Generalized Dirac-Pauli equation and neutrino quantum states in matter

Alexander Studenikin[*], Alexei Ternov[†]

Abstract

Starting with the Dirac-Pauli equation for a massive neutrino in an external magnetic field, we propose a new quantum equation for a neutrino in the presence of background matter. On this basis the quantum theory of a neutrino moving in the background matter is developed: i) for the particular case of matter with constant density the exact solutions of this new equation are found and classified over the neutrino spin states, ii) the corresponding energy spectrum is also derived accounting for the neutrino helicity. Using these solutions we develop the quantum theory of the spin light of neutrino ($SL\nu$) in matter. The $SL\nu$ radiation rate and total power are derived for different linear and circular polarizations of the emitted photons. Within the solid base of the developed quantum approach, the existence of the neutrino self-polarization effect in matter is also shown.

Recently in a series of our papers [1] we have developed the quasi-classical approach to the massive neutrino spin evolution in the presence of external electromagnetic fields and background matter. In particular, we have shown that the well known Bargmann-Michel-Telegdi (BMT) equation [2] of the electrodynamics can be generalized for the case of a neutrino moving in the background matter and under the influence of external electromagnetic fields. The proposed new equation for a neutrino, which simultaneously accounts for the electromagnetic interaction with external fields and also for the weak interaction with particles of the background matter, was obtained from the BMT equation by the following substitution of the electromagnetic field tensor $F_{\mu\nu} = (\mathbf{E}, \mathbf{B})$:

$$F_{\mu\nu} \to E_{\mu\nu} = F_{\mu\nu} + G_{\mu\nu}, \tag{1}$$

[*]Address: Department of Theoretical Physics, Moscow State University, 119992 Moscow, Russia, e-mail:studenik@srd.sinp.msu.ru .

[†]Department of Theoretical Physics, Moscow Institute for Physics and Technology, 141700 Dolgoprudny, Russia, e-mail:A_Ternov@mail.ru

where the tensor $G_{\mu\nu} = (-\mathbf{P}, \mathbf{M})$ accounts for the neutrino interactions with particles of the environment. The substitution (1) implies that in the presence of matter the magnetic $\mathbf{B}$ and electric $\mathbf{E}$ fields are shifted by the vectors $\mathbf{M}$ and $\mathbf{P}$, respectively:

$$\mathbf{B} \to \mathbf{B} + \mathbf{M}, \quad \mathbf{E} \to \mathbf{E} - \mathbf{P}. \tag{2}$$

We have also shown how to construct the tensor $G_{\mu\nu}$ with the use of the neutrino speed, matter speed, and matter polarization four-vectors.

Within the developed quasi-classical approach to the neutrino spin evolution we have also considered [3–5] a new type of electromagnetic radiation by a neutrino moving in the background matter and/or gravitational fields which we have named the "spin light of neutrino" ($SL\nu$). The $SL\nu$ originates, however, from the quantum spin flip transitions and for sure it is important to revise the calculations of the rate and total power of the $SL\nu$ in matter using the quantum theory. Note that within the quantum theory the radiation emitted by a neutrino moving in a magnetic field was also considered in [6].

In this paper we should like to present a reasonable step forward, which we have made recently, in the study of the neutrino interaction in the background matter and external fields. On the basis of the generalization of the Dirac-Pauli equation of the quantum electrodynamics we propose a new quantum equation for the neutrino wave function with effects of the neutrino-matter interaction being accounted for. This new equation establishes the basis for the quantum treatment of a neutrino moving in the presence of the background matter. In the limit of the constant matter density, we get the exact solutions of this equation, classify them over the neutrino helicity states and determine the energy spectrum, which depends on the helicity. Then with the use of these wave functions we develop the quantum theory of the $SL\nu$ and calculate the rate and power of the spin-light radiation in matter accounting for the emitted photons polarization. The existence of the neutrino-spin self-polarization effect [3,5] in matter is also confirmed within the developed quantum approach [1].

To derive the quantum equation for the neutrino wave function in the background matter we start with the well-known Dirac-Pauli equation for a neutral fermion with non-zero magnetic moment. For a massive neutrino moving in an electromagnetic field $F_{\mu\nu}$ this equation is given by

$$\Big(i\gamma^{\mu}\partial_{\mu} - m - \frac{\mu}{2}\sigma^{\mu\nu}F_{\mu\nu}\Big)\Psi(x) = 0, \tag{3}$$

[1] The neutrino-spin self-polarization effect in the magnetic field was discussed in [6].

where m and μ are the neutrino mass and magnetic moment[2], $\sigma^{\mu\nu} = i/2\big(\gamma^{\mu}\gamma^{\nu} - \gamma^{\nu}\gamma^{\mu}\big)$. It worth to be noted here that Eq.(3) can be obtained in the linear approximation over the electromagnetic field from the Dirac-Schwinger equation, which in the case of the neutrino takes the following form [6]:

$$(i\gamma^{\mu}\partial_{\mu} - m)\Psi(x) = \int M^{F}(x', x)\Psi(x')dx', \qquad (4)$$

where $M^{F}(x', x)$ is the neutrino mass operator in the presence of the external electromagnetic field.

For the case of the external magnetic filed, the Hamiltonian form of the equation (3) reads

$$i\frac{\partial}{\partial t}\Psi(\mathbf{r}, t) = \hat{H}_{F}\Psi(\mathbf{r}, t), \qquad (5)$$

where

$$\hat{H}_{F} = \hat{\boldsymbol{\alpha}}\mathbf{p} + \hat{\beta}m + \hat{V}_{F}, \hat{V}_{F} = -\mu\hat{\beta}\hat{\boldsymbol{\Sigma}}\mathbf{B}, \qquad (6)$$

and $\mathbf{B}$ is the magnetic field vector. We use the Pauli-Dirac representation of the Dirac matrices $\hat{\boldsymbol{\alpha}}$ and $\hat{\beta}$, in which

$$\hat{\boldsymbol{\alpha}} = \begin{pmatrix} 0 & \hat{\boldsymbol{\sigma}} \\ \hat{\boldsymbol{\sigma}} & 0 \end{pmatrix} = \gamma_{0}\boldsymbol{\gamma}, \quad \hat{\beta} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \gamma_{0}, \quad \hat{\boldsymbol{\Sigma}} = \begin{pmatrix} \hat{\boldsymbol{\sigma}} & 0 \\ 0 & \hat{\boldsymbol{\sigma}} \end{pmatrix}, \qquad (7)$$

where $\hat{\boldsymbol{\sigma}} = (\sigma_1, \sigma_2, \sigma_3)$ and σ denotes the Pauli matrixes.

Now let us consider the case of a neutrino moving in matter without any electromagnetic field in the background. The quantum equation for the neutrino wave function can be obtained from (3) with application of the substitution (1) which now becomes

$$F_{\mu\nu} \to G_{\mu\nu}. \qquad (8)$$

Thus, we get the quantum equation for the neutrino wave function in the presence of the background matter in the form

$$\Big(i\gamma^{\mu}\partial_{\mu} - m - \frac{\mu}{2}\sigma^{\mu\nu}G_{\mu\nu}\Big)\Psi(x) = 0, \qquad (9)$$

that can be regarded as the generalized Dirac-Pauli equation. The generalization of the neutrino quantum equation for the case when an electromagnetic field is present, in addition to the background matter, is obvious.

[2]For the recent studies of a massive neutrino electromagnetic properties, including discussion on the neutrino magnetic moment, see Ref. [7]

The detailed discussion on the evaluation of the tensor $G_{\mu\nu}$ is given in [1]. We consider here, for simplicity, the case of the unpolarized matter composed of the only one type of fermions of a constant density. For a background of only electrons we get

$$G^{\mu\nu} = \gamma\rho^{(1)} n \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\beta_3 & \beta_2 \\ 0 & \beta_3 & 0 & -\beta_1 \\ 0 & -\beta_2 & \beta_1 & 0 \end{pmatrix}, \tag{10}$$
$$\gamma = (1-\boldsymbol{\beta}^2)^{-1/2},$$
$$\rho^{(1)} = \frac{\tilde{G}_F}{2\sqrt{2}\mu}, \tilde{G}_F = G_F(1+4\sin^2\theta_W),$$

where $\boldsymbol{\beta} = (\beta_1, \beta_2, \beta_3)$ is the neutrino three-dimensional speed, n denotes the number density of the background electrons. From (10) and the two equations, (3) and (9), it is possible to see that the term $\gamma\rho^{(1)}n\boldsymbol{\beta}$ in Eq.(9) plays the role of the magnetic field $\mathbf{B}$ in Eq.(3). Therefore, the Hamiltonian form of (9) is

$$i\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \hat{H}_G \Psi(\mathbf{r},t), \tag{11}$$

where

$$\hat{H}_G = \hat{\boldsymbol{\alpha}}\mathbf{p} + \hat{\beta} m + \hat{V}_G, \tag{12}$$

and

$$\hat{V}_G = -\mu\frac{\rho^{(1)} n}{m}\hat{\beta}\boldsymbol{\Sigma}\mathbf{p}, \tag{13}$$

here $\mathbf{p}$ is the neutrino momentum. From (13) it is just straightforward that the potential energy in matter depends on the neutrino helicity.

The form of the Hamiltonian (12) ensures that the operators of the momentum, $\hat{\mathbf{p}}$, and helicity, $\boldsymbol{\Sigma}\mathbf{p}/p$, are integrals of motion. That is why for the stationary states we can write

$$\Psi(\mathbf{r},t) = e^{-i(Et-\mathbf{pr})}u(\mathbf{p},E), \quad u(\mathbf{p},E) = \begin{pmatrix} \varphi \\ \chi \end{pmatrix}, \tag{14}$$

where $u(\mathbf{p},E)$ is independent on the coordinates and time and can be expressed in terms of the two-component spinors φ and χ. Substituting (14) into Eq.(11), we get the two equations

$$(\boldsymbol{\sigma}\mathbf{p})\chi - \big(E - m + \alpha(\boldsymbol{\sigma}\mathbf{p})\big)\varphi = 0, \tag{15}$$
$$(\boldsymbol{\sigma}\mathbf{p})\varphi - \big(E - m + \alpha(\boldsymbol{\sigma}\mathbf{p})\big)\chi = 0.$$

Suppose that φ and χ satisfy the following equations,

$$(\boldsymbol{\sigma}\mathbf{p})\varphi = sp\varphi, \quad (\boldsymbol{\sigma}\mathbf{p})\chi = sp\chi, \tag{16}$$

where $s = \pm 1$ specify the two neutrino helicity states. Upon the condition that the set of Eqs.(15) has a non-trivial solution, we arrive to the energy spectrum of a neutrino moving in the background matter:

$$E = \sqrt{\mathbf{p}^2(1+\alpha^2) + m^2 - 2\alpha mps}, \qquad \alpha = \frac{\mu\rho^{(1)}}{m}n = \frac{1}{2\sqrt{2}}\tilde{G}_F\frac{n}{m}. \tag{17}$$

It is important that the the neutrino energy in the background matter depends on the state of the neutrino longitudinal polarization (helicity), i.e. the left-handed and right-handed neutrinos with equal momentum have different energies.

The obtained expression (17) for the neutrino energy can be transformed to the form

$$E = \sqrt{\mathbf{p}^2 + m^2\Big(1 - s\frac{\alpha p}{m}\Big)^2}. \tag{18}$$

It is easy to see that the energy spectrum of a neutrino in vacuum, which is derived on the basis of the Dirac equation, is modified in the presence of matter by the formal shift of the neutrino mass

$$m \to m\Big(1 - s\frac{\alpha p}{m}\Big). \tag{19}$$

The procedure, similar to one used for the derivation of the solution of the Dirac equation in vacuum, can be adopted for the case of the neutrino moving in matter. We apply this procedure to the equation (11) and arrive to the final form of the wave function of a neutrino moving in the background matter:

$$\Psi_{\mathbf{p},s}(\mathbf{r},t) = \frac{e^{-i(Et-\mathbf{p}\mathbf{r})}}{2L^{\frac{3}{2}}}\begin{pmatrix} \sqrt{1+\frac{m-s\alpha p}{E}}\ \sqrt{1+s\frac{p_3}{p}} \\ s\sqrt{1+\frac{m-s\alpha p}{E}}\ \sqrt{1-s\frac{p_3}{p}}\ e^{i\delta} \\ s\sqrt{1-\frac{m-s\alpha p}{E}}\ \sqrt{1+s\frac{p_3}{p}} \\ \sqrt{1-\frac{m-s\alpha p}{E}}\ \sqrt{1-s\frac{p_3}{p}}\ e^{i\delta} \end{pmatrix}, \tag{20}$$

where L is the normalization length and $\delta = \arctan p_y/p_x$. In the limit of vanishing density of matter, when $\alpha \to 0$, the wave function of Eq.(20) transforms to the vacuum solution of the Dirac equation.

The proposed new quantum equation (9) for a neutrino moving in the background matter and the obtained exact solutions (20) establish a basis for a new method in the study of different processes with participation of neutrinos in the presence of matter. As an example, we should like to use the new method in the study of the spin light of neutrino ($SL\nu$) in matter and to develop the *quantum* theory of this effect. Within the quantum approach, the corresponding Feynman diagram of the $SL\nu$ in matter is the standard one-photon emission diagram with the initial and final neutrino states described by the "broad lines" that account for the neutrino interaction with matter. From the usual neutrino magnetic moment interaction, it follows that the amplitude of the transition from the neutrino initial state ψ_i to the final state ψ_f, accompanied by the emission of a photon with a momentum $k^\mu = (\omega, \mathbf{k})$ and a polarization $\mathbf{e}^*$, can be written in the form

$$S_{fi} = -\mu\sqrt{4\pi} \int d^4x \bar{\psi}_f(x)(\hat{\mathbf{\Gamma}}\mathbf{e}^*)\frac{e^{ikx}}{\sqrt{2\omega L^3}}\psi_i(x), \tag{21}$$

where ψ_i and ψ_f are the corresponding exact solutions of the equation (9) given by (20), and

$$\hat{\mathbf{\Gamma}} = i\omega\big\{\gamma^0\big[\mathbf{\Sigma}\times\boldsymbol{\varkappa}\big] + i\gamma^0\gamma^5\mathbf{\Sigma}\big\}. \tag{22}$$

Here $\boldsymbol{\varkappa} = \frac{\mathbf{k}}{\omega}$ is the unit vector pointing in the direction of the emitted photon propagation.

The integration in (23) with respect to time yields

$$S_{fi} = -\mu\sqrt{\frac{2\pi}{\omega L^3}}2\pi\delta(E_f - E_i + \omega)\int d^3x \bar{\psi}_f(\mathbf{r})(\hat{\mathbf{\Gamma}}\mathbf{e}^*)e^{i\mathbf{kr}}\psi_i(\mathbf{r}), \tag{23}$$

where the delta-function stands for the energy conservation. Performing the integrations over the spatial co-ordinates, we can recover the delta-functions for the three components of the momentum. Finally, we get the law of energy-momentum conservation for the considered process,

$$E_i = E_f + \omega, \quad \mathbf{p}_i = \mathbf{p}_f + \boldsymbol{\varkappa}. \tag{24}$$

Let us suppose that the weak interaction of the neutrino with the electrons of the background is indeed weak. Here we should like to note that Eq.(9) is derived under the assumption that the matter term is small. This condition is similar to the condition of smallness of the electromagnetic term in the Dirac-Pauli equation (3) in the electrodynamics. In this case, we can expand the energy (18) over $\alpha\frac{pm}{E_0^2} \ll 1$ and in the liner approximation get

$$E \approx E_0 - sm\alpha\frac{p}{E_0}, \tag{25}$$

where $E_0 = \sqrt{p^2+m^2}$. Then from the law of the energy conservation (24) we get for the energy of the emitted photon

$$\omega = E_{i_0} - E_{f_0} + \Delta, \;\; \Delta = \alpha m \frac{p}{E_0}(s_f - s_i), \tag{26}$$

where the indexes i and f label the corresponding quantities of the neutrino initial and final states. From Eqs.(26) and the law of the momentum conservation, in the linear approximation over α, we obtain

$$\omega = (s_f - s_i)\alpha m \frac{\beta}{1-\beta\cos\theta}, \tag{27}$$

where θ is the angle between $\varkappa$ and the direction of the neutrino speed $\boldsymbol{\beta}$.

From the above consideration it follows that the only possibility for the $SL\nu$ to appear is provided in the case when the neutrino initial and final states are characterized by $s_i = -1$ and $s_f = +1$, respectively. Thus we conclude, on the basis of the quantum treatment of the $SL\nu$ in matter, that in this process the left-handed neutrino is converted to the right-handed neutrino (see also [3]) and the emitted photon energy is given by

$$\omega = \frac{1}{\sqrt{2}}\tilde{G}_F n \frac{\beta}{1-\beta\cos\theta}. \tag{28}$$

Note that the photon energy depends on the angle θ and also on the value of the neutrino speed β. In the case of $\beta \approx 1$ and $\theta \to 0$ we confirm the estimation for the emitted photon energy given in [3].

Now let us derive the $SL\nu$ rate and radiation power using the quantum theory. In the case of a neutrino moving along the OZ-axes, the solution (20) for the states with $s=-1$ and $s=+1$ can be rewritten in the form,

$$\Psi_{\mathbf{p},s=-1}(\mathbf{r},t) = \frac{e^{-i(Et-\mathbf{pr})}}{\sqrt{2}L^{\frac{3}{2}}} \begin{pmatrix} 0 \\ -\sqrt{1+\frac{m+\alpha p}{E}} \\ 0 \\ \sqrt{1-\frac{m+\alpha p}{E}} \end{pmatrix}, \tag{29}$$

and

$$\Psi_{\mathbf{p},s=+1}(\mathbf{r},t) = \frac{e^{-i(Et-\mathbf{pr})}}{\sqrt{2}L^{\frac{3}{2}}} \begin{pmatrix} \sqrt{1+\frac{m-\alpha p}{E}} \\ 0 \\ \sqrt{1-\frac{m-\alpha p}{E}} \\ 0 \end{pmatrix}. \tag{30}$$

We now put these wave functions into Eq.(23) and calculate the spin light transition rate in the linear approximation over the parameter $\alpha\frac{pm}{E_0^2}$. Finally, for the rate we get

$$\Gamma_{SL} = 8\mu^5 (n\rho^{(1)}\beta)^3 \int \frac{S\sin\theta}{(1-\beta\cos\theta)^4} d\theta, \tag{31}$$

where

$$S = (\cos\theta - \beta)^2 + (1-\beta\cos\theta)^2. \tag{32}$$

The corresponding expression for the radiation power is

$$I_{SL} = 16\mu^6 (n\rho^{(1)}\beta)^4 \int \frac{S\sin\theta}{(1-\beta\cos\theta)^5} d\theta. \tag{33}$$

Performing the integrations in Eq.(31) over the angle θ, we obtain for the rate

$$\Gamma_{SL} = \frac{2\sqrt{2}}{3}\mu^2 \tilde{G}_F^3 n^3 \beta^3 \gamma^2. \tag{34}$$

This result exceeds the value of the neutrino spin light rate derived in [3] by a factor of two because here the neutrinos in the initial state are totally left-handed polarized, whereas in [3] the case of initially unpolarized neutrinos (i.e., an equal mixture of the left- and right-handed neutrinos) is considered. From Eq.(33) we get for the total radiation power,

$$I_{SL} = \frac{2}{3}\mu^2 \tilde{G}_F^4 n^4 \beta^4 \gamma^4. \tag{35}$$

Using Eq.(23) we can also derive the $SL\nu$ rate and total power in matter accounting for the photon polarization. If $\mathbf{j}$ is the unit vector pointing in the direction of the neutrino propagation, then we can introduce the to vectors

$$\mathbf{e}_1 = \frac{[\varkappa \times \mathbf{j}]}{\sqrt{1-(\varkappa\mathbf{j})^2}}, \quad \mathbf{e}_2 = \frac{\varkappa(\varkappa\mathbf{j}) - \mathbf{j}}{\sqrt{1-(\varkappa\mathbf{j})^2}}, \tag{36}$$

which specify the two different linear polarizations of the emitted photon. For these vectors it is easy to get

$$\mathbf{e}_1 = \{\sin\phi, -\cos\phi\}, \quad \mathbf{e}_2 = \{\cos\phi\cos\theta, \sin\phi\cos\theta, -\sin\theta\}. \tag{37}$$

Note that the vector $\mathbf{e}_1$ is orthogonal to $\mathbf{j}$. Decomposing the neutrino transition amplitude (23) in contributions from the two linearly polarized photons, one can obtain the power of the process with radiation of the polarized photons. For the two linear polarizations determined by the vectors $\mathbf{e}_1$ and $\mathbf{e}_2$, we get

$$I_{SL}^{(1),(2)} = 16\mu^6(n\rho^{(1)}\beta)^4 \int \frac{\sin\theta}{(1-\beta\cos\theta)^5}\begin{pmatrix} S^{(1)} \\ S^{(2)} \end{pmatrix} d\theta, \tag{38}$$

where

$$S^{(1)} = (\cos\theta - \beta)^2, \quad S^{(2)} = (1-\beta\cos\theta)^2. \tag{39}$$

Finally, performing the integration over the angle θ we get the power of the radiation of the linearly polarized photons

$$\begin{pmatrix} I_{SL}^{(1)} \\ I_{SL}^{(2)} \end{pmatrix} = \begin{pmatrix} \frac{1}{3} \\ 1 \end{pmatrix} \frac{1}{2}\mu^2\tilde{G}_F^4 n^4\beta^4\gamma^4. \tag{40}$$

It is also possible to decompose the radiation power for the circular polarized photons. We introduce the two unit vectors for description of the photons with the two opposite circular polarizations,

$$\mathbf{e}_l = \frac{1}{\sqrt{2}}(\mathbf{e}_1 \pm i\mathbf{e}_2) \tag{41}$$

where $l = \pm 1$ correspond to the right and left photon circular polarizations, respectively. Then for the power of the radiation of the circular-polarized photons we get

$$I_{SL}^{(l)} = 16\mu^6(n\rho^{(1)}\beta)^4 \int \frac{\sin\theta}{(1-\beta\cos\theta)^5} S^{(l)} d\theta, \tag{42}$$

where

$$S^{(l)} = \frac{1}{2}(S^{(1)} + S^{(2)}) - l\sqrt{S^{(1)}S^{(2)}}. \tag{43}$$

Integration over the angle θ in (42) yields

$$I_{SL}^{(l)} = \frac{1}{3}\mu^2\tilde{G}_F^4 n^4\beta^4\gamma^4\Big(1 - \frac{1}{2}l\beta\Big). \tag{44}$$

In conclusion, we have shown how the Dirac-Pauli equation for a massive neutrino in an external electromagnetic field can be modified in order to account for the effect of the neutrino-matter interaction. On the basis of the new equation the quantum treatment of a neutrino moving in the presence of the background matter has been realized. In the limit of the constant density of matter, we have obtained the exact solutions of this new equation for different neutrino helicity states and also determined the neutrino energy spectrum, which depends on the helicity. Then we have developed the quantum theory of the $SL\nu$ in matter and calculated its rate and power accounting for the emitted photons polarization. We have also confirmed, within the solid base of the developed quantum approach, the existence of the neutrino-self polarization effect [3, 5] in the process of the spin light radiation of a neutrino moving in the background matter. The $SL\nu$ radiation and the corresponding neutrino self-polarization effect, due to the significant dependence on the matter density, are expected to be important in different astrophysical dense media and in the early Universe.

One of the authors (A.S.) is thankful to Victor Matveev and Valery Rubakov for the invitation to participate in the 13th International Seminar on High Energy Physics, and also to all of the organizers for their hospitality in Pushkinskie Gory.

References

[1] A.Egorov, A.Lobanov, A.Studenikin, Phys.Lett. B491 (2000) 137; A.Lobanov, A.Studenikin, Phys.Lett.B515 (2001) 94; M.Dvornikov, A.Studenikin, JHEP 09 (2002) 016; A.Studenikin, Phys.Atom.Nucl. 67 (2004) 993, hep-ph/0306280, hep-ph/0407010.

[2] V.Bargmann, L.Michel, V.Telegdi, Phys.Rev.Lett. 2 (1959) 435.

[3] A.Lobanov, A.Studenikin, Phys.Lett.B564 (2003) 27.

[4] M.Dvornikov, A.Grigoriev, A.Studenikin, Int.J.Mod.Phys.D (2004), in press, hep-ph/0406114.

[5] A.Lobanov, A.Studenikin, Phys.Lett.B (2004), in press, hep-ph/0408026.

[6] A.Borisov, V.Zhukovsky, A.Ternov, Sov.Phys.J. 31 (1988) 228.

[7] M.Dvornikov, A.Studenikin, Phys.Rev.D 69 (2004) 073001; JETP 99 (2004) 254.