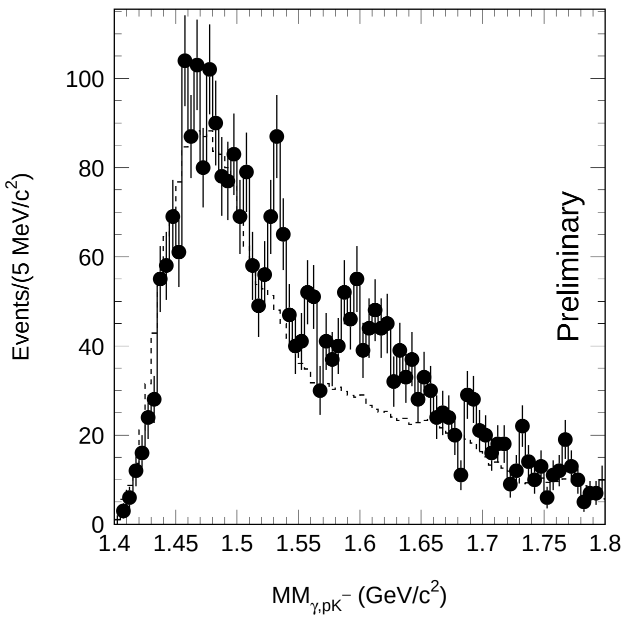


Figure 7. The LEPS data on deuterium.The missing mass distribution $MM_{\gamma,K-p}$ with events selected with the invariant mass M_{K-p} near the Λ^*. A peak is seen at a mass of 1530 MeV, and is interpreted as the Θ^+.

lected with $M_{K-p} \sim M_{\Lambda^*}$, indicating that the process $\gamma D \to \Lambda^*\Theta^+$ may be observed. The mass distribution also shows an excess of events near 1600 MeV/c^2.

5.4 Results from Belle

$qqqq\bar{q}$

New results by the Belle collaboration have

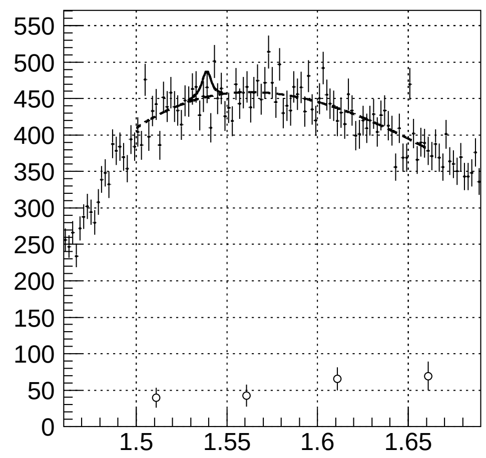


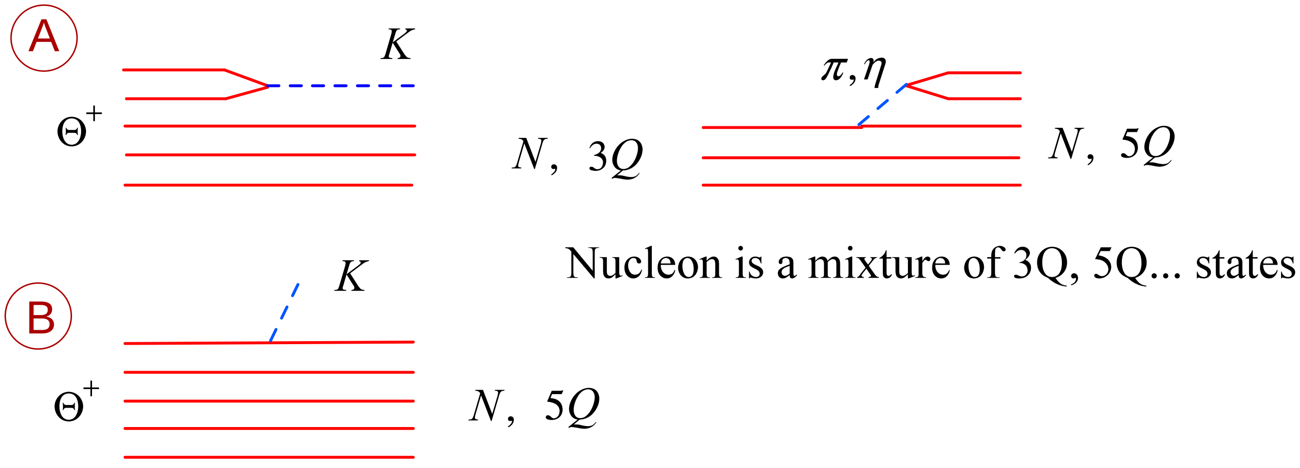
Figure 8. The Belle pK_s^0 mass spectrum.

per limit would be $\Gamma_{\Theta+} < 1$ MeV (90% c.l.). The latter value confirms the limit derived in previous analyses.

5.5 BaBar results in quark fragmentation

The BaBar collaboration at SLAC searches for the Θ^+ as well as the Ξ^{--} pentaquark states directly in e^+e^- collisions [24], mostly in the quark fragmentation region. With high statistics no signal is found for either $\Theta^+(1540)$ or $\Xi^{--}(1862)$, and upper limits

Θ^+ decay



Both processes contribute in the decay

Both diagrams A and B contribute to the decay, and they depend on the reference frame; only their sum is Lorentz invariant.

In the Infinite Momentum Frame (IMF) only the second diagram B survives, as vector and axial currents with a finite momentum transfer do not create or annihilate quarks with infinite momenta. The baryon matrix elements are thus non-zero only between Fock components with **equal number** of quarks and antiquarks.

Normalization of the $3Q$ and $5Q$ components

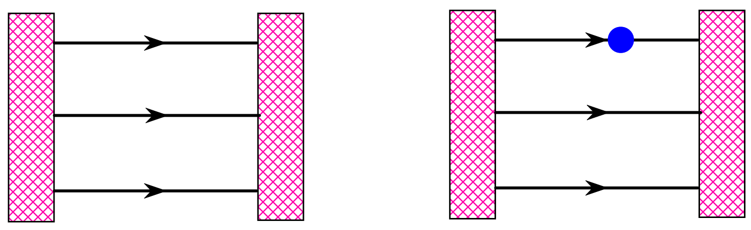


Figure 5: Graphs showing the normalization of a 3-quark component of a baryon (left) and the matrix element of a local operator denoted by a circle (right).

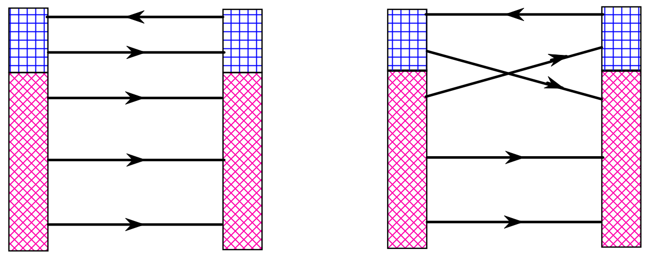


Figure 6: Direct (left) and exchange (right) contributions to the normalization of the 5-quark component of a baryon. The upper rectangles denote $Q\bar{Q}$ pairs.

$$\mathcal{N}_N^{(3)} = 1, \qquad \mathcal{N}_N^{(5)} \approx 0.4\,!$$

Momentum carried by antiquarks in the nucleon at low virtuality, roughly,

$$\frac{0 \cdot 1 + \frac{1}{5} \cdot 0.4}{1 + 0.4} \approx 6\%$$

Baryon matrix elements related to the $5Q$ components

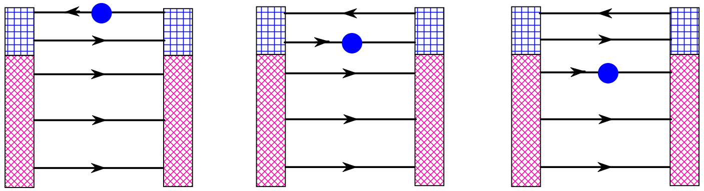


Figure 7: Direct contributions to the matrix element of an operator computed by DD and Petrov (2005).

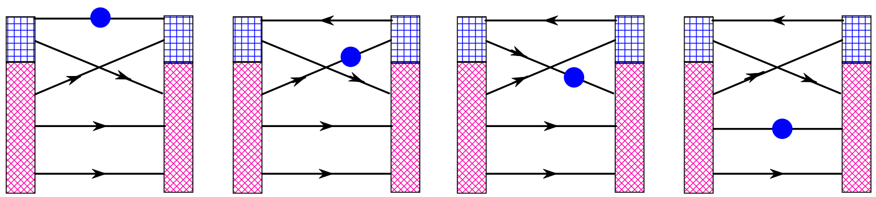


Figure 8: Exchange contributions to the matrix element in the 5-quark component of a baryon computed by Lorcé (2006).

Observables

$$g_A^{(3)}(N) = \frac{A^{(3)}(N)}{\mathcal{N}^{(3)}(N)} = \frac{5}{3} = 1.67 \qquad (g_A(N)^{\rm exper} = 1.27)$$

$$g_A^{(5)}(N) = \frac{A^{(3)}(N)+A^{(5)}(N)+\ldots}{\mathcal{N}^{(3)}(N)+\mathcal{N}^{(5)}(N)+\ldots} = 1.44 \quad (1.32 \text{ when all } \bar{Q}Q \text{ pairs are summed up})$$

$$g_A^{(5)}(\Theta \to KN) = \frac{A^{(5)}(\Theta\to KN)+\ldots}{\sqrt{N_\Theta^{(5)}+\ldots}\sqrt{\mathcal{N}^{(3)}(N)+\mathcal{N}^{(5)}(N)+\ldots}} = 0.15 \Longrightarrow \Gamma_\Theta = 2.3\,\mathrm{MeV} \text{ !!}$$

This is computed as if the decay is with zero momentum transfer. In reality the width must be **smaller** due to additional formfactor suppression.

Conclusions

1. Ordinary baryons are **not** made of 3 quarks only but have a substantial component with the additional $\bar{Q}Q$ pairs. In particular, $1/3$ of the time the nucleon is made of 5 quarks. For some observables, the effect is 20% but for some other it changes the naive $3Q$ result by a factor 3-4.

2. One adds additional $\bar{Q}Q$ pairs in the form of nearly massless Goldstone bosons, which costs little energy. This is why the $5Q$ component of the nucleon is large, and why Θ^+ is light!

3. We have presented a compact and universal technique, how to write explicitly the $3Q, 5Q, 7Q\ldots$ wave functions of the octet, decuplet and antidecuplet baryons, based on the Relativistic Mean Field Approximation.

4. The exotic Θ^+ width is proportional to the number of additional $\bar{Q}Q$ pairs in $nucleons$ and is thus naturally suppressed as compared to the expected widths of baryons with the dominant $3Q$ component. No-free-parameter estimate: $\Gamma_\Theta \sim 2\,\mathrm{MeV}$, and can easily be less!

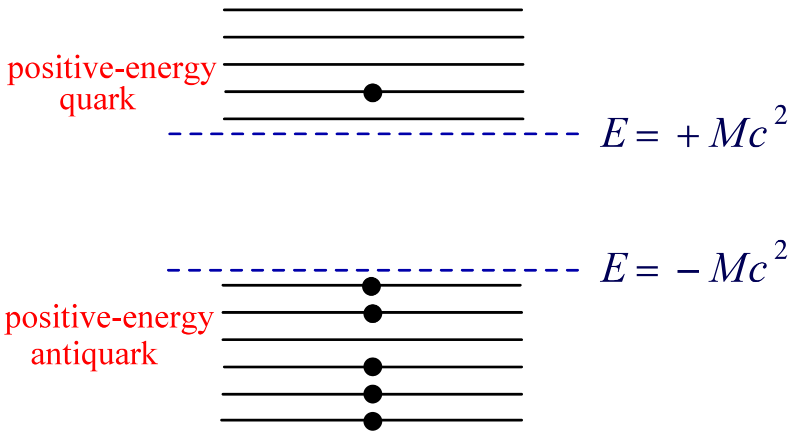


Figure 9: Vector, axial, tensor mesons are particle-hole excitations of the vacuum. They are made of a quark with positive energy and an antiquark with positive energy, hence their mass is roughly $2M$.

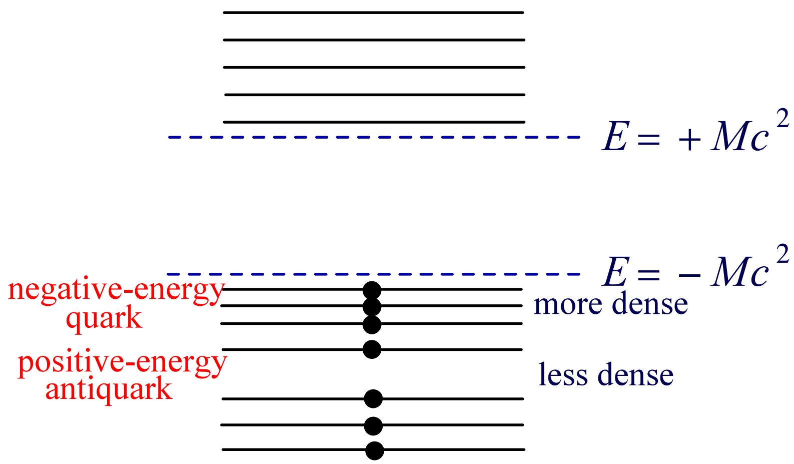


Figure 10: Pseudoscalar mesons are *not* particle-hole excitations but a collective re-arrangement of the vacuum. They are made of an antiquark with positive energy and a quark with *negative* energy, hence their mass is roughly zero, $(M - M) = 0$.