On generation of Abelian magnetic fields in $SU(3)$ gluodynamics at high temperature

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Abstract

The vacuum state of $SU(3)$ gluodynamics at high temperature is investigated. A consistent approach including the calculation of either the spontaneously generated constant chromomagnetic isotopic H_3 and hypercharge H_8 fields or the polarization operator of charged gluons in this background is applied. It is shown within the effective potential including the one-loop plus daisy diagrams that the specific values of the fields deliver a global minimum to free energy. The spectrum of the transversal charged modes is stable at high temperature due to a gluon magnetic mass accounting for the vacuum fields. This leads to stable chromomagnetic fields in the deconfinement phase of QCD. A comparison with the results of other approaches is done.

1 Introduction

The deconfinement phase transition remains the most topical problem of QCD at finite temperature. Nowadays a general believe is that the formation of the magnetic monopole condensate at low temperature and its evaporation at high temperature are responsible for this phenomenon (see, for instance, the papers [1], [2] and references therein). The present day status of this problem is characterized by the fact that the results obtained by the former method are in agreement with that of the latter one and complement each other, although some discrepancies exist. The most important discrepancy concerns the properties of the high temperature phase. As it is well known from lattice calculations, due to asymptotic freedom at high temperature the deconfining phase is to be a gas of free quarks and gluons - quark-gluon plasma. No other macroscopic parameters except temperature are expected. However, in the continuum calculations the generation of the classical chromomagnetic field of order $gH \sim g^4T^2$ was observed (see [4] and references therein). This spontaneously created field is a reflection of infrared dynamics of the non-Abelian gauge fields at finite temperature. The field stabilization due to the electrostatic potential (so-called the A_0 condensate) [3] and radiation corrections to the charged gluon spectrum [5] has been investigated for the $SU(2)$ gluodynamics. According to the picture derived in these papers the vacuum at high temperature is to be a stable, magnetized state. The noted discrepancy as well as some properties of the vacuum at high temperature have been discussed recently by Meisinger and Ogilvie [6]. To eliminate the classical field in the deconfiming phase these authors have introduced a gluon magnetic mass on heuristic grounds. Then it was observed that to have a zero field at high temperature the value of the magnetic mass substituted into the one-loop effective potential (EP) must be of order $\sim g^2T$. However, the spontaneous creation of the chromomagnetic field is mutually related with infrared properties of the non-Abelian gauge fields , that should be taken into consideration. This important aspect of the gauge field dynamics at high temperature needs in a more detailed investigation. Moreover, the correlation corrections accounting for long distance effects should be included to find a consistent picture at high temperature .

In the present paper the restored phase of QCD at high temperature is investigated within an approach including two stages of calculation. First, the consistent EP of the Abelian constant chromomagnetic fields - isotopic , H_3, and hypercharge, H_8 taking into consideration the one-loop and the daisy diagrams, which include the gluon magnetic mass insertions, is computed and the field configuration which is spontaneously generated is determined. This potential is real due to the daisies of the charged gluons, which cancel the imaginary part entering the one-loop part of the EP. As it is occurred, a specific combination of both fields is formed. Second, the one-loop polarization operator (PO) of charged gluons in these external fields is calculated. It is averaged over the gluon tree level states in the fields in order to find the radiation corrections to the spectrum. In this way the effective Debye's and magnetic masses of gluons are derived. Then the vacuum magnetic field strengths are used to check whether or not the charged gluon spectrum (and therefore the magnetized vacuum) at finite temperature is stable. As it is found, this is the case and the non trivial vacuum is favorable at high temperature in the wide interval of temperature above the decofinement transition temperature T_d. Hence we come to the conclusion that the scenario with the magnetized vacuum generated due to infrared dynamics of gauge fields at high temperature results in a consistent picture. The higher loop corrections can be included perturbatively within an external field problem.

2 The spontaneous generation of chromomagnetic fields

We investigate the spontaneous vacuum magnetization in the high temperature $SU_c(3)$ gluodynamics, charged sector of which is described by the Lagrangian:

$$
\begin{aligned}
L_{ch.gl.} = \sum_{r=1}^{3} \Big(-\frac{1}{2} W^{+}_{r\mu\nu} W^{-}_{r\mu\nu} - (D^{*}_{\mu} W^{+}_{r\mu})(D_{\nu} W^{-}_{r\nu}) - \\
-\frac{1}{2} c_r g^2 W^{+}_{r\mu} W^{-}_{r\nu} W^{+}_{r\lambda} W^{-}_{r\rho} \Gamma_{\mu\nu\lambda\rho} \Big) + \\
+ ig(F_{3\mu\nu} + Q^{3}_{\mu\nu}) W^{+}_{1\mu} W^{-}_{1\nu} + ig Q^{3}_{\mu} (W^{+}_{1\nu}(\partial_\mu W^{-}_{1\nu} - \partial_\nu W^{-}_{1\mu}) - (h.c.)) + \\
+ i\sqrt{\frac{3}{2}} g (\lambda_2 F_{8\mu\nu} + Q^{8}_{\mu\nu} + \frac{1}{\sqrt{6}} Q^{3}_{\mu\nu}) W^{+}_{2\mu} W^{-}_{2\nu} \\
+ i\sqrt{\frac{3}{2}} g (Q^{8}_{\mu} + \frac{1}{\sqrt{6}} Q^{3}_{\mu}) (W^{+}_{2\nu}(\partial_\mu W^{-}_{2\nu} - \partial_\nu W^{-}_{2\mu}) - (h.c.)) + \\
+ i\sqrt{\frac{3}{2}} g (\lambda_3 F_{8\mu\nu} + Q^{8}_{\mu\nu} - \frac{1}{\sqrt{6}} Q^{3}_{\mu\nu}) W^{+}_{3\mu} W^{-}_{3\nu} \\
+ i\sqrt{\frac{3}{2}} g (Q^{8}_{\mu} - \frac{1}{\sqrt{6}} Q^{3}_{\mu}) (W^{+}_{3\nu}(\partial_\mu W^{-}_{3\nu} - \partial_\nu W^{-}_{3\mu}) - (h.c.)) + L_{gh}, \quad (1)
\end{aligned}
$$

where $W^{+}_{r\mu\nu} = D^{*}_{r\mu} W^{+}_{r\nu} - D^{*}_{r\nu} W^{+}_{r\mu}$, $W^{-}_{r\mu\nu} = D_{r\mu} W^{-}_{r\nu} - D_{r\nu} W^{-}_{r\mu}$, $D_{r=1\ \mu} = \partial_\mu + igB_{3\mu}$, $D_{r=2,3\ \mu} = \partial_\mu + i\sqrt{\frac{3}{2}}\lambda_{r=2,3} g B_{8\mu}$, are covariant derivatives, $\Gamma_{\mu\nu\lambda\rho} = \delta_{\mu\nu}\delta_{\lambda\rho} - \delta_{\mu\lambda}\delta_{\nu\rho}$,
$\lambda_{r=2} = 1 + \frac{1}{\sqrt{6}}\frac{H_3}{H_8}$, $\lambda_{r=3} = 1 - \frac{1}{\sqrt{6}}\frac{H_3}{H_8}$ $c_r = 1, \frac{7}{4}, \frac{5}{4}$ for $r = 1, 2, 3$, respectively. For this purpose we apply the method of the effective Lagrangian.

The effective Lagrangian of constant chromomagnetic fields H_3 and H_8 at finite temperature can be written in the form:

$$L_{eff} = L^{(1)} + L^{(ring)} + \dots, \tag{2}$$

where the first term represents the one-loop contribution of the charged gluons:

$$\begin{aligned} L^{(1)} = & -\frac{gH_3}{2\pi\beta} \sum_{l=-\infty}^{\infty} \int\limits_{-\infty}^{\infty} \frac{dp_3}{2\pi} \sum_{n,\sigma} \ln[\beta^2 G_{r=1}^{-1}(p_3, H_3, T)] \\ & - \sum_{r=2,3} \sqrt{\frac{3}{2}} \lambda_r \frac{gH_8}{2\pi\beta} \sum_{l=-\infty}^{\infty} \int\limits_{-\infty}^{\infty} \frac{dp_3}{2\pi} \sum_{n,\sigma} \ln[\beta^2 G_r^{-1}(p_3, H_3, H_8, T)]. \end{aligned} \tag{3}$$

Here r marks the index of the charged basis , G_r is the corresponding propagator of the charged gluons in the external fields H_3 and H_8. The second term in Eq. (2) presents the contribution of daisy or ring diagrams of the charged gluons,

$$\begin{aligned} L_{ch}^{(ring)} = & -\frac{gH_3}{2\pi\beta} \sum_{l=-\infty}^{\infty} \int\limits_{-\infty}^{\infty} \frac{dp_3}{2\pi} \times \\ & \times \sum_{n,\sigma} \ln[1 + G_{r=1}(\epsilon_n^2, H_3, T) \Pi^{r=1}(H_3, T, n, \sigma)] \\ & - \sum_{r=2,3} \sqrt{\frac{3}{2}} \lambda_r \frac{gH_8}{2\pi\beta} \sum_{l=-\infty}^{\infty} \int\limits_{-\infty}^{\infty} \frac{dp_3}{2\pi} \times \\ & \times \sum_{n,\sigma} \ln[1 + + G_r(\epsilon_n^2, H_3, H_8, T) \Pi^r(H_3, H_8, T, n, \sigma)], \end{aligned} \tag{4}$$

and of neutral gluons,

$$\begin{aligned} L_{neut}^{(ring)} = & -\frac{1}{2\beta} \sum_{l=-\infty}^{\infty} \int\limits_{-\infty}^{\infty} \frac{dp_3}{2\pi} \ln[\omega_l^2 + \overline{p}^2 + \Pi'(H_3, H_8, T)] \\ & -\frac{1}{2\beta} \sum_{l=-\infty}^{\infty} \int\limits_{-\infty}^{\infty} \frac{dp_3}{2\pi} \ln[\omega_l^2 + \overline{p}^2 + \Pi''(H_3, H_8, T)]. \end{aligned} \tag{5}$$

These expressions include the PO of charged gluons averaged over physical states, which are also dependent on $H = H_3, H_8$, the level number $n = 0, 1, \ldots$, the spin projection $\sigma = \pm 1$, and the Debye masses of the neutral gluons Q^3_μ, Q^8_μ,

$$\Pi'(H,T) = \Pi'_{00}(k = 0, H, T),$$
$$\Pi''(H,T) = \Pi''_{00}(k = 0, H, T),$$

respectively. The averaged values of charged gluon PO taken in the state $n = 0$ and $\sigma = +1$ give the magnetic masses of the transversal modes. The quantities $\Pi'(H,T)$ and $\Pi''(H,T)$ are the zero-zero components of the corresponding neutral gluon polarization operators calculated in the external fields $H = H_3, H_8$ at finite temperature and taken at zero momentum. The ring contribution to the L_{eff} has to be calculated when the vacuum magnetization at non-zero temperature is investigated. These diagrams account for long range correlations at finite temperature [4].

The detailed evaluations of the one-loop effective Lagrangian in finite-temperature $SU(2)$ gluonodynamics can be found in Ref.[4]. Performing them for our case we arrive at the following result for the high-temperature limit of $L^{(1)}$:

$$\begin{aligned} L^{(1)} = -\frac{H_3^2}{2} - \frac{11}{32}\frac{g^2}{\pi^2}H_3^2 \ln[\frac{T}{\mu}] + \frac{1}{3\pi}(gH_3)^{\frac{3}{2}}\frac{T}{3\pi} \\ -\frac{H_8^2}{2} - \frac{11}{16}\frac{g^2}{\pi^2}H_8^2 \ln[\frac{T}{\mu}] + (\lambda_2^{\frac{3}{2}} + |\lambda_3|^{\frac{3}{2}})(\frac{3}{2})^{\frac{3}{4}}(gH_8)^{\frac{3}{2}}\frac{T}{3\pi} \\ +i[(gH_3)^{\frac{3}{2}} + (\lambda_2^{\frac{3}{2}} + |\lambda_3|^{\frac{3}{2}})(\frac{3}{2})^{\frac{3}{4}}(gH_8)^{\frac{3}{2}}]\frac{T}{2\pi}, \end{aligned} \tag{6}$$

where $T \gg \sqrt{gH_{3,\,8}} \gg \mu$, μ is a renormalization point. The imaginary part in the expression (6) signals for the vacuum instability and must be considered carefully. Namely, as it will be shown below, the inclusion of ring diagrams , L^{ring}, leads to canceling the imaginary parts so that the whole expression L_{eff} becomes real. To see this, let us consider the contribution of the ring diagrams which correspond to the unstable modes of the charged gluons:

$$L^{(ring)}_{unstable} = -\frac{gH_3}{2\pi\beta}\sum_{l=-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{dp_3}{2\pi}\times$$
$$\times \ln[1+(\omega_l^2+p_3^2-gH_3)^{-1}\Pi^{r=1}(H_3,T)]$$
$$- \sum_{r=2,3}\sqrt{\frac{3}{2}}\lambda_r\frac{gH_8}{2\pi\beta}\sum_{l=-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{dp_3}{2\pi}\times$$
$$\times \ln[1+(\omega_l^2+p_3^2-\sqrt{\frac{3}{2}}\lambda_r gH_8)^{-1}\Pi^{r}(H_3,H_8,T)], \quad (7)$$

where $\omega_l = 2\pi l T,\ \ l = 0, \pm 1, \ldots,$ are the Matsubara frequencies. To obtain (7) one has merely to put $n = 0$ and $\sigma = +1$ in the expression for L^{ring}_{ch}. An elementary integration gives

$$L^{(ring)}_{unstable} = -\frac{gH_3T}{2\pi}[\Pi^{r=1}(H_3,T)-gH_3]^{\frac{1}{2}}$$
$$- \sum_{r=2,3}\sqrt{\frac{3}{2}}\lambda_r\frac{gH_8T}{2\pi}[\Pi^{r}(H_3,H_8,T)-\sqrt{\frac{3}{2}}\lambda_r gH_8]^{\frac{1}{2}}$$
$$-i[(gH_3)^{\frac{3}{2}}+(\lambda_2^{\frac{3}{2}}+|\lambda_3|^{\frac{3}{2}})(\frac{3}{2})^{\frac{3}{4}}(gH_8)^{\frac{3}{2}}]\frac{T}{2\pi}. \quad (8)$$

From Eq.(6) and (8) it is seen that imaginary parts are cancelled out in the total. The final effective Lagrangian L_{eff} is real if the relations

$$\Pi^{r=1}(H_3,T) > gH_3,$$

and

$$\Pi^{r=2,3}(H_3,H_8,T) > \sqrt{\frac{3}{2}}\lambda_{2,3}gH_8$$

hold.

In one-loop order the neutral gluon contribution is a trivial H- independent constant which can be omitted. However, these fields are long range states and give H-dependent effective Lagrangian through the correlation corrections depending on the temperature and external fields. Below, only the longitudinal neutral modes are included because their Debye's masses are nonzero. The corresponding effective Lagrangian is easily calculated and has the form [4]

$$
\begin{aligned}
L^{(ring)}_{neut} &= -\frac{T^2}{24}[\Pi'(H_3,H_8,T)+\Pi''(H_3,H_8,T)] \\
&+\frac{T}{12\pi}[\Pi'(H_3,H_8,T)+\Pi''(H_3,H_8,T)]^{\frac{3}{2}} \\
&-\frac{1}{32\pi^2}[\Pi'(H_3,H_8,T)+\Pi''(H_3,H_8,T)]^2 \\
&(\log[4\pi T[\Pi'(H_3,H_8,T)+\Pi''(H_3,H_8,T)]^{-\frac{1}{2}}+\frac{3}{4}-\gamma]).
\end{aligned} \tag{9}
$$

Evaluating the Debye masses of the neutral gluons Q^3_μ, Q^8_μ gives the following results (see for details [4]):

$$
\begin{aligned}
(m'_D)^2 &= \Pi'_{00}(k=0,H,T) = \tfrac{2}{3}g^2T^2 - \tfrac{g^2T}{\pi}\times \\
&\times[(gH_3)^{\frac{1}{2}} + (\lambda_2^{\frac{1}{2}} + |\lambda_3|^{\frac{1}{2}})(\frac{3}{2})^{\frac{1}{4}}(gH_8)^{\frac{1}{2}}]] \\
(m''_D)^2 &= \Pi''_{00}(k=0,H,T) \\
&= \tfrac{2}{3}g^2T^2 - \tfrac{g^2T}{\pi}(\lambda_2^{\frac{1}{2}} + |\lambda_3|^{\frac{1}{2}})(\tfrac{3}{2})^{\frac{1}{4}}(gH_8)^{\frac{1}{2}}.
\end{aligned} \tag{10}
$$

Substituting expressions (10) into $L^{(ring)}_{neut}$ we obtain the correlation corrections due to neutral gluons:

$$
L^{(ring)}_{neut} = \frac{g^2T^3}{24\pi}[(gH_3)^{\frac{1}{2}} + 2(\lambda_2^{\frac{1}{2}} + |\lambda_3|^{\frac{1}{2}})(\frac{3}{2})^{\frac{1}{4}}(gH_8)^{\frac{1}{2}}], \tag{11}
$$

where the H-independent terms were skipped. Thus, the vacuum magnetization at high temperature $T \gg \sqrt{gH_{3,8}}$ will be investigated within the following effective Lagrangian:

$$
\begin{aligned}
L^{(eff)} &= -\frac{H_3}{2} - \frac{H_8}{2} - \frac{11}{32}\frac{g^2}{\pi^2}H_3^2\ln[\frac{T}{\mu}] - \frac{11}{16}\frac{g^2}{\pi^2}H_8^2\ln[\frac{T}{\mu}] \\
&+[(gH_3)^{\frac{3}{2}} + (\lambda_2^{\frac{3}{2}} + |\lambda_3|^{\frac{3}{2}})(\frac{3}{2})^{\frac{3}{4}}(gH_8)^{\frac{3}{2}}]\frac{T}{3\pi} \\
&-\frac{gH_3T}{2\pi}[\Pi^{r=1}(H_3,T) - gH_3]^{\frac{1}{2}} \\
&-\sum_{r=2,3}\sqrt{\frac{3}{2}}\lambda_r\frac{gH_8T}{2\pi}[\Pi^r(H_3,H_8,T) - \sqrt{\frac{3}{2}}\lambda_r gH_8]^{\frac{1}{2}} \\
&+\frac{g^2T^3}{24\pi}[(gH_3)^{\frac{1}{2}} + 2(\lambda_2^{\frac{1}{2}} + |\lambda_3|^{\frac{1}{2}})(\frac{3}{2})^{\frac{1}{4}}(gH_8)^{\frac{1}{2}}] + O(g^3).
\end{aligned} \tag{12}
$$

This expression includes the contributions of $L^{(1)}$ as well as $L^{ring}_{unstable}$ and L^{ring}_{neut}. Notice that the quantity $L^{ring}_{unstable}$ has the order $g^{\frac{9}{4}}$ in coupling constant g , whereas the order of L^{ring}_{neut} is $g^{\frac{5}{2}}$. In other wards, the contribution of the neutral gluons does not play essential role in generation of external fields and this part cannot be taken into account. This is natural on general grounds because the neutral gluon field is stable at the tree and the one-loop levels. So, one has not to expect any role of this sector in the field generation. Thus, in this approximation, $L^{(eff)}$ is equal to the effective Lagrangian (12) without the terms in the last line.

Our problem is divided into two separate parts: first, one has to calculate the spontaneously generated fields in the vacuum and, second, to compute $\Pi^r(H, T, n, \sigma)$, which are the average values of the charged gluon PO taken in the tree level states.

To derive the strengths of the generated fields one has to solve the set of the stationary equations:

$$\frac{\partial L^{(eff)}}{\partial H_3} = 0,$$

$$\frac{\partial L^{(eff)}}{\partial H_8} = 0.$$

There are three nontrivial solutions:

$$H_3 = 0, \quad H_8 = (\frac{3}{2})^{\frac{3}{2}} \, \frac{g^3 T^2}{\pi^2} \tag{13}$$

$$H_3 = \frac{1}{4} \, (1 + \frac{1}{\sqrt{2}})^2 \, \frac{g^3 T^2}{\pi^2}, \quad H_8 = 0 \tag{14}$$

and

$$H_3 = 0.2976 \, \frac{g^3 T^2}{\pi^2}, \quad H_8 = 0.9989 \, (\frac{3}{2})^{\frac{3}{2}} \, \frac{g^3 T^2}{\pi^2}. \tag{15}$$

The terms of L^{eff} that depend on the magnetic masses of the transversal modes are not included in the expressions (13)-(15) because their contributions are of higher order in g. Note that the latter of these configurations corresponds to the minimum of the EP and we,therefore, conclude that both chromomagnetic fields have to arise spontaneously at high temperature.

In determining the solutions (13) -(15), the logarithmic terms $\sim \ln[\frac{T}{\mu}]$ were omitted as negligibly small. This approximation is appropriate for the region $T \geq T_d$, where T_d is the deconfinement phase transition temperature.

However, in the limit $T \to \infty$, the logarithmic terms become large and should be accounted for. To analyze the asymptotic region the solutions of the above stationary equations must be rewritten in terms of the effective coupling constant

$$g^2_{eff} \approx (\frac{11}{16\pi^2} \ln[\frac{T}{\mu}])^{-1}.$$

For this purpose one has merely to eliminate the tree-level terms in the $L^{(eff)}$. It turns out that, at $T \gg T_d$, the field configuration

$$gH_3 = \frac{1}{4}\,(1 + \frac{1}{\sqrt{2}})^2\,\frac{g^4_{eff}T^2}{\pi^2}, \quad gH_8 = 0 \tag{16}$$

delivers the global minimum of the EP. That is only the isotopic chromomagnetic field is generated at asymptotically high temperatures. If the temperature decreases, the hypercharge H_8 field appears below some temperature T_0, and in the deconfinement region $T_0 > T \geq T_d$, where the terms $\sim \ln[\frac{T}{\mu}]$ are small in comparison with the tree-level ones, both fields are present.

3 Gluon polarization operator

The next question that must be answered is whether the obtained in (15) chromomagnetic fields H_3 and H_8 are stable. This sufficiently complicate problem requires an explicit calculation of the polarization operator of the charged gluons in the external fields H_3 and H_8 .

To calculate the PO we make use of the proper time representation and the Schwinger operator formalism. The PO of the charged gluons in a chromomagnetic field at nonzero temperature can be written as

$$\Pi^r_{\mu\nu} = \frac{g^2}{\beta}\; c_r \sum_{k_4} \int \frac{d^3k}{(2\pi)^3} \Pi^r_{\mu\nu}(k, P), \quad r = 1, 2, 3, \tag{17}$$

where

$$\begin{aligned}
\Pi^r_{\mu\nu}(k, P) &= k^{-2}\Big\{\Gamma_{\mu\alpha,\rho} G_{r\ \alpha\beta}(P-k)\Gamma_{\nu\beta,\rho} + (P-k)_\mu D(P-k)k_\nu + \\
&\quad + k_\mu D(P-k)(P-k)_\nu + \\
&\quad + k^2\Big[G_{r\ \mu\nu}(P-k) - 2G_{r\ \nu\mu}(P-k) + \delta_{\mu\nu} G_{r\ \rho\rho}(P-k)\Big]\Big\}, \\
\Gamma_{\mu\alpha,\rho} &= \delta_{\mu\alpha}(2P-k)_\rho + \delta_{\alpha\rho}(2k-P)_\mu + \delta_{\mu\rho}(P+k)_\alpha,
\end{aligned}$$

where

$$G_{r=1\ \mu\nu}(P) = -[P^2 + 2igF_{3\mu\nu}]^{-1},$$

$$G_{r=2,3\ \mu\nu}(P) = -[P^2 + \sqrt{6}i\lambda_{r=2,3}gF_{8\mu\nu}]^{-1}, \qquad D(P) = -\frac{1}{P^2},$$

are the propagators of the charged gluons and corresponding ghosts, $\beta = \frac{1}{T}$, $k_4 = \frac{2\pi l}{\beta}$, $l = 0, \pm 1, \pm 2, \ldots$, $P_\mu = i\partial_\mu + gB_{3\mu}$ for $r = 1$ and $P_\mu = i\partial_\mu + \sqrt{\frac{3}{2}}\lambda_r gB_{8\mu}$ for $r = 2, 3$, respectively, the constant c_r is defined above. We restrict our consideration to the case of a high temperature limit. In Eq.(17) this limit corresponds to the $l = 0$ term in the sum over k_4 . To evaluate the expression for the PO we used the Schwinger proper-time method modified for the case of the high temperature (see Ref. [5] for details). Thus, averaging over the physical states the quantities for the gluon PO and the Debye mass squared of charged gluons and performing integrations in the high-temperature limit, $\frac{gH}{T^2} \ll 1$, we obtain

$$
\begin{aligned}
&\Pi^{r=1}(P_4=0,P_3=0,H_3,T,n,\sigma=+1)=\\
&\qquad=\frac{g^2}{4\pi}\sqrt{gH_3}T((4n+11.44)+i(10n+7)),\\
&\Pi^{r=2,3}(P_4=0,P_3=0,H_3,H_8,T,n,\sigma=+1)=\\
&\qquad=\frac{3g^2}{8\pi}c_{r=2,3}\ (\frac{3}{2})^{\frac{3}{4}}\ \sqrt{\lambda_{r=2,3}}\ \sqrt{gH_8}\times\\
&\qquad\times T((4n+11.44)+i(10n+7)),\\
&\Pi^{r=1}(P_4=0,P_3=0,H_3,T,n,\sigma=-1)=\\
&\qquad=\frac{g^2}{4\pi}\sqrt{gH_3}T((4n+15.62)+i(2n+9.69)),
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
&\Pi^{r=2,3}(P_4=0,P_3=0,H_3,H_8,T,n,\sigma=-1)=\\
&\qquad=\frac{3g^2}{8\pi}c_{r=2,3}\ (\frac{3}{2})^{\frac{3}{4}}\ \sqrt{\lambda_{r=2,3}}\ \sqrt{gH_8}\times\\
&\qquad\times T((4n+15.62)+i(2n+9.69)),\\
&\Pi_{44}^{r=1}(P_4=0,P_3=0,H_3,T,n)=\\
&\qquad=\frac{g^2T^2}{2}+\frac{g^2}{4\pi}\sqrt{gH_3}T((4n+6)+i(6n+9)),\\
&\Pi_{44}^{r=2,3}(P_4=0,P_3=0,H_3,H_8,T,n)=\\
&\qquad=g^2T^2+\frac{3g^2}{8\pi}c_{r=2,3}\ (\frac{3}{2})^{\frac{3}{4}}\ \sqrt{\lambda_{r=2,3}}\ \sqrt{gH_8}\times\\
&\qquad\times\ T((4n+6)+i(6n+9)).
\end{aligned}
\tag{19}
$$

From Eqs. (15) and (18) it is seen that the real parts of the PO are positive in the ground and excited states. The imaginary parts in the expressions $\Pi(P_4=0,P_3=0,H,T,n,\sigma)$ and Π_{44} occur due to the nonanalyticity of a number of terms in the correspondent integrals. The imaginary part of the radiation corrections describes the decay of the state owing to transitions to the states with lower energies.

The imaginary part of Π_{44} for $n=0$ describes the Landau damping of the ground state plasmon quasi-particles. It is important to note that the imaginary parts entering the Π_{44} and the $\Pi(P_4=0,P_3=0,H,T,n=0,\sigma=+1)$ are of the same order of magnitude. Since a spin interaction does not affect the former correction and whereas the tachyonic state in the field is excited just due to the spin interaction of charged gluons, one has to conclude that the non-zero imaginary part of the latter function does not correspond to the instability of the chromomagnetic fields and also describes a usual damping of states at finite temperature. Thus, to verify whether or not the radiation corrections in the external fields stabilize the spectrum at high temperature one should calculate the gluon effective mass squared determined by the real part of the function $\Pi(P_4 = 0, P_3 = 0, H, T, n = 0, \sigma = +1)$ at one-loop level. If it is positive - the spectrum, and hence the vacuum, is stable.

4 Discussion

We investigated the QCD restored phase. As it was found from the EP accounting for the one-loop plus daisy diagrams the vacuum with non-zero Abelian chromomagnetic fields H_3 and H_8 is favorable energetically. It is stable due to the magnetic masses of charged gluons which have to be included into consideration when the value of the background fields is estimated. These masses are computed from the gluon one-loop polarization operator in the external fields. As it occurs, the charged gluon spectrum in the fields accounting for the tree plus the one-loop corrections is stable in a wide interval of temperature above the T_d. Important notice, in the field presence the gluon magnetic mass $m^2_{magn.} \sim g^2 (gH)^{1/2} T$ is generated in one-loop order, in contrast to what happens in the case of the trivial vacuum where the mass $m^2_{magn.} \sim g^4 T^2$ is a non-perturbative effect. Since the order of the magnetic vacuum field is $(gH)^{1/2} \sim g^2 T$, the order of the magnetic mass squared is $\sim g^4 T^2$. That is, the order of the magnetic mass is the same in both calculation methods. Clearly that the former case is also non-perturbative because the field is taken into account exactly through the Green functions. If one accounts for the magnetic field perturbatively, zero result follows [7]. Of course, the stabilization of the gluon spectrum by radiation corrections in the fields is an interesting fact which could not be expected beforehand. It was observed already in $SU(2)$ gluodynamics [8] and here for the $SU(3)$ gauge group. One may believe that this is the case for other non-Abelian gauge groups and therefore the stabilization is the reflection of the intrinsic dynamics of fields. Note here that other mechanism of the field stabilization was discussed in Ref.[3] which takes into account the generation of the electrostatic gauge field potential (A_0 condensate). But this picture was not investigated consistently since the common generation of the A_0 and magnetic fields has not been considered. Some aspects of the influence of the

A_0 condensate on magnetic field have been investigated recently in one-loop order in Ref. [6].

Consider in more detail the values of the gluon magnetic mass determined in different calculations and compare these with the value of the vacuum magnetic field. In recent paper [6] in $SU(2)$ gluodynamics to stabilize the one-loop effective potential the gluon magnetic mass $m_{magn.}$ of order $\sim cg^2T$ was introduced on heuristic grounds. Then, in particular, it has been found that for $m_{magn.} \geq 0.388g^2T$ the effective potential has a global minimum at $H = 0$. Hence it was concluded that a sufficiently heavy magnetic mass leads to the trivial vacuum of the deconfinement phase. This critical value is close to the magnetic masses determined in a number of lattice simulations : 0.505 g^2T [9], 0.360 g^2T [10]. It is interesting to compare our result for the magnetic mass identified with the effective mass of gluon $M^2_{eff.}(H) = 11,44\frac{g^2}{4\pi}(gH)^{1/2}T - gH$ and the field $(gH)^{1/2} = \frac{g^2}{2\pi}T$ for $SU(2)$ sector. Simple estimate gives $M_{eff.} = 0.345g^2T$ that is close to the value derived by Philipsen. For this values we observed the stabilization of the magnetized vacuum. On the other hand, this value is insufficient to have a zero vacuum field, if the approach of the paper [6] is adopted. Moreover, if one takes into account the structure of the magnetic mass, $m^2_{magn.} \sim \sqrt{gH}g^2T$, there is no possibility to have zero for the generated field. So, we believe that the magnetized vacuum has to be considered seriously not an artificial mathematical fact.

Other important point which we are going to discuss is the influence of higher loop contributions. First we note that the one loop plus daisy graphs account for the long distance contributions and give the main effects. It was realized already in the related problem on the electroweak phase transition in strong magnetic fields [11], [12]. The results obtained within this EP are in a good agreement with that of found in the nonperturbative approach Refs. [13], [14] (see also recent survey [15]). The most important feature of this approximation is that the EP is real at sufficiently high temperatures and therefore the spontaneously generated magnetic fields are stable. We believe that the higher loop corrections will not change qualitatively the results obtained and the results on the stable magnetize vacuum survive. To check our results the resummations on the super daisy level have to be carried out. That is the problem for future.

It is interesting to compare our results with that of obtained in lattice calculations by Cea and Cosmai [16], [17]. In the former paper the creation of the colour Abelian chromomagnetic was investigated by means of the lattice Schrödinger functional. It was observed that at $T = 0$ the applied external chromomagnetic field is completely screened by the vacuum. At finite temperature the applied field is supported by the temperature and is increased with the growth of temperature. That is in correspondence with our calculations. In the latter paper the influence of the external fields on the deconfinement phase transition has been investigated and an intimate connection between Abelian chromomagnetic field and colour confinement was observed. This interesting result is not directly related with obtained in the present paper because we do not consider the field as external one. From our results it follows that in the deconfinement phase the Abelian chromomagnetic fields have to present. So, we have to answer how the spontaneous vacuum magnetization affects the temperature of the phase transition.

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