Seesaw, Susy and SO(10)

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Abstract

The seesaw mechanism can explain why the neutrino masses are so tiny with respect to the charged fermion masses. In the canonical version it is the presence of the right-handed neutrino that is responsible for it. In renormalizable grand unified theories with left-right gauge symmetry it is possible to show quite generically that there is another type of seesaw contribution, mediated by a heavy weak triplet. In this talk I will show that such non-canonical seesaw mechanism can very nicely connect $b-\tau$ Yukawa unification with the large atmospheric neutrino mixing angle in the context of a SO(10) grand unified theory. Also, a fit to the available low energy masses and mixings points towards the domination of this non-canonical contribution with respect to the canonical one. Finally I will explicitly present the minimal supersymmetric SO(10) model with all the above nice features.

1 Introduction

The charged and neutral fermion sectors in the SM have quite different structures: 1) though very different among themselves, the charged fermions have much larger masses than neutrinos, 2) the mixing angles in the quark sector tend to be much smaller than the corresponding ones in the leptonic (neutrino) sector. While the first point can be easily accounted with the famous seesaw mechanism [1], the second issue is more controversial.

The first nontrivial framework that could be predictive in both charged fermion and neutrino sectors is SO(10) grandunification. This is because it automatically incorporates the righthanded neutrino, thus providing for a theory of the seesaw mechanism. Most important, the model connects the neutrino and charged fermion mass matrices, providing some nontrivial relations between them at the grandunification scale. Then, with the assumption of the desert, motivated by gauge coupling unification [2], one can extrapolate the values of these masses and mixings down to low energies and here compare them with experimental data. I will describe in some details the origin and form of the neutrino mass matrix, and connect it to charged fermion masses in the context of a SO(10) grand unified theory.

2 Seesaw

The left-handed neutrino ν ($\equiv \nu_L$) is a component of a weak $\mathrm{SU}(2)_L$ doublet with $B-L=-1$. If left-right symmetry is assumed, than a right-handed neutrino ν^c ($\equiv C\bar{\nu}_R^T$) must exist as part of a doublet of $\mathrm{SU}(2)_R$ gauge symmetry, with $B-L=+1$. Since ν^c is a standard model singlet, its mass is given by the scale of $\mathrm{SU}(2)_R$ (and $B-L$) breaking, which can be achieved in a renormalizable version by the vev of a $\mathrm{SU}(2)_R$ triplet (with $B-L=-2$), $M_{\nu_R} \propto \langle \Delta_R \rangle$. This, Majorana mass term, must then be added to the Dirac mass $M_{\nu_D} \propto \langle \Phi \rangle$ term with Φ a $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ bidoublet:

$$L_m = -\nu^{cT} M_{\nu_R} \nu^c + \nu^{cT} M_{\nu_D} \nu + \text{ } h.c. \, . \tag{1}$$

After integrating out the heavy ν^c one gets the famous seesaw [1] formula for the light neutrino mass:

$$M_\nu = -M_{\nu_D}^T M_{\nu_R}^{-1} M_{\nu_D} \, , \tag{2}$$

which clearly connects the smallness of the neutrino mass with the largeness of the $\mathrm{SU}(2)_R$ breaking scale.

There is however another contribution to the seesaw [3], several times neglected, but as we will see, of great potential importance. It comes essentially from the following arguments: as we said, the large right-handed Majorana mass comes from the vev of a $\mathrm{SU}(2)_R$ triplet via the term $\nu^{cT}\Delta_R \nu^c$, so due to left-right symmetry an analog term $\nu^T \Delta_L \nu$ must exist, with Δ_L a triplet under $\mathrm{SU}(2)_L$. With these two interaction terms plus the usual Dirac term $\nu^{cT}\Phi\nu$ one can easily show that the one-loop box diagram contribution to $\Delta_R \Phi^2 \Delta_L$ is UV divergent, so that such a term must be present already at tree level. Thus, the potential for the triplets looks like

$$V = -M^2 \left(\Delta_R^2 + \Delta_L^2 \right) + \Delta_R \Phi \Delta_L \, , \tag{3}$$

with M a large mass (not much less than M_{GUT}). The last term represents a tadpole for the lefthanded triplet:

$$\langle \Delta_L \rangle \approx \frac{\langle \Delta_R \Phi^2 \rangle}{M^2} \approx \frac{M_W^2}{M} , \tag{4}$$

since $\langle \Delta_R \rangle \approx M$ and $\langle \Phi \rangle \approx M_W$. So the term

$$\nu^T \langle \Delta_L \rangle \nu \tag{5}$$

gives another contribution to the seesaw. All together we thus have

$$M_N = -M_{\nu_D}^T M_{\nu_R}^{-1} M_{\nu_D} + M_{\nu_L} , \tag{6}$$

where the first term represents the type I or canonical seesaw formula mediated by the $\mathrm{SU}(2)_L$ singlet ν^c, while the second term is the $\mathrm{SU}(2)_L$ triplet contribution to the type II or non-canonical see-saw formula.

Of course, in general the matrices in generation space M_{ν_L}, M_{ν_R} and M_{ν_D} are arbitrary, so the above formula is not very useful. It is thus important to connect the above matrices to the charged fermion sector, which is experimentally better known. For this one needs a framework. We will choose the most economical one, i.e. a SO(10) grand unified theory.

3 SO(10)

Although GUTs are not theories of flavours, they still put some constraints on the possible Yukawa interactions, since different SM fields live in the same representations. In SO(10) all the light fermions of each generation plus the right-handed neutrino are grouped together in the 16-dimensional spinorial representation, while in the most economical version the two light Higgs doublets live in a fundamental 10-dimensional complex Higgs representation. There is thus only one Yukawa matrix in generation space (3×3). A U(3) rotation of the 16's in generation space can diagonalize it, giving one good prediction ($y_b = y_\tau = y_t$) and several bad ones (equality of charged lepton, up and down quark Yukawas of the second and first generations, and no mixing at all). A nontrivial quark mixing could be achieved adding a new 10-dimensional Higgs, but that would not help in improving the relations among Yukawas of the first two generations. To see it, one can remember that the 10-dimensional Higgs gets a vev in the $(2, 2, 1)$ direction of the

Pati-Salam $SU(2)_L \times SU(2)_R \times SU(4)_C$ subgroup. This means that such a vev cannot break $SU(4)_C$ and thus the equality between leptons and quarks. Thus, a nontrivial multiplet in $SU(4)_C$ (and bidoublet in the left-right sector) is needed to get such a splitting. An ideal possibility is given by the Higgs in the representation $\overline{126}$ (5 index antisymmetric and anti-self-dual) of SO(10): its vev in the $SU(3)_C$ colour singlet state of the $(2, 2, 15)$ direction is a possible candidate. The Yukawa terms in the Lagrangian can thus be written as [4]

$$L_Y = 10_H 16^T Y_{10} 16 + \overline{126}_H 16^T Y_{126} 16 \ . \tag{7}$$

These terms can be decomposed in the usual Pati-Salam basis as

$$\begin{aligned}
10_H &= = (\mathbf{2}, \mathbf{2}, \mathbf{1}) + (1, 1, 6) \ , && (8)\\
16 &= (2, 1, 4) + (1, 2, \overline{4}) \ , && (9)\\
\overline{126}_H &= (\mathbf{1}, \mathbf{3}, \mathbf{10}) + (\mathbf{3}, \mathbf{1}, \overline{\mathbf{10}}) + (\mathbf{2}, \mathbf{2}, \mathbf{15}) + (1, 1, 6) \ , && (10)
\end{aligned}$$

where the fields in boldface have $SU(3)_C \times U(1)_{em}$ singlets and can thus generate a nonzero vev. The two colour singlet bidoublets are actually needed to develop a vev in order to fit the light fermion masses, as we have just seen. On top of that, the $SU(2)_R$ triplet $(1, 3, 10)$ above is ideally suited to give a large Majorana mass to the heavy right-handed neutrino ν^c from $(2, 1, \overline{4})$: from here we can see the double role that the 126-dimensional Higgs can play. Finally, the $SU(2)_L$ triplet in $(3, 1, \overline{10})$, if nonzero as conjectured in the previous section, can contribute to the type II see-saw formula.

Denoting $v_{10}^{u,d}$ and $v_{126}^{u,d}$ the vevs from the bidoublets in 10_H and $\overline{126}_H$ and $v_{L,R}$ the vevs from the L or R triplets, one gets from (7) the following expressions for the fermion masses:

$$\begin{aligned}
M_U &= v_{10}^u Y_{10} + v_{126}^u Y_{126} \ , && (11)\\
M_D &= v_{10}^d Y_{10} + v_{126}^d Y_{126} \ , && (12)\\
M_{\nu_D} &= v_{10}^u Y_{10} - 3 v_{126}^u Y_{126} \ , && (13)\\
M_E &= v_{10}^d Y_{10} - 3 v_{126}^d Y_{126} \ , && (14)\\
M_{\nu_R} &= \langle (1, 3, 10)_{\overline{126}} \rangle Y_{126} = v_R Y_{126} \ , && (15)\\
M_{\nu_L} &= \langle (3, 1, \overline{10})_{\overline{126}} \rangle Y_{126} = v_L Y_{126} \ . && (16)
\end{aligned}$$

Notice that the factor -3 from the $\overline{126}_H$ contribution in the lepton sector with respect to the quark one comes because the $SU(3)_C$ colour singlet direction in 15 of SU$(4)_C$ is proportional to $B - L \propto diag(1,1,1,-3)$. Also, the above relations are valid at the GUT scale only, so that the known experimental values of the masses and mixings must first be run up to that energy using the renormalization group equations.

Expressions (11) and (12) are needed to evaluate the matrices Y_{10} and Y_{126} in terms of the better known M_U and M_D. Their expressions are then used in (14) as well as in the various neutrino matrices (13), (15) and (16) to be used in (6). Defining

$$x = \frac{v_{10}^d v_{126}^u}{v_{10}^u v_{126}^d} \ , \ y = \frac{v_{10}^d}{v_{10}^u} \ , \ \alpha = \frac{16 \left(v_{126}^u\right)^2}{v_L v_R x^2} \ , \ \beta = \frac{4 v_{126}^d}{v_L} \ , \tag{17}$$

one gets two matrix equations:

$$(1-x) M_E = 4y M_U - (3+x) M_D \ , \tag{18}$$

$$\begin{aligned} \beta M_N &= -\alpha \left[\frac{3(1-x)M_D + (1+3x)M_E}{4}\right] (M_D - M_E)^{-1} \\ &\times \left[\frac{3(1-x)M_D + (1+3x)M_E}{4}\right] + (M_D - M_E) \ . \end{aligned} \tag{19}$$

4 Type I versus type II seesaw

What we want to find out is which type of seesaw is compatible with data. To make the analysis simpler, let us study the situation of the 2^{nd} and 3^{rd} generations only, as well as no CP violation (real parameters) [5].

Equation (18) has 6 known masses ($m_{\tau,\mu}$, $m_{t,c}$, $m_{b,s}$), one known mixing angle ($\theta_q = \theta_{cb}$), but 3 unknown parameters, x, y and the angle between the orthogonal matrices that diagonalize M_E and M_D. Since the matrices are all symmetric (due to its SO(10) origin), we have 3 equations, just enough. After determining these parameters one can attack equation (19). We are interested mainly in the leptonic mixing angle and this is determined as a function of one single parameter, α. In the limit $\alpha \to \infty$ one remains with a pure type I seesaw, while in the opposite case $\alpha \to 0$ and the seesaw is of type II.

To get a feeling let us first consider the idealized situation of small second generation masses ($0 \approx m_2 \ll m_3$), but still finite quark mixing θ_q (although

$\ll 1$). It is easy to show that the leptonic atmospheric mixing angle is given by

$$\tan 2\theta_l = \frac{\sin 2\theta_q}{2\sin^2\theta_q - \Delta} \;, \tag{20}$$

with

$$\Delta = \frac{1}{1-9\alpha}\left[-5\alpha + (1-4\alpha)\,\epsilon\right] \quad , \quad \epsilon = \frac{m_b - m_\tau}{m_b} \quad . \tag{21}$$

In the limit $\alpha \to \infty$ one gets $\Delta = (5+4\epsilon)/9$, while the opposite case $\alpha \to 0$ gives $\Delta = \epsilon$. Since the parameter ϵ is experimentally small (approximate $b-\tau$ unification), the small α regime is the one that predicts a large atmospheric angle: type II seesaw is thus favoured in these type of models.

Now we restore a finite mass of the second generation, but keep in mind that they are small, i.e. that

$$\frac{m_c}{m_t} \;,\; \frac{m_s}{m_b} \;,\; \frac{m_\mu}{m_\tau} \;,\; \theta_q \;\approx\; O(\delta) \;,\; \delta \approx 10^{-2} \;. \tag{22}$$

Writing in a schematic way (coefficients of order 1 are not explicitly written) the neutrino mass matrix is

$$M_N \approx -\alpha \begin{pmatrix} \delta & \delta/\epsilon \\ \delta/\epsilon & 1/\epsilon \end{pmatrix} + \begin{pmatrix} \delta & \delta \\ \delta & \epsilon \end{pmatrix} \;, \tag{23}$$

where the first (second) matrix is the type I (II) seesaw contribution. The atmospheric mixing angle is

$$\tan 2\theta_l \approx \frac{\delta\,(1+\alpha/\epsilon)}{\delta(1+\alpha) + \epsilon + \alpha/\epsilon} \;. \tag{24}$$

If we further assume that $\epsilon \approx O(\delta)$ [1], we get

$$\theta_l \approx O(1) \iff \alpha \leq O(\delta^2) \;. \tag{25}$$

The dominant type I seesaw ($\alpha \to \infty$) is clearly excluded, since it would predict a small mixing angle $\theta_l \approx O(\delta)$. Notice that without $b-\tau$ unification (small ϵ) θ_l would be small in any case.

[1] This is not really compatible with the small $\tan\beta$ (≈ 10) regime, in which corrections are under control. In this case a realistic $\epsilon \approx 0.1$, but (20) becomes much more complicated. I thank Zurab Berezhiani for remarks and an enlightening discussion on this point.

It is possible to understand why a large atmospheric mixing angle emerges in type II seesaw [6]: the point is that the triplet contribution to the Majorana neutrino mass is proportional to Y_{126} (16), but from (12) and (14) this Yukawa is proportional to the difference $M_D - M_E$, so that in pure type II seesaw

$$M_N \propto M_D - M_E \ . \tag{26}$$

Assuming small mixings one immediately obtains approximately

$$M_N \propto \left(\begin{array}{cc} 0 & 0 \\ 0 & m_b - m_\tau \end{array} \right) \ , \tag{27}$$

so that an unexpected connection between large atmospheric neutrino angle and $b - \tau$ unification emerges from type II seesaw [6]:

$$\text{large} \ \ \theta_{atm} \ \Longleftrightarrow \ b - \tau \ \ \text{unification} \ . \tag{28}$$

5 SUSY

Up to now we did not specify which is the grandunification theory we were considering: the only requirements were that the Yukawas came from the interactions of matter 16-dimensional representations with two complex Higgses - 10_H and $\overline{126}_H$, and that they got some nonzero vevs in the relevant directions. In order to show that this is possible and to find these vevs as functions of parameters in the Lagrangian, we need to write down an explicit model. I will here shortly describe the minimal supersymmetric SO(10) model, which has all the nice above features and is still consistent with data.

As we saw, in order to fit the fermion masses we need two Higgs representations, 10_H and $\overline{126}_H$. In supersymmetry, the presence of $\overline{126}$ is particularly welcome, since its vev does not break R-parity. In fact $(1, 3, 10)$, which vev gives a large mass to the right-handed neutrino, has $B - L = -2$. R-parity is given by

$$R = (-1)^{3(B-L)+2S} \tag{29}$$

and since the spin S of any vev is 0, the ν^c mass is R-parity even and so does not break it at the large scale [7]. One can show that this is true all the way to the electroweak scale, i.e. R-parity is exact [8]. This means among others that the lightest supersymmetric partner is stable and is thus a good candidate for dark matter.

Now, in supersymmetry we need more than just 10_H and $\overline{126}_H$, since the right-handed neutrino mass must be very large, on the order of the GUT scale or so. This would strongly break supersymmetry by the D-terms, so another Higgs - 126_H (5 index antisymmetric and self-dual) - must be used to cancel it. SO(10) symmetry allow the matter 16 to be coupled in the superpotential only to 10, $\overline{126}$ and 120 (3 index antisymmetric) dimensional representations, so this new 126_H cannot change (7). Finally we need to get two (practically) massless Higgs doublets, while heavy all colour triplets. This can be achieved by first introducing the Higgs representation 210_H (4 index antisymmetric) and then by one fine-tuning of the parameters in the superpotential.

Such a renormalizable theory, with three matter 16, and with the Higgs sector made of 10_H, 126_H, $\overline{126}_H$ and 210_H [9] has the correct symmetry breaking pattern [10] and has been shown to be the minimal grand unified theory [11], i.e. the most predictive one.

6 Further developments

The three generation generalization [12] roughly confirms the consistency of the type II seesaw with data, and predicts a quite large $U_{e3} \approx 0.16$. Type I see-saw domination seems however still possible, especially after CP violating phases are considered [13].

An interesting and necessary check of the model is to calculate the proton decay rates of $d = 5$ operators. A preliminary study [14] shows that these decays could easily be too fast and thus rule out the minimal model. The question is of course of great importance, and a detailed analyses requires the knowledge of the SO(10) Clebsch-Gordan coefficients [15].

Another issue is the gauge coupling unification: it has been shown, that at least in some part of parameter space threshold corrections are under control in spite of the large representations involved [16].

Finally, nonminimal models with 120 dimensional Higgses have been considered [17]: such models are less restrictive and fit slightly better the available experimental data. They represent interesting variations and generalizations of the above mentioned models.

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