Hierarchical fermionic mass pattern, LED and family number conservation

J.-M. Frère[1], M.V. Libanov[1,2],
E.Ya. Nugaev[2], and S.V Troitsky[2]

[1] Service de Physique Théorique,
CP 225, Université Libre de Bruxelles,
B–1050, Brussels, Belgium;
[2] Institute for Nuclear Research
of the Russian Academy of Sciences,
60th October Anniversary Pr. 7a,
Moscow 117312 Russia

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Abstract

In 5+1 dimensions, we consider a vortex-like solution on a two-dimensional sphere. We study fermionic zero modes in the background of this solution and relate them to the replication of fermion families in the Standard Model. In particular, using a compactified space removes the need for the difficult localisation of gauge fields. We calculate probabilities of flavour violating processes mediated by Kaluza-Klein modes of gauge bosons in a model where three generations of the Standard Model fermions arise from a single generation in (5+1) dimensions. We discuss a distinctive feature of the model: while the processes in which the generation number G changes are strongly suppressed, the model is constrained by those with $\Delta G = 0$, for instance $K \to \mu^{\pm} e^{\mp}$. We conjecture that the overall structure is generic, and work out possible signatures at colliders, compatible with rare decays data.

1 Introduction

The interest to models with more than three spatial dimensions was revived after the work [1] (see Ref. [2] for a review of theories with large extra dimensions (LED)), where elegant solution of gauge hierarchy problem have been supposed. Later, another hierarchy problem, the problem of hierarchical fermionic mass pattern of the Standard Model (SM) was considered in the model with one extra dimension [3]. However, model with two extra dimensions, which was suggested in [4, 5] and studied in [6] is more interesting. The feature of this model is the reduction of fermionic sector to one generation, i.e. one spinor field of theory in $(5+1)$ dimensions corresponds to three chiral fermions of effective theory (SM) in $(3+1)$ dimensions. Due to this fact number of free parameters of the model can be significantly reduced. Different generations of effective low energy theory correspond to different value of the angular momentum in transverse dimensions. In the presence of mixing terms this rotational symmetry is slightly broken.

To incorporate into the model gauge fields we considered theory on manifold with compact extra dimensions [8]. Size of compact extra dimensions, R, can be considered as a scale of gauge fields localisation (possible methods of gauge fields localisation are discussed in [7]). The unification of different generations of the SM fermions in single generation results to processes with flavour violation, which are mediated by Kaluza-Klein (KK) exitations of ordinary gauge fields of SM. The mass of the first flavour violating exitation of, for example Z-boson, Z', is proportional to $1/R$, so, a consideration of rare decays gives us a limit on the size of gauge fields localisation. Approximate symmetry in $(5+1)$ dimensions results to additional suppression of flavour changing neutral currents (FCNC) with changing number of generation, $\Delta G \neq 0$. As was shown in [9], the strongest bound is arised from the branching of the decay $K_L \to \mu e$. The possible signatures of a new FCNC bosons (Z', γ') for collider physics were discussed in [10]. In this talk we summarize our results.

2 Origin of hierarchy in quark mass matrices

In what follows we restrict ourselves to the model of Ref. [5]. In this section we give a brief description of this construction. Our notations coincide with those used in Refs. [4, 5]. In particular, six-dimensional coordinates x_A are labeled by capital Latin indices $A, B = 0, \dots, 5$. Four–dimensional coordinates x_μ are labeled by Greek indices $\mu, \nu = 0, \dots, 3$. The Minkowski metric is $\eta_{AB} = \mathrm{diag}(+, -, \dots, -)$. We use also chiral representation for six-dimensional Dirac Γ-matrices (see Ref. [4]). Beside this we introduce polar coordinates r, and θ in the x_4, x_5 plane. Effective $(3+1)$ theory arises after integration on extra dimensions.

Localization of fermions of the SM is based on localization of fermion zero modes on a two-dimensional vortex with winding number n. In particular, in the paper [4] the case of global vortex (i.e. group of symmetry of the vortex $U_g(1)$ is global) with winding number $n = 3$ was considered. In this setup three localized fermionic zero modes appear due to specific Yukawa coupling to the vortex scalar Φ. Coupling of fermions to the SM Higgs doublet H results in four-dimensional effective fermion masses. Intergeneration mixings occur due to explicit breaking of $U_g(1)$ symmetry. The latter point is the main drawback of this model since it does not allow to gauge $U_g(1)$ symmetry. In the paper [5] this problem was overcome. The price for unbroken $U_g(1)$, however, is the necessity to invoke higher dimension operators in the scalar-fermion interactions[1]. More precisely, to obtain n fermionic generations we consider the vortex with winding number 1 but fermions coupled to the vortex scalar raised to the n-th power. In both cases (global and gauged $U_g(1)$ symmetry) the hierarchical pattern of the fermion masses occurs due to different profiles of the fermionic wave functions in the transverse extra dimensions. In the core of the vortex, where field Φ is negligible, the Dirac equation for zero modes is equivalent to Laplace equation. Holomorphic solutions of the last have structure

$$(re^{\mathrm{i}\theta})^{\mathrm{k}}, \tag{1}$$

where $k = 0, 1, 2....$ So, if classical background field $H = H(r)$ has step-like profile in extra dimensions, overlaps of wave functions has natural hierarchical structure. The interesting classical solution for system of vortex fields and H was obtained numerically. With one more scalar field (see Ref. [5] for details) mixing terms for quark mass matrices can be obtained. In the both cases of compact and infinite extra dimensions the result for quark mass matrices is the following

$$M_D \propto \begin{pmatrix} \delta^4 & \epsilon_d\delta^3 & 0 \\ 0 & \delta^2 & \epsilon_d\delta \\ 0 & 0 & 1 \end{pmatrix}, \qquad M_U \propto \begin{pmatrix} \delta^4 & 0 & 0 \\ \epsilon_u\delta^3 & \delta^2 & 0 \\ 0 & \epsilon_u\delta & 1 \end{pmatrix}, \tag{2}$$

where ϵ_u, ϵ_d are small (complex) parameters of mixing and $\delta \sim 1$. After diagonalization we reproduced quark masses and appropriate expression for Cabibbo-Kobayashi-Maskawa (CKM) matrix.

3 Incorporation of the model with gauge fields

One of the principal issues of models with LED is the localization of the Standard Model gauge fields. One of possible ways to avoid this problem is to consider the transverse extra dimensional space as a compact manifold and to allow gauge fields to propagate freely in the extra dimensions. It is however not a straightforward task to put a vortex and fermionic zero modes in a finite volume space. Indeed, in a flat space with a boundary, "stray" fermionic zero modes of opposite chirality appear in addition to the chiral zero modes localized in the core. This happens because of the finite volume of the bulk: the modes which would have been killed by the normalization

[1]The requirement of renormalizability of the theory does not make much sense anyway in the six-dimensional models, since even usual Yukawa scalar-fermion-fermion coupling is non-renormalizable.

condition in the case of infinite space, survive in a finite volume. Even imposing the physical boundary condition which corresponds to the absence of the fermionic current outside the boundary does not help since the current of the zero modes lack radial components.

In the paper [8] we consider the vortex and the fermions taking a two dimensional sphere as extra dimensions. Keeping in mind the application of this system to the model with LED we will refer to the coordinates on the sphere as to the fourth and the fifth coordinates on this $M^4 \times S^2$ manifold, where M^4 represents our four-dimensional Minkowski space. Our results, however, do not depend on the number of ordinary dimensions. The main features of the "flat" models intact in the spherical case. As a consequence, expression (2) for quarks mass matrices is still alive.

KK exitations of gauge fields are eigen functions of Laplace operator on S^2 (interactions with fermions and Higgs field are considered on perturbation theory). It is worth to mention that only modes with nontrivial angular dependence result to flavour violation (angular dependence of fermions of different generations is described by Eq.(1)). Zero modes of gauge fields are constants and correspond to usual bosons of SM and they do not violate flavour.

Interaction of the zero modes of the fermions with the KK modes of the gauge field with orbital moment l is obtained by integration over S^2. At large enough l gauge field excitation several times oscillates on the size of the fermionic zero modes and as a result overlap integral with smooth function of fermionic zero mode is suppressed exponentially. At $l \sim 1$ couplings in effective $(3+1)$ theory are similar to couplings between usual gauge bosons and fermions of the SM. Additionally, one should note that without account of inter-generation mixings, the generation number G is exactly conserved. Indeed, the integration over θ in the Lagrangian results in the corresponding selection rules: no vector boson has both diagonal and off-diagonal couplings simultaneously. This forbids all processes with nonzero change of G; the probabilities of the latters in the full theory are thus suppressed by powers of the mass-matrix mixing parameter, $(\epsilon\alpha)^{\Delta G}$ (α is determined in [9], the largest $\alpha \sim \delta$). However, the amplitudes of processes with $\Delta G = 0$ but lepton and quark flavours violated separately are suppressed only by the mass squared of the Kaluza-Klein modes. The best studied among these processes are kaon decays $K_L^0 \to \mu e$ and $K^+ \to \pi^+ e^- \mu^+$, forbidden in the Standard Model with massless neutrinos.

4 Signatures for flavour physics

The strongest restriction on FCNC currents usually arises from consideration of additional contributions to neutral kaon mass difference or parameter of CP-violation ϵ_K. However, both of this processes has $\Delta G = 2$, so, they are additionally suppressed and one have to sistematically consider all processes with flavour violation.

Angular excitation of, for example, the first KK mode of Z-boson behaves in six dimensions as

$$Z' \sim e^{\pm i\theta}.$$

After integration in extra dimensions we obtain an effective four-dimensional Lagrangian with complex vector field Z', which generates "horizontal" transitions between families in which the generation number changes by one unit.

Such transitions are of course severely limited by the high mass of the excitations, but also, in the first approximation (neglecting of the K-M mixing), they do conserve the family number. For instance, the following processes are possible:

$$s + \overline{d} \Rightarrow Z' \Rightarrow s + \overline{d},$$
$$s + \overline{d} \Rightarrow Z' \Rightarrow \mu + \overline{e},$$
$$s + \overline{d} \Rightarrow Z' \Rightarrow \tau + \overline{\mu}$$

The first process in the first order in Z' exchange thus conserves strangeness (and only small corrections linked to Cabibbo mixing would affect this), but the second, while conserving "family number", is a typical FCNC interaction, violating both strangeness and electron number. While the last reaction is only possible in collisions, the study of rare K_L decay puts the strongest restriction on the mass and coupling constant of the Z' [9] (similar relations hold for the photon and gluon angular excitations). Discovery of this decay without signs of rare processes which violate the generation number (such as μe-conversion) would support significantly the models discussed here. Future experiments on the search of lepton-flavour violating kaon decays are thus of great importance (see Ref. [11] for relevant discussion).

We conjecture that the same structure would remain intact in other implementations. In Ref. [9] we supposed that the wave functions of fermions and the first KK mode of the gauge boson overlap strongly. Then the effective Lagrangian for the interaction between fermions and flavour-changing bosons contains the same coupling constant, as interaction with the lowest KK-modes, i.e. the usual gauge bosons. However, in particular models the profiles of fermionic wave functions can be shifted, which means more freedom in couplings. Let us denote the absolute value of the overlap integral in extra dimensions between the wave functions ψ_i, ψ_j of the fermions of generations i, j and the wave function $\psi_{Z'}$ of the Z excitation as

$$\left| \int \psi_{Z'} \psi_i \psi_j d^2x \right| = \kappa_{ij}.$$

Then $(\bar{e}, \mu)$-interaction through Z' is described by:

$$\frac{g_{EW} \kappa_{12}}{2 \cos\theta_W} Z'_\mu \left[\frac{1}{2} \bar{e} \gamma_\mu \gamma_5 \mu - \left(\frac{1}{2} - 2 \sin^2\theta_W \right) \bar{e} \gamma_\mu \gamma_5 \mu \right].$$

The structure of this term coincides with the interaction of $\bar{e}, e$ and Z in SM with the strength $g = g_{EW}\kappa_{12}$. Interactions of other leptons and quarks arise in a similar way.

The main restriction on the mass scale of the model with $\kappa_{ij} \simeq \delta_{i,i+1}$ arises from the limit on the branching ratio for the process $K_L \to \bar{\mu}e$. Taking into account that κ_{12} can be different from 1, the strongest restriction from the rare processes gives [9]

$$M_{Z'} \gtrsim \kappa_{12} \cdot 100\text{TeV}.$$

In the simplest case when all $\kappa_{ij} \sim \kappa \cdot \delta_{i,i+1}$,

$$\kappa \lesssim \frac{M_{Z'}}{100\text{TeV}}.$$

The decay width of the excited Z and photon results mainly from their decay into fermions (with the possibility of model-dependent additional scalar decay channels), and, by simple counting of modes, is estimated as

$$\Gamma(Z') = \kappa^2 \cdot \frac{M_{Z'}}{M_Z} \cdot 12.5 \cdot \Gamma_{Z\to\overline{\nu}\nu} \cong \kappa^2 \cdot \frac{M_{Z'}}{M_Z} \cdot 1.8\text{GeV}.$$

Similarly, the width of the first photon angular excitation is given by

$$\Gamma(\gamma') = \frac{16}{3}\kappa^2 \sin^2 2\theta_W \cdot \frac{M_{\gamma'}}{M_Z} \cdot \Gamma_{Z\to\bar{\nu}\nu} \cong \kappa^2 \cdot \frac{M_{\gamma'}}{M_Z} \cdot 1.3\text{GeV}.$$

The first KK excitation of the gluon is wider due to the larger coupling constant,

$$\Gamma(G') \cong \kappa^2 \frac{M_{G'}}{M_Z} \cdot 7.2\text{GeV}.$$

A typical value of Γ is of order 10^{-3}GeV for $\kappa \simeq 10^{-2}$ and $M_{Z'} = 1$TeV. In what follows we will assume that the masses of all the FCNC bosons are equal:

$$M_{Z'} = M_{\gamma'} = M_{g'} = M,$$

as in the case of spherical model of Ref [9].

The vector bosons discussed here can, in principle, be observed at colliders due to the flavour-changing decay modes into (μe) and $(\tau\mu)$ pairs. The corresponding process is very similar to the Drell-Yan pair production. A typical feature of the latter is the suppression of the cross section with increasing of the resonance mass at a fixed center-of-mass energy.

The flavour-changing decays of this kind have a distinctive signature: antimuon and electron (or their antiparticles) with equal and large transverse momenta in the final state.

We estimate the number of events for the case of pp-collisions with the help of the CompHEP package [12]. For our calculation we use the expected

LHC value of 100fb^{-1} for luminosity and $\sqrt{s} = 14$TeV. The number of (μ^+e^-) events is presented at Fig.1 for different values of the vector bosons mass M and κ adjusted to $\kappa = M/(100\text{TeV})$. The same plot for (μ^-e^+) pairs is given at Fig.2.

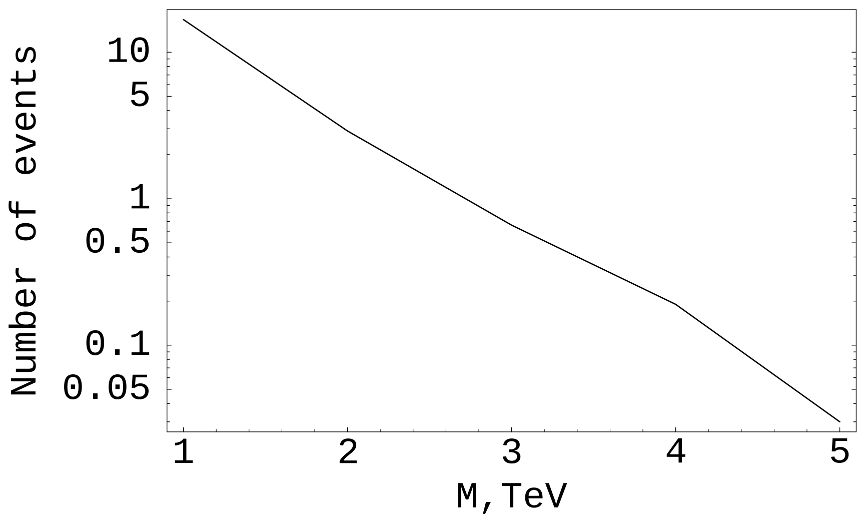

Figure 1: Fig. 1. Number of events for (μ^+e^-) pairs production as a function of the vector bosons mass M, with $\kappa = M/(100\text{TeV})$.

Note that production of (μ^+e^-) pairs is more probable than (μ^-e^+) because the former process can use valence u and d-quarks in the proton, while the second only involves partons from the sea. The same numbers are representative also for the $(\mu^-\tau^+)$ channel.

There are also other signatures of FCNC effects, in particular, with hadronic final states, when $(\bar{t}, c)$ or $(\bar{b}, s)$ jets are produced. The dominant contribution to these processes arises from the interactions with higher KK modes of gluons, which have large coupling constant. For the mass of $M_{G'} = 1$TeV we estimate the number of events as $N = 1.2 \cdot 10^3$. But potentially large SM backgrounds should be carefully considered for such channels.

5 Conclusions

In a class of multidimensional models with one vector-like fermionic family, the low-energy effective theory describes three chiral families in four dimensions. Hierarchy of fermionic masses appears due to different profiles of the fermionic wave functions in extra dimensions. We found the set of the parameters of the model which reproduces well all known fermion masses and mixings, without hierarchy among parameters.

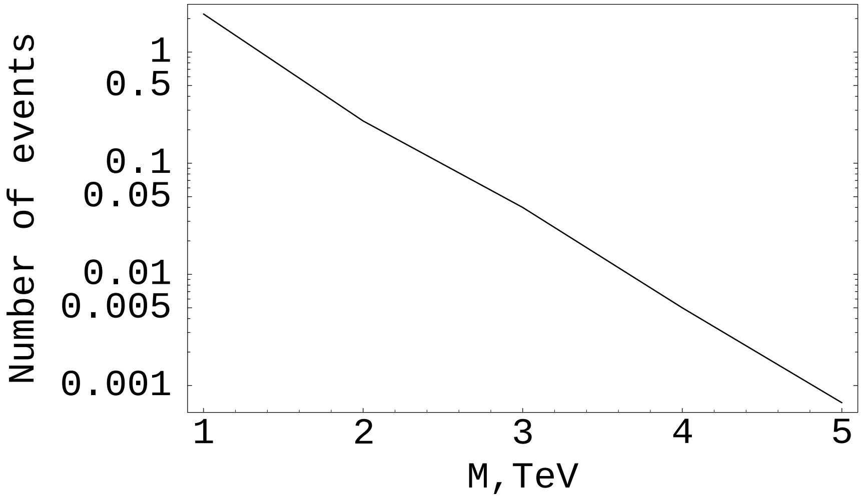

Figure 2: Fig. 2. Number of events for $(\mu^- e^+)$ pairs production as a function of the vector bosons mass M, with $\kappa = M/(100\text{TeV})$.

We have considered FCNC effects in models with approximate family-number conservation, mediated by the heavy vector bosons in a class of models. From our estimations, there is a reason for searching for such FCNC bosons with masses of order 1TeV at LHC. The main signature is the production of $(\mu^+ e^-)$ or $(\mu^- \tau^+)$ pairs with equal and large transverse momenta of leptons. Production of $(\bar{t}c)$ quarks is more probable, but less clear-cut due to the large background from SM processes.

On the other hand, the models with heavy vector bosons, whose interactions conserve the family number, can be tested in experiments studing rare processes. The strongest and the least model-independent limit on the mass of these bosons arises from the limit on $K_L \to \mu^\pm e^\mp$ branching ratio (in this process, the family number does not change). Discovery of this decay without signs of rare processes which violate the generation number (such as μe-conversion) would support significantly the models discussed here. Future experiments on the search of lepton-flavour violating kaon decays are thus of great importance (see Ref. [11] for relevant discussion).

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