A New Bound State $6t + 6\bar{t}$ and the Fundamental-Weak Scale Hierarchy in the Standard Model

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October 18, 2004

Abstract

The multiple point principle, according to which several vacuum states with the same energy density exist, is put forward as a fine-tuning mechanism predicting the exponentially huge ratio between the fundamental and weak scales in the Standard Model (SM). Using renormalisation group equations for the SM, we obtain the effective potential in the 2-loop approximation and investigate the existence of its postulated second minimum at the fundamental scale. A prediction is made of the existence of a new bound state of 6 top quarks and 6 anti-top quarks, formed due to Higgs boson exchanges between pairs of quarks/anti-quarks. This bound state is supposed to condense in a new phase of the SM vacuum. The existence of three vacuum states (new, weak and fundamental) solves the hierarchy problem in the SM.

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1. Cosmological Constant and Multiple Point Principle

In the present talk we suggest a scenario, using only the pure SM, in which an exponentially huge ratio between the fundamental (Planck) and electroweak scales results: $\frac{\mu_{fund}}{\mu_{ew}} \sim e^{40}$.

In such a scenario it is reasonable to assume the existence of a simple and elegant postulate which helps us to explain the SM parameters: couplings, masses and mixing angles. In our model such a postulate is based on a phenomenologically required result in cosmology: the cosmological constant is zero, or approximately zero, meaning that the vacuum energy density is very small. *A priori* it is quite possible for a quantum field theory to have several minima of its effective potential as a function of its scalar fields. Postulating zero cosmological constant, we are confronted with a question: is the energy density, or cosmological constant, equal to zero (or approximately zero) for all possible vacua or it is zero only for that vacuum in which we live?

This assumption would not be more complicated if we postulate that all the vacua which might exist in Nature, as minima of the effective potential, should have approximately zero cosmological constant. This postulate corresponds to what we call the Multiple Point Principle (MPP) [1].

The MPP postulates: *there are many vacua with the same energy density or cosmological constant, and all cosmological constants are zero, or approximately zero.*

In the present talk we want to use this principle to solve the hierarchy problem in the SM.

2. The renormalisation group equation for the effective potential

The renormalisation group (RG) improvement of the effective potential, which is a function of the scalar field ϕ obeys the Callan-Symanzik equation (see Refs.[2]):

$$(M\frac{\partial}{\partial M} + \beta_{m^2}\frac{\partial}{\partial m^2} + \beta_\lambda\frac{\partial}{\partial \lambda} + \beta_g\frac{\partial}{\partial g} + \gamma\,\phi\frac{\partial}{\partial \phi})V_{eff}(\phi) = 0. \qquad (1)$$

Here M is a renormalisation mass parameter, β_{m^2}, β_λ, β_g are the RG functions for mass, scalar field self-interaction and gauge couplings, respectively; γ is the anomalous dimension, g_i are gauge coupling constants: $g_i = (g', g, g_3)$ for $U(1)_{Y(hypercharge)}$, $SU(2)$ and $SU(3)$ groups of the SM.

From now on $h \stackrel{def}{=} g_t$ *is the top-quark Yukawa coupling constant. And we neglect all Yukawa couplings of light fermions.*

In the loop expansion of the V_{eff}:

$$V_{eff} = V^{(0)} + \sum_{n=1} V^{(n)}, \tag{2}$$

we have $V^{(0)}$ as a tree-level potential of the SM.

The breaking $SU(2)_L \times U(1)_Y \to U(1)_{em}$ is achieved in the SM by the Higgs mechanism, giving masses to the gauge bosons $W^{\pm}$, Z, the Higgs boson and the fermions.

With one Higgs doublet of $SU(2)_L$, we have the following tree–level Higgs potential:

$$V^{(0)} = -m^2 \Phi^+ \Phi + \frac{\lambda}{2} (\Phi^+ \Phi)^2. \tag{3}$$

The vacuum expectation value of the Higgs field Φ is:

$$< \Phi > = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \tag{4}$$

where

$$v = \sqrt{\frac{2m^2}{\lambda}} \approx 246 \text{ GeV}. \tag{5}$$

Introducing a four-component real field ϕ:

$$\Phi^+ \Phi = \frac{1}{2} \phi^2, \tag{6}$$

we have the following tree-level potential:

$$V^{(0)} = -\frac{1}{2} m^2 \phi^2 + \frac{1}{8} \lambda \phi^4. \tag{7}$$

The masses of the gauge bosons W and Z, a fermion with flavor f and the physical Higgs boson H are expressed in terms of the VEV parameter v:

$$\begin{aligned} M_W^2 &= \frac{1}{4} g^2 v^2, \quad M_Z^2 = \frac{1}{4} (g^2 + g'^2) v^2, \\ m_f &= \frac{1}{\sqrt{2}} h_f v, \quad M_H^2 = \lambda v^2, \end{aligned} \tag{8}$$

where h_f are the Yukawa couplings with the flavor f.

3. The second minimum of the effective potential in the 2-loop approximation

In our paper [3] we have calculated the 2–loop effective potential in the limit:

$$\phi^2 >> v^2, \qquad \phi^2 >> m^2, \tag{9}$$

using the SM renormalisation group equations in the 2-loop approximation given by Ref.[4]. We have obtained:

$$V_{eff}(2-loop) = (\frac{\lambda}{8} + At + Bt^2)\phi^4, \tag{10}$$

where $t = \log(\mu/M) = \log(\phi/M)$ is the evolution variable,

$$A = \frac{1}{8}(\beta_\lambda^{(1)} + \beta_\lambda^{(2)}) + \frac{\lambda}{2}(\gamma^{(1)} + \gamma^{(2)} + (\gamma^{(1)})^2) + \frac{1}{8}\gamma^{(1)}\beta_\lambda^{(1)}, \tag{11}$$

and

$$B = \frac{1}{4}\gamma^{(1)}(\beta_\lambda^{(1)} + 4\lambda\,\gamma^{(1)}) + \frac{3}{32\pi^2}\lambda\,\beta_\lambda^{(1)} + \frac{3}{256\pi^2}\beta_{g'}^{(1)}(g'^3 + g'g^2) + \frac{3}{256\pi^2}\beta_g^{(1)}(3g^3 + g'^2 g) - \frac{3}{16\pi^2}\beta_h^{(1)}h^3. \tag{12}$$

Assuming the existence of the two minima of the effective potential in the simple SM, we have taken the cosmological constants for both vacua equal to zero, in accord with the MPP.

Then we have the following illustrative qualitative picture:

Here the first minimum:

$$\phi_{min1} = v = 246 GeV \tag{13}$$

is the standard "Electroweak scale minimum", in which we live, and the second one is the non-standard "Fundamental scale minimum", if it exists.

4. The Multiple Point Principle requirements

The MPP requirements for the two degenerate minima in the SM are given by the following equations:

$$V_{eff}(\phi_{min1}) = V_{eff}(\phi_{min2}) = 0, \tag{14}$$

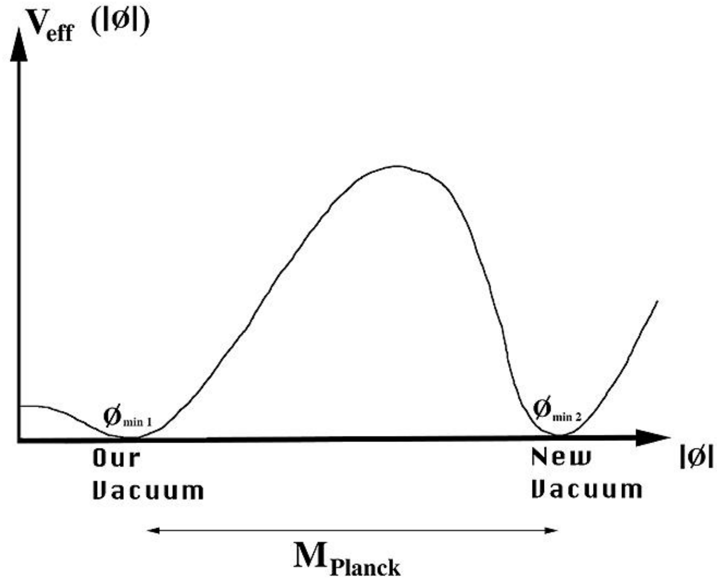

Figure 1:

$$V'_{eff}(\phi_{min1}) = V'_{eff}(\phi_{min2}) = 0, \tag{15}$$

$$V''_{eff}(\phi_{min1}) > 0, \qquad V''_{eff}(\phi_{min2}) > 0, \tag{16}$$

where

$$V'(\phi) = \frac{\partial V}{\partial \phi^2}, \qquad V''(\phi) = \frac{\partial^2 V}{\partial (\phi^2)^2}. \tag{17}$$

As was shown in Ref.[5], the degeneracy conditions of MPP give the following requirements for the existence of the second minimum in the limit $\phi^2 >> m^2$:

$$\lambda_{run}(\phi_{min2}) = 0, \tag{18}$$

and

$$\lambda'_{run}(\phi_{min2}) = 0, \tag{19}$$

what means:

$$\beta_\lambda(\phi_{min2}, \lambda = 0) = 0. \tag{20}$$

Using these requirements and the renormalisation group flow the authors of Ref.[5] computed quite precisely the top quark (pole) and Higgs boson masses:

$$M_t = 173 \pm 4 \; GeV \qquad \text{and} \qquad M_H = 135 \pm 9 \; GeV. \tag{21}$$

Let us consider now the searching for the fundamental scale given by these requirements.

5. The top-quark Yukawa coupling evolution and the second minimum of the effective potential

The position of the second minimum of the SM effective potential essentially depends on the running of gauge couplings and on the top-quark Yukawa coupling evolution.

Starting from the experimental results [6], we have:

$$M_t = 174.3 \pm 5.1 \text{ GeV}, \tag{22}$$

$$M_Z = 91.1872 \pm 0.0021 \,\text{GeV}, \tag{23}$$

and for QCD α_s we have:

$$\alpha_3(M_Z) \equiv \alpha_s(M_Z) = 0.117 \pm 0.002. \tag{24}$$

For the running top quark Yukawa coupling constant considered at the pole mass of t-quark M_t the experiment gives:

$$h(M_t) \approx 0.95 \pm 0.03. \tag{25}$$

Establishing the running of gauge couplings g', g, g_3, exactly $\alpha_Y(t)$, $\alpha_2(t)$ and $\alpha_3(t)$, in accord with the present experimental data [6], and using all experimental results with their uncertainties we have constructed the evolutions of the inverse top-quark Yukawa constant: $y(t) = \alpha_h^{-1}(t) = 4\pi h^{-2}(t)$ for different experimental uncertainties (see Fig.2).

Three bunches 1(middle), 2(up), 3(down) of curves correspond respectively to the three values of $h(M_t) = 0.95, 0.92, 0.98$ given by experiment. The spread of each bunch corresponds to the experimental values of $\alpha_3(M_Z) = 0.117 \pm 0.002$. (upper and lower curves correspond to $\alpha_3(M_Z) = 0.115$ and $\alpha_3(M_Z) = 0.119$ respectively).

The curve y1 for $y = \alpha_h^{-1}(t)$ was calculated from the requirement (20): $\beta_\lambda(\phi_{min2}, \lambda = 0) = 0$. The intersection of the curve y1 with the evolution of $\alpha_h^{-1}(t)$ for the experimentally established central values: $\alpha_s(M_Z) = 0.117$ and $h(M_t) = 0.95$ gives us the position of the second minimum of the SM effective potential at

$$\phi_{min2} \approx 10^{19} \ GeV. \tag{26}$$

In general, the experimental uncertainties lead to the following second minimum position interval:

$$\phi_{min2} \approx 10^{16} - 10^{22} \ GeV. \tag{27}$$

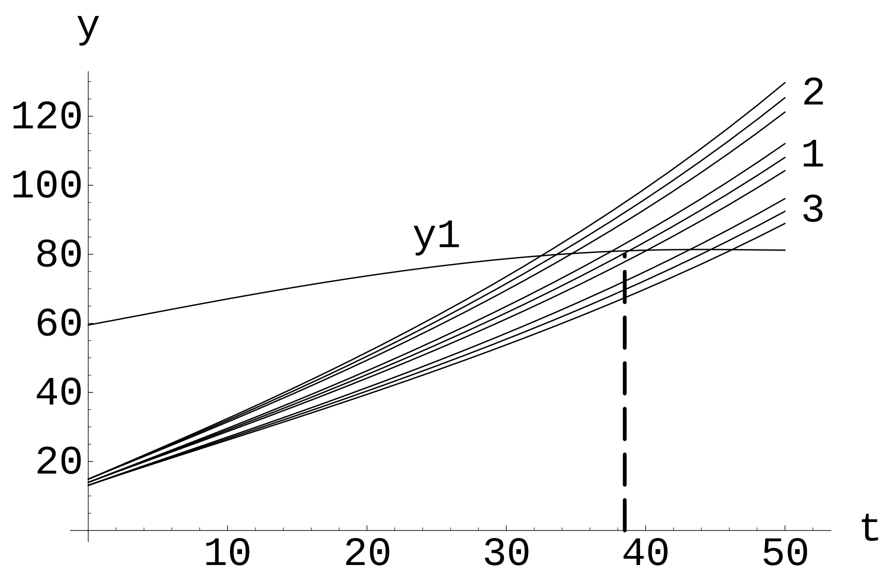

Figure 2:

Just this position of the second minimum with given uncertainties predicts the Froggatt-Nielsen's result [5]: $M_H = 135 \pm 9\ GeV$.

The shape of the second minimum at $\mu = 10^{19}$ GeV is described by the curve of Fig.3 where we have used the following designation:

$$V \stackrel{def}{=} \frac{(16\pi)^4}{24}(\phi_{min2}^{-4})V_{eff}, \tag{28}$$

In this scenario the new physics begins at the scale $\sim 10^{19}$ GeV.

6. A new bound state $6t + 6\bar{t}$, three phases in the SM and the hierarchy problem

The MPP is helpful in solving the fine-tuning problems, in particular, the problem of the electroweak scale being so tiny compared to the Planck scale.

As is well-known, the quadratic divergencies occur order by order in the square of the SM Higgs mass, requiring the bare Higgs mass squared to be fine-tuned again and again as the calculation proceeds order by order. If the cut-off reflects new physics entering near the Planck scale Λ_{Planck}, then these quadratic divergencies become about 10^{34} times bigger than the final mass

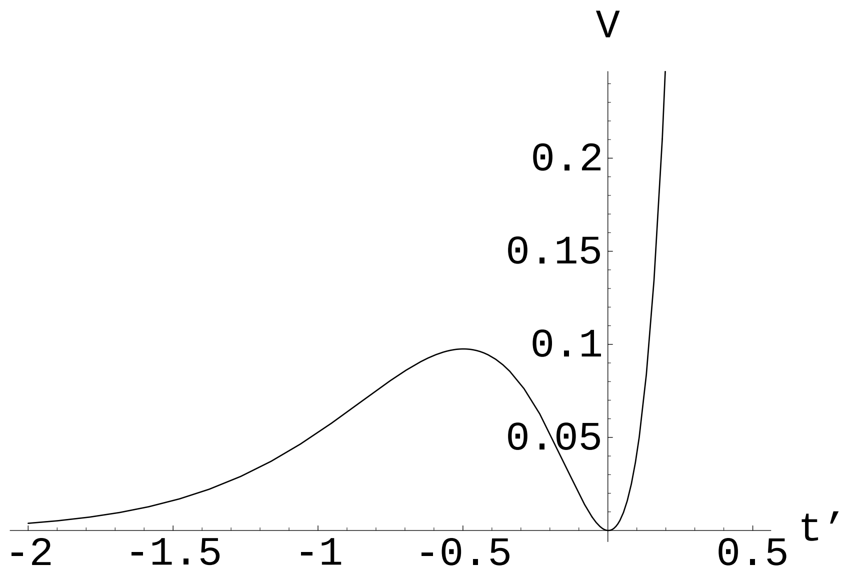

Figure 3:

squared of the Higgs particle:

$$(\frac{\Lambda_{Planck}}{\Lambda_{electroweak}})^2 \sim (10^{17})^2 = 10^{34}.$$

It is clear that an explanation for such a fine-tuning is quite needed.

Supersymmetry solves the technical hierarchy problem, removing the divergencies by having a cancellation between fermion and boson contributions. But the problem of origin of the huge scale ratio still remains. For example, it exists in the form why the soft supersymmetry breaking terms are small compared to the fundamental scale Λ_{Planck}.

At first sight, it looks difficult to get an explanation of the cancellation of the quadratic divergencies by fine-tuning, based on the MPP, which predicts the existence of vacua with degenerate energy densities. The difficulty is that, from dimensional arguments, the energy density, or cosmological constant, tends to become dominated by the very highest frequencies and wave numbers relevant the Planck scale in our case. In fact, the energy density has the dimension of energy to the fourth powers, so the modes with Planck scale frequencies contribute typically $(10^{17})^4 = 10^{68}$ times more than those at the electroweak scale.

Therefore, the only hope of having any sensitivity to electroweak scale physics is the existence of two degenerate phases in the SM, which are identical with respect to the modes higher than electroweak scale frequencies, but deviate by their physics at the electroweak scale. So, in order to solve the large scale ratio problem using our MPP we need to have a model with two different phases that only deviate by the physics at the electroweak scale.

What could that now be?

It is obvious that it is necessary to seek a condensation of any strongly bound states with a binding so strong, in fact, as to make this bound state tachyonic and to condense it into the vacuum.

As was shown in papers [7-9], such a bound state can be $6t + 6\bar{t}$. Here Higgs scalar particle exchange has an important special feature. Unlike the exchange of gauge particles, which lead to alternative signs of the interaction, many top-anti-top constituents put together lead to attraction in all cases due to the Higgs scalar boson exchange. This attraction of t and anti-t quarks by the Higgs exchange is independent of colour.

The bound state of a top quark and an anti-top quark (toponium) is mainly bound by gluon exchange which is comparable with the Higgs exchange. But if we now add more top or anti-top quarks, then the Higgs exchange continues to attract while the gluon exchange saturates and gets less significant. The maximal binding energy comes from S-wave $6t + 6\bar{t}$ ground state. The reason is that the t-quark has 2 spin states and 3 colour states. This means that by Pauli principle only 6 t-quarks can be put in an S-wave function, together with 6 anti-t-quarks. So, in total, we have 6 + 6 = 12 t-constituents together in relative S-waves.

If we try to put more t and $\bar{t}$ quarks together, then some of them will go into a P-wave and the pair binding energy ($E_{binding}$) will decrease by at least a factor of 4.

Calculating the pair binding energy using the Bohr formula for atomic energy levels (here t and 11t-nucleus), C.D.Froggatt and H.B.Nielsen [7,8] have obtained the following expression for the mass squared of the new bound state $6t + 6\bar{t}$:

$$m^2_{bound} \approx (12m_t)^2(1 - \frac{33}{8\pi^2}h^4 + ...), \qquad (29)$$

which gives the critical value of h at $m^2_{bound} = 0$: $h_{crit} \approx 1.24$. Taking into account a possible correction due to the Higgs field quantum fluctuations [9], we obtained the following result:

$$h_{crit} \approx 1.06 \pm 0.18, \qquad (30)$$

what is comparable with the experimental value of the top Yukawa coupling constant at the electroweak scale: $h_{exper}(M_t) \equiv g_{t,exper}(M_t) \approx 0.95 \pm 0.03$.

7. The fundamental-electroweak scale hierarchy in the SM

The requirement of the degeneracy of the three vacua (new, electroweak and fundamental) solves the hierarchy problem in the SM.

The central experimental values $h(M_t) = 0.95$ and $\alpha_3(M_Z) = 0.117$, together with the vacuum degeneracy conditions (18,20), predict a second minimum at $\phi_{min2} \approx 10^{19}$ GeV.

The existence of the second vacuum at $\phi_{min2} \approx 10^{19}$ GeV gives a huge ratio between the fundamental and electroweak scales:

$$\frac{\mu_{(fund)}}{\mu_{(ew)}} \sim 10^{17},$$

what leads to the prediction of an exponentially huge scale ratio:

$$\frac{\mu_{(fund)}}{\mu_{(ew)}} \sim e^{40},$$

in the absence of new physics between the electroweak and fundamental scales (with the exception of neutrinos).

8. Acknowledgements

The speaker (L.V.L.) thanks the financial support by Russian Foundation for Basic Research, project No.02-02-17379.

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