

Two- and three-dimensional vacuum defects in $SU(2)$ gluodynamics

A. V. Kovalenko[†], M. I. Polikarpov[†],
S. N. Syritsyn[†] and V. I. Zakharov^{*}

April 29, 2005

[†] *Institute of Theoretical and Experimental Physics,
B. Chermushkinskaya 25, Moscow, 117259, Russia*

^{*} *Max-Planck Institut für Physik,
Föhringer Ring 6, 80805, München, Germany*

Abstract

We review briefly lattice evidence for existence of lower-dimension vacuum defects in $SU(2)$ gluodynamics. On one hand, the defects are known to be crucial for confinement, or physics in infrared. On the other hand, the defects have non-trivial properties in ultraviolet, exhibiting an infrared-ultraviolet fine tuning. We illustrate first these properties on the example of central vortices, or two dimensional vacuum defects. Most recently, evidence was found for existence of three-dimensional domains whose total volume scales in physical units. Technically, the domains are defined in terms of $Z(2)$ projection of original gauge fields. The volume can be viewed also as the minimal volume bound by the center vortices. We argue that the three-dimensional domains are closely related to confinement.

1 Introduction

It is a general trend in modern theoretical physics to consider extended objects, like strings and membranes. Usually, one applies these ideas to hypothetical, high-dimensional completions of the four-dimensional world. However, lower-dimensional vacuum structures might have been observed also as excitations within 4d gauge theories [1, 2]. A difficulty with presenting and even the lattice data is that at present time there is no well developed framework which would predict such structures. Observationally they appear to be ‘quantum’ or lattice branes with huge entropy and action which balance each other, for reviews see [3].

The vacuum fluctuations in point are in fact discussed since long and nothing else but lattice monopoles and P-vortices, for review see, e.g., [4, 5]. However traditionally one has been emphasizing the relevance of these fluctuations in the infrared, or to the confinement. What seems to have been revealed more recently [1, 2] is that these structures have also highly nontrivial ultraviolet properties. In particular, they appear to be infinitely thin in terms of distribution of the non-Abelian action associated with them. It is primarily this observation which allows to claim them to be physical objects of lower dimension. Namely, monopoles appear to be 1d dimensional objects (closed trajectories) while the central vortices are 2d branes.

The main emphasis here is on vacuum fluctuations observed by using the so called central projection. The best known example of fluctuations of this type are P-vortices. Most recently this projection allowed to observe three dimensional vacuum structures percolating through the vacuum of the 4d $SU(2)$ gauge theory [7, 8]. Since the observation is very recent the confinement-related phenomenology of the new objects is not so much developed as in case of the monopoles or central vortices. Nevertheless one can argue that the three dimensional structures are probably related to the confinement as well.

2 P-vortices

2.1 Z_2 gauge theory

There is no clear theoretical picture of confinement in the non-Abelian case (for simplicity we concentrate on the SU(2) gluodynamics)¹. On the other hand, in case of $U(1)$ and Z_2 gauge theories the proof of confinement is quite straightforward, see [9] and [10], respectively. In the former case it is percolation of the monopoles, known also as dual-superconductor model. In the latter case it is percolation of the central vortices (see also below). Thus, it is tempting to assume that degrees of freedom crucial for confinement are retained if one narrows, or projects the original non-Abelian fields to their Abelian or center-group subspace. In this talk we will concentrate on the center-group, or Z_2 projection.

The gauge Z_2 theory can be defined only on the lattice. The variables are links

$$Z_\mu(x) = \pm 1$$

where x is a point in space-time while μ indicates the direction of the link. The action depends only on the values of the plaquettes which are defined as product of the corresponding links and, obviously, take on the values ± 1 as well. The partition function is defined as

$$Z = \sum \exp(-\beta A^-) \quad (1)$$

where β is a constant and A^- is the total area of all the negative plaquettes for a given configuration of the links.

It is quite clear that at large β , $\beta \rightarrow \infty$ the area A^- is negligible while in the opposite limit, $\beta \rightarrow 0$ the number of negative plaquettes is not suppressed at all. Respectively, there are two phases corresponding to the strong and weak couplings. For us, it is crucial that the strong-coupling phase is confining. Indeed, let us calculate the Wilson loop in this phase. For a given configuration of the Z_2 fields the Wilson line is the product

$$W = \prod Z_W = \prod (Plaquettes)_A \quad (2)$$

where Z_W are the values of the link variables along the Wilson loop while $(Plaquettes)_A$ are the values of the plaquettes covering an area bounded by

¹By ‘confinement’ we understand here existence of a linearly rising potential for external heavy quarks.

the Wilson loop. The replacement of the product of the links by the product of the plaquettes is an identity based on the fact that $(Z_\mu(x))^2 = 1$.

Now, let us assume for a moment that the plaquettes take on the values ± 1 randomly. Which means, in turn that we can calculate the expectation value of the Wilson loop as a product of the expectation values of individual plaquettes:

$$\langle W \rangle = \langle \text{Plaquette} \rangle^A, \quad (3)$$

where A is the area bounded by the Wilson loop. Denote by p the probability for a plaquette to be negative. Then $(1-p)$ is the probability for a plaquette to be positive and

$$\langle \text{Plaquette} \rangle = (1-p) \cdot 1 + p \cdot (-1) = (1-2p), \quad (4)$$

and

$$\langle W \rangle = \exp(-\sigma \cdot A), \quad (5)$$

where σ is the string tension equal to

$$\sigma = \frac{|\ln(1-2p)|}{a^2}, \quad (6)$$

where a is the lattice spacing.

It is useful to introduce notion of P- or central- vortices. The vortices are defined on the dual lattice. Namely, the plaquettes belonging to the P-vortices are orthogonal to the negative plaquettes on the original lattice. In four dimensions the plaquettes on the dual and original lattices intersect at one point which coincides in fact with the centers of the two plaquettes. In other words, the P-vortices pierce negative plaquettes. Moreover, one can show that P-vortices are closed surfaces.

Now we can come back to our assumption that the plaquettes covering the area bounded by the Wilson loop take on the values ± 1 randomly. Let us distinguish between finite and infinite clusters of P-vortices. Finite P-vortices pierce the area in a correlated way, that is twice, and do not produce any non-trivial effect on the Wilson loop. It is only infinite, or percolating cluster which determines the probability p entering Eq (4).

2.2 Thick vortices

At first sight, consideration of the Z_2 gauge theory does not help us at all to understand confinement in the realistic case of non-Abelian theories. Indeed,

negative plaquettes which play the crucial role in the Z_2 case are unphysical in the non-Abelian case ².

Nevertheless, one can argue that the center group plays a central role in the non-Abelian case as well [11]. Indeed, consider heavy quarks in the fundamental and adjoint representations. It is only for the quarks in the fundamental representation that we expect the potential to grow linearly at large distances. The color charge of heavy quarks in the adjoint representation is screened by gluons and the heavy quark potential flattens out at large distances.

Thus, quarks with half integer color isotopic spin are confined while quarks with integer spin are not confined. Clearly, it is the center of the group which distinguishes between integer and non-integer spins.

Thus, the picture with percolating P-vortices would be still helpful to explain confinement. However, in the non-Abelian case one usually thinks about the confining fields in terms of ‘soft’ fields ³, with gauge potential of order $A_\mu \sim \Lambda_{QCD}$. On a very qualitative level one could think in terms of effective lattice with the ‘lattice size’ of order Λ_{QCD}^{-1} . Then we could apply the picture learned from the Z_2 example. These vortices could be called ‘thick’ vortices. Roughly speaking, in the confining phase the Z_2 symmetry is not violated and the probabilities of a large Wilson loop to be positive or negative are the same ⁴.

It is worth mentioning that the notion on importance of the group center in formulating a criterion of confinement can be made precise [11]. Also, theory of the thick vortices is well developed, for review see, e.g., [13, 5].

2.3 Central projection

For a long time, the notion of the thick vortices was used mostly in theoretical constructions. Indeed, it is very difficult to suggest ways to detect them, let it be on the lattice. However, a few years ago a method of central

²Indeed, there is no continuum field corresponding to the negative plaquettes and according to the standard wisdom they could be thrown away altogether.

³We will partly question this picture below.

⁴At finite temperature T , the time direction is periodic. At some temperature the Wilson line is not long enough to develop negative values, the Z_2 symmetry is broken and there is phase transition to deconfinement [12].

projection was proposed which seemed to allow to tag thick vortices, for history and details see [5].

The basic idea is to extract from a full configuration of non-Abelian fields Z_2 degrees of freedom which might be responsible for the confinement. This is achieved by projecting the standard link variables $U_\mu(z)$ into $Z_\mu(x)$ variables. ‘Projection’ means now simply replacement. For the projected fields to memorize basic properties of the original field configuration one chooses the norm of the projected fields as close as possible to the norm of the original fields.

In more detail, one uses first gauge invariance to minimize the functional

$$R = \sum_{x,\mu} |Tr U_\mu(x)|^2 \quad , \quad (7)$$

where the sum is over the whole of the lattice. Then one maps the $SU(2)$ link variables $U_\mu(X)$ to Z_2 elements by replacing

$$Z_\mu(x) = sign Tr[U_\mu(x)] \quad (8)$$

The discovery was that the linear quark potential (or string tension) evaluated in terms of the projected fields was rather close to the actual value obtained in the full $SU(2)$ theory. In any case, the property of the confinement was certainly there in terms of the projected fields as well.

The P-vortices defined constructed on the projected fields are infinitely thin by definition, the same as in case of Z_2 gauge theory. However, as far as the total area of the central vortices is concerned, there is a striking difference from the Z_2 case. Namely, according to the the measurements the total area scales in the physical units:

$$A_{tot} \approx 4 (fm)^{-2} V_4 \quad (9)$$

where V_4 is the total volume of the lattice. Observation (9) implies, in particular, that the fraction of all plaquettes which belong to the P-vortices is proportional to $(a \cdot \Lambda_{QCD})^2$. Which is a spectacular phenomenon observationally.

2.4 Lattice branes

The main theoretical difficulty of interpreting the lattice results is the use of the central projection. Indeed, the projection is defined in a highly non-local

way and there is no direct relation between the original and projected fields. And this makes any theoretical discussion very qualitative, at the very best. Most common interpretation of the central vortices detected through the central projection is that they tag thick vortices discussed above, for review see [5]. In this way one gets qualitative explanation of appearance of the Λ_{QCD} scale in (9).

On the other hand, the picture of the central vortex following only loosely thick vortices also has problems with explaining (9). Indeed it introduces in fact a kind of conspiracy between infrared and ultraviolet properties. Imagine that we are tending $a \rightarrow 0$. Then the number of plaquettes covering a cross section of the hypothetical thick vortex is growing as $1/a^2$. And nevertheless the projection picks up exactly one negative plaquette per thick vortex.

This kind of a puzzle led to a search for ‘gauge invariant identity’ of the plaquettes belonging to the P-vortices. The search brought an unexpected result [1]: the P-vortices are distinguished by an ultraviolet divergent action. Namely, the non-Abelian action associated with the central vortices is equal to:

$$S_{vort} \approx 0.53 \frac{A_{vort}}{a^2} \quad (10)$$

where $S_{vort} = \beta(1 - \frac{1}{2} \langle \text{Tr} U_P^{vort} \rangle)$ is measured on the plaquettes U_P^{vort} dual to P-vortices. Moreover, Eq (10) refers actually to the excess of the action with respect to the average over the whole lattice.

Another amusing result is that the excess (10) vanishes already on the plaquettes next to the P-vortices. Thus, P-vortices have vanishing thickness in terms of the distribution of the non-Abelian action. They are physical two-dimensional objects. To distinguish them from hypothetical thick vortices (which carry action of order $\Lambda_{QCD}^2 \cdot (Area)$) one can call the vortices revealed through projection lattice branes.

Let us emphasize again that (10) implies that P vortices are suppressed by huge action, $\exp(-S) \sim \exp(-const \cdot (Area)_{tot}/a^2)$, where the total area does not change as the lattice spacing tends to zero in the continuum limit. The probability to observe the vortex, however, is a product of the action factor and entropy:

$$W(vortex) \sim e^{(-const \cdot A/a^2)} \cdot (Entropy) \quad (11)$$

Thus, the very existence of the central vortices implies that their entropy grows as an exponent of $(Area)/a^2$. This dependence of the entropy on the

lattice spacing was indeed confirmed by measurements [14]. The number in front of the factor $(Area)/a^2$ cannot be measured independently, however.

3 Three-dimensional defects

3.1 Removal of P-vortices

There is no regular way to search for lower-dimensional defects. Historically, the monopoles and P-vortices emerged as candidates for confining field configurations. The physical idea behind introduction of the monopoles and vortices is, as mentioned above, that $U(1)$ or Z_2 degrees of freedom are responsible for the confinement.

There no further replicas of this idea. However, there exists another remarkable observation [15] which might shed light on the nature of the confinement. Namely, removal of the P-vortices eliminates both confinement and spontaneous violation of the chiral symmetry.

In more detail, one determines first $Z_\mu(x)$ projected fields (see above) and then replaces the original link variables $U_{x,\mu}$ by new matrices $\tilde{U}_{x,\mu}$ defined as

$$\tilde{U}_\mu(x) \equiv U_\mu(x) \cdot Z_\mu(x) \quad , \quad (12)$$

where the Z factors are ± 1 . Note that the substitution (12) changes the sign only of the plaquettes pierced by P-vortices. The result [15] is that if one evaluates the Wilson loop using the modified links (12) then the confining potential disappears.

To give a feeling on the significance of the numerical results, let us mention that we have repeated calculation of Ref [15] for $\beta = 2.4$. While the string tension in the original theory equals to

$$\chi = (0.093 \pm 0.012)/a^2 \quad , \quad (13)$$

where a is the lattice spacing, for the modified link variables we found:

$$\chi = (0.0034 \pm 0.0096)/a^2 \quad (14)$$

The result is indeed impressive and its theoretical appreciation could probably be crucial for understanding the nature of the confining fields.

3.2 Minimization of negative links

Usually the procedure just described is dubbed as removal of the P-vortices. There is a puzzle, however. Indeed, the total area of the vortices scales in physical units and in this sense they represent $d=2$ defects. Furthermore, the Wilson line is obviously a $d=1$ subspace. However, in four dimensions, $d = 4$, the subspaces $d=1$ and $d=2$ do not intersect at all, generally speaking. For a local change (like (12)) to affect the Wilson line one should change fields at least on a $d=3$ subspace. Thus, no local change of the plaquettes on the vortices can eliminate confinement.

The resolution of the paradox is that the field modification (12) affects originally links, not plaquettes directly. And it would be too naive to assume that it is only change of the plaquettes that counts. In quantum mechanics, potentials are also significant, as demonstrated by the Aharonov-Bohm effect (for related discussions see, in particular, [17]).

If we turn to consideration of links then the change (12) may look, to the contrary, as an enormous modification of the original fields. Indeed, the Z -projected fields, $Z_\mu(x) = \pm 1$ fall onto $Z_\mu(x) = -1$ in about half of all the cases. And one could argue that it is not surprising that we lose confinement by changing potentials on a half of the lattice.

Thus, it seems reasonable to ask what is the minimal number of links which are to be changed to eliminate confinement through the procedure (12). In other words, let us introduce ‘Landau-gauge’ in terms of the projected fields. Minimizing the number of negative links one might hope to get a gauge invariant, or physically significant result. In fact, this idea goes back to the paper in Ref. [16].

The results of the measurements are represented in Fig. 1. We find, indeed, that the negative links, after the minimization, occupy a 3d volume:

$$V_3 \approx 2(fm)^{-1} \cdot V_4, \quad (15)$$

where V_4 is the lattice volume. Note also that Fig. 1 summarizes results of two series of measurements. The difference between them is definition of the Z_2 projection. Namely, the procedure described above (see discussion of (7)) is called Direct Maximal Center Projection (DMCP). Another possibility is to project first the original fields into the closest $U(1)$ fields configuration and then perform the Z_2 projection. The latter projection is called Indirect Maximal Center Projection (IMCP), for references and details see, e.g., [8].

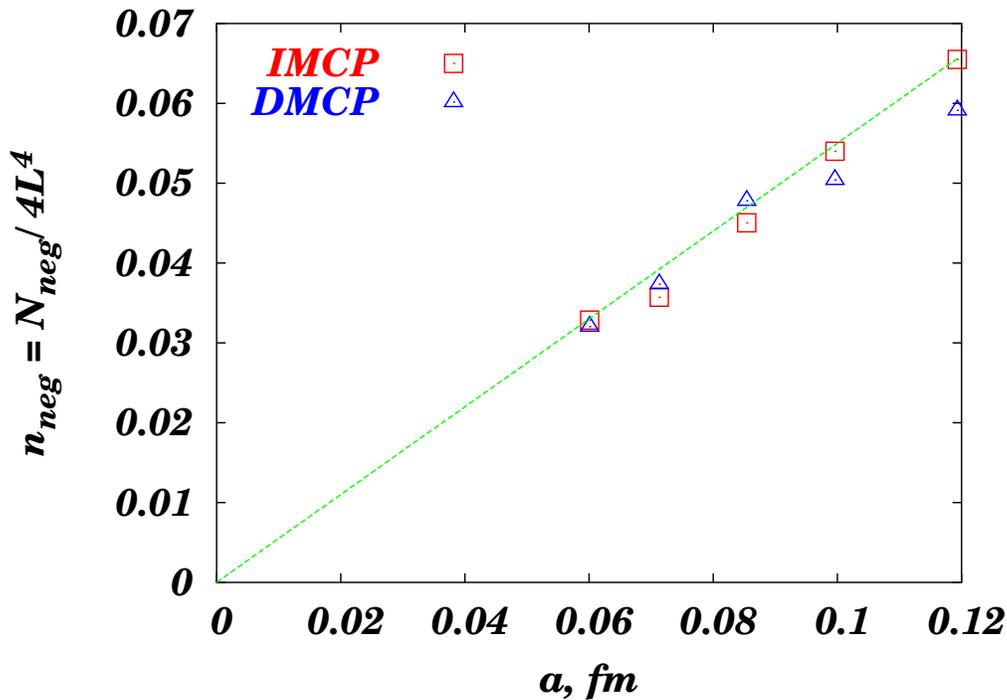


Figure 1: Probability for a link to be negative in $Z(2)$ gauge field configuration after $Z(2)$ Landau gauge fixing.

3.3 Confining fields in the ultraviolet

Thus, the minimization of the number of the negative links reveals another vacuum defect, that is is 3d volume. Relation of these defects to the the confinement is encoded in the statement that if we change in the ultraviolet the links belonging to this volume, confinement disappears! Here, by ‘change in the ultraviolet’ we understand multiplying the link variables by (-1).

Let us also emphasize that the change affects the number of links which is a fraction of order $(a \cdot \Lambda_{QCD})$ from the total and is vanishing in the continuum limit. It might be the most important message to the continuum theory obtained on the lattice: confinement can be uncoded from the Yang-Mills theory by changing field on a submanifold which is vanishing in the continuum limit. It seems quite obvious that none of the attempts on explaining confining in the continuum theory is consistent with this observation ⁵.

⁵Of course, there is always a possibility that we have not reached yet, on the existing

We can also now evaluate the average number of intersections of the Wilson line with the 3d volume:

$$\langle N_{intersection} \rangle \approx 0.5 \frac{P_W}{fm} , \quad (16)$$

where P_W is the perimeter of the Wilson line and there is no dependence on the lattice spacing. Note also that if we would change randomly the sign of links variables occupying the same volume as (15) we could change only the perimeter dependence of the Wilson loop which is not linked to the confinement.

There is another way to demonstrate that the fields $A_\mu \sim 1/a$ are crucial for the confinement. Let us introduce another change of the links. Namely replace the original links $U_\mu(x)$ by unit matrix if the link belongs to the minimal volume of the negative links:

$$U_\mu(x) \rightarrow I , \text{ if } Z_\mu(x) = -1 \text{ in the } Z_2 \text{ Landau gauge} . \quad (17)$$

Then according to our measurements, for $\beta = 2.4$ the string tension changes considerably and equals to:

$$\chi \approx 0.057/a^2 , \quad (18)$$

compare (13). This is a preliminary result, however.

Again the result (18), if confirmed, reveals features of the confining fields which have never been thought of in the continuum theory. Indeed, we just ‘forget’ to account for a single link out of the number of order $\sim (2fm)/a$ and still the string tension drops by order unit. Thus, if confirmed, (18) would prove that fields of order $A_\mu \sim 1/a$ are crucial for the confinement.

4 Conclusions

To summarize, the lattice evidence suggests that it is vacuum defects of lower dimension which are responsible for the confinement. The defects exhibit fine tuning between the infrared and ultraviolet. For example, the total area of

lattices, values of the lattice spacing a which can consistently be considered as being small. However, this would also imply that the confinement which has been observed so far on the lattice is just by coincidence looks as the confinement considered in the continuum theory.

the 2d defects scales in the physical units (see (9)) while their action is ultra-violet divergent, see (10). In other words, there is self-tuning of the confining field configurations which exhibit power dependence both on the infrared scale (Λ_{QCD}) and the ultraviolet scale (lattice spacing a). Theoretically, this phenomenon has no explanation yet. One check, however, that the ultraviolet divergence in the action (10) is in no contradiction with the asymptotic freedom [18]. Moreover, the self-tuning could have been possibly explained if a dual formulation of the Yang-Mills theories were found [19].

Acknowledgements

The authors are grateful to F.V. Gubarev for useful discussions. A.V.K., M.I.P. and S.N.S. are partially supported by grants RFBR 02-02-17308, RFBR 01-02-17456, DFG-RFBR 436 RUS 113/739/0, RFBR-DFG 03-02-04016, INTAS-00-00111, and CRDF award RPI-2364-MO-02. V.I.Z. is partially supported by grant INTAS-00-00111 and by DFG grant “From lattice to hadron phenomenology”.

One of the authors (V.I.Z.) is grateful to the organizers of the International Seminar ‘Quarks-04’ for the invitation to attend the conference and hospitality.

References

- [1] F.V. Gubarev, A.V. Kovalenko, M.I. Polikarpov, S.N. Syritsyn, and V.I. Zakharov, *Phys. Lett.* **B574** (2003) 136 (hep-lat/0212003); “*Selftuning of the P-vortices*”. hep-lat/0309034.
- [2] V.G. Bornyakov, M.N. Chernodub, F.V. Gubarev, M.I. Polikarpov, T. Suzuki, A.I. Veselov, and V.I. Zakharov, *Phys. Lett.* **B537** (2002) 291.
- [3] V.I. Zakharov, “*Confining fields in lattice SU(2)*”, hep-ph/0312210
V.I. Zakharov, *Nucl. Phys. Proc. Suppl.* **121** (2003), 325.
- [4] M.N. Chernodub, F.V. Gubarev, M.I. Polikarpov, and A.I. Veselov, *Progr. Theor. Phys. Suppl.* **131** (1998) 309.
- [5] J. Greensite, *Prog. Part. Nucl. Phys.* **51** (2003) 1.

- [6] V.G. Bornyakov, P.Yu. Boyko, M.I. Polikarpov, and V.I. Zakharov, *Nucl. Phys.* **B672** (2003) 222.
- [7] A.V. Kovalenko, M.I. Polikarpov, S.N. Syritsyn, and V.I. Zakharov, “Three dimensional vacuum domains in four dimensional $SU(2)$ gluodynamics” hep-lat/0408014.
- [8] S.N. Syritsyn, M.I. Polikarpov, and V.I. Zakharov, “A novel probe of the vacuum of the lattice gluodynamics”, hep-lat/0402018.
- [9] A.M. Polyakov, *Phys. Lett.* **B59** (1975) 82.
- [10] G. Mack and V.B. Petcova, *Ann. Phys.* **123** (1979) 447;
G. Mack, *Phys. Rev. Lett.* **45**, (1980) 1378.
- [11] A.M. Polyakov, *Phys. Lett.* **72B** (1978) 477;
G. ’t Hooft, *Nucl. Phys.* **B138**, 1 (1978).
- [12] A.M. Polyakov, “Confinement and liberation”, hep-th/0407209.
- [13] E.T. Tomboulis, hep-lat/0311022.
- [14] A.V. Kovalenko, M.I. Polikarpov, S.N. Syritsyn, and V.I. Zakharov, *Nucl. Phys. Proc. Suppl.* **129** (2004) 665 (hep-lat/0309032);
A.V. Kovalenko, M.I. Polikarpov, S.N. Syritsyn, and V.I. Zakharov, “Properties of P vortex- and monopole-clusters in lattice $SU(2)$ gauge theories”, hep-lat/0402017.
- [15] P. de Forcrand and M. D’Elia, *Phys. Rev. Lett.* **82** (1999) 4582.
- [16] F.V. Gubarev, L. Stodolsky and V.I. Zakharov, *Phys. Rev. Lett.* **86** (2001) 2220.
- [17] M.G. Alford and F. Wilczek, *Phys. Rev Lett* **62** (1989) 1071;
M.G. Alford, J. March–Russel, and F. Wilczek, *Nucl. Phys.* **B337** (1990) 695;
J. Preskill and L.M. Krauss, *Nucl. Phys.* **B341** (1990) 50;
M.I. Polikarpov, U.J. Wiese, and M.A. Zubkov, *Phys. Lett.* **B309** (1993) 133.
E.T. Akhmedov, M.N. Chernodub, M.I. Polikarpov, and M.A. Zubkov, *Phys. Rev.* **D53** (1996) 2087.

- [18] V.I. Zakharov, “*Nonperturbative match of the ultraviolet renormalon*”, hep-ph/0309178.
- [19] V.I. Zakharov, *Hints on the dual variables from lattice $SU(2)$ gluodynamics*”, in “Wako 2003, Color confinement and hadrons in quantum chromodynamics” pp. 112-123 (hep-ph/0309301).