Chiral Symmetry Breaking in QCD

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Abstract

Chiral Symmetry Breaking (CSB) is derived in QCD starting from the QCD Lagrangian and using Field Correlators Method (FCM). The kernel in the resulting equations responsible for CSB is directly connected to confinement, and therefore both phenomena occur and vanish together as supported by lattice data.

Chiral Lagrangian and quark-meson Lagrangian are derived with explicit coefficients and compared to standard expressions. Spectrum of Nambu-Goldstone mesons and their radial excitations is calculated in good agreement with experiment.

1 Introduction

Chiral Symmetry Breaking (CSB) is known to govern the low-energy properties of hadrons [1]. The Effective Chiral Lagrangians (ECL) have been proposed [2, 3] before the advent of QCD, and the phenomenon of CSB and Nambu-Goldstone theorem was established more than 40 years ago [4].

When the QCD era began, it was natural to ask whether CSB can be derived from the QCD Lagrangian. The direct way being difficult, some indirect arguments have been found in favor of CSB. In 1971 Dolgov and Zakharov [5] and in 1980 Coleman and Witten [6] noticed that the analytic properties of the anomalous triple correlator of currents cannot be ensured unless there is a pseudoscalar pole (massless in the chiral limit). This signifies CSB at least in the large N_c limit, but tells nothing about the mechanism of CSB and field configurations responsible for it. For the latter instantons have been proposed and studied in numerous papers. It was shown that instantons indeed can explain CSB both qualitatively and quantitatively, if Effective Quark Lagrangian (EQL) is taken in the 'tHooft form and instanton density corresponds to the gluon condensate value [8]. Another evidence which seemingly favored instanton model, is the quark mode distribution, which should be nonzero for small eigenvalues [9] and was measured on the lattice (see [10] and refs therein). On more theoretical ground the instantons have been shown to create the stochastic

matrix ensemble for eigenvalues [11], and the so-called Wigner's semicircle for the spectrum which automatically ensures CSB.

The measurements however revealed a more sophisticated picture with correlations not specific for instantons (see [12] and refs therein) so the dynamics of CSB should be rather complicated. In addition the ansatz of the 'tHooft determinant for the EQL in the instanton model was seriously questioned [13] and more complicated EQL was computed directly in the model [14, 15] with yet unknown properties.

The most serious drawback of instanton model is that it does not ensure confinement, hence it may be responsible for only one part of total picture. Therefore in [15] another model was suggested, containing instantons together with background vacuum fields ensuring confinement. The model was shown to give a reasonable qualitative picture, but instantons are essential element producing CSB with density comparable to the confining background. However in [16] it was found that the Casimir scaling for charges in different SU(3) representations strongly limits the density of instantons, thus making them not the chief source of CSB. Therefore background fields themselves should ensure CSB, and what was found recently in this field is reported below. In 1997 it was proved [17] that CSB occurs in the example of heavy-light system due to the same field configurations (field correlators) which create the confining string. Recently [18, 20] the dynamics of CSB was studied in more general context, and ECL [18], Nambu-Goldstone spectrum [19] and CSB order parameters [20] have been obtained from the QCD Lagrangian without extra parameters. It is a purpose of the present talk to give a brief outline of the derivation and results of these investigations. The plan of the talk is as follows. In section 2 the derivation of the Effective Quark Lagrangian (EQL), Effective Quark-Meson Lagrangian (QML) and ECL are given, in section 3 properties of those are discussed and the Gell-Mann-Oakes-Renner (GOR) relations are derived, as well as the spectrum of Nambu-Goldstone mesons and their radial excitations. In section 5 the role of QML in the description of the decays and channel coupling is described, the concluding section summarizes the results.

2 Derivation of Effective Lagrangian

One starts with the QCD partition function in the Euclidean space-time.

$$Z = \int DAD\psi D\psi^+ e^{-S_0(A)+\int {}^f\psi^+(i\hat{\partial}+im+g\hat{A}){}^f\psi d^4x} \tag{1}$$

where $S_0(A) = \frac{1}{4}\int (F^a_{\mu\nu}(x))^2 d^4x$, m is the current quark mass (mass matrix $\hat{m}$ in SU(3)), and the quark operator ${}^f\psi_{a\alpha}(x)$ has flavor index $a(f = 1, ...n_f)$, color index $a(a = 1, ...N_c)$ and Lorenz bispinor index $\alpha(\alpha = 1, 2, 3, 4)$, and use the contour gauge to express $A_\mu(x)$ in terms of $F_{\mu\nu}$. One has for the contour

$z_\mu(s,x)$ starting at point x and ending at $Y = z(0,x)$

$$\begin{aligned} A_\mu(x) &= \int_0^1 ds \frac{\partial z_\nu(s,x)}{\partial s} \frac{\partial z_\rho(s,x)}{\partial x_\mu} F_{\nu\rho}(z(s)) \\ &\equiv \int_Y^x d\Gamma_{\mu\nu\rho}(z) F_{\nu\rho}(z). \end{aligned} \tag{2}$$

Integrating out the gluonic fields $A_\mu(x)$, one obtains

$$Z = \int D\psi D\psi^+ e^{\int {}^f\psi^+ (i\hat{\partial} + im) {}^f\psi d^4x} e^{L^{(2)}_{EQL} + L^{(3)}_{EQL} + \ldots} \tag{3}$$

where the EQL proportional to $\langle\langle A^n \rangle\rangle$ is denoted by $L^{(n)}_{EQL}$,

$$\begin{aligned} L^{(2)}_{EQL} = \frac{g^2}{2} \int d^4x d^4y \; {}^f\psi^+_{a\alpha}(x) \; {}^f\psi_{b\beta}(x) \; {}^g\psi^+_{c\gamma}(y) \; {}^g\psi_{d\varepsilon}(y) \times \\ \times \langle A^{(\mu)}_{ab}(x) A^{(\nu)}_{cd}(y) \rangle \gamma^{(\mu)}_{\alpha\beta} \gamma^{(\nu)}_{\gamma\varepsilon} \end{aligned} \tag{4}$$

Average of gluonic fields can be computed using (2) as

$$\begin{aligned} & g^2 \langle A^{(\mu)}_{ab}(x) A^{(\nu)}_{cd}(y) \rangle \\ & \quad = \frac{\delta_{bc}\delta_{ad}}{N_c} \int_0^x du_i \alpha_\mu(u) \int_0^y dv_k \alpha_\nu(v) D(u-v) (\delta_{\mu\nu}\delta_{ik} - \delta_{i\nu}\delta_{k\mu}), \end{aligned} \tag{5}$$

where $D(x)$ is the correlator $\langle F(x)F(0) \rangle$. As it was argued in [17] the dominant contribution at large distances from the static antiquark is given by the color-electric fields, therefore we shall write down explicitly $L^{(2)}_{EQL}(el)$ for this case, i.e. taking $\mu = \nu = 4$. As a result one has

$$\begin{aligned} L^{(2)}_{EQL}(el) = \frac{1}{2N_c} \int d^4x \int d^4y \; {}^f\psi^+_{a\alpha}(x) \; {}^f\psi_{b\beta}(x) \; {}^g\psi^+_{b\gamma}(y) \; {}^g\psi_{a\varepsilon}(y) \times \\ \times \gamma^{(4)}_{\alpha\beta} \gamma^{(4)}_{\gamma\varepsilon} J(x,y) \end{aligned} \tag{6}$$

where $J(x,y)$ is

$$J(x,y) = \int_0^x du_i \int_0^y dv_i D(u-v), \quad i = 1,2,3. \tag{7}$$

One can form bilinears $\Psi^{fg}_{\alpha\varepsilon} \equiv {}^f\psi^+_{a\alpha} \; {}^g\psi_{a\varepsilon}$ and project using Fierz procedure given isospin and Lorentz structures, $\Psi^{fg}_{\alpha\varepsilon} \to \Psi^{(n,k)}(x,y)$. With the help of the standard bosonization trick

$$e^{-\Psi\tilde{J}\Psi} = \int (\det \tilde{J})^{1/2} D\chi \exp[-\chi\tilde{J}\chi + i\Psi\tilde{J}\chi + i\chi\tilde{J}\Psi] \tag{8}$$

$$Z = \int D\psi D\psi^+ D\chi \exp L_{QML} \tag{9}$$

one obtains the effective Quark-Meson Lagrangian (QML)

$$L^{(2)}_{QML} = \int d^4x \int d^4y \, \{ \, {}^f\psi^+_{a\alpha}(x) \\ [(i\hat{\partial} + im)_{\alpha\beta}\delta(x-y) + iM^{(fg)}_{\alpha\beta}(x,y)] \, {}^g\psi_{a\beta}(y) - \\ -\chi^{(n,k)}(x,y)\tilde{J}(x,y)\chi^{(n,k)}(y,x)\Big\} \quad (10)$$

and the effective quark-mass operator is

$$M^{(fg)}_{\alpha\beta}(x,y) = \sum_{n,k} \chi^{(n,k)}(x,y)\bar{O}^{(k)}_{\alpha\beta}t^{(n)}_{fg}\tilde{J}(x,y). \quad (11)$$

The QML in Eq.(10) $L^{(2)}_{QML}$ contains functions $\chi^{(n,k)}$ which are integrated out in (11), and the standard way is to find $\chi^{(n,k)}$ from the stationary point of $L^{(2)}_{QML}$. Limiting oneself to the scalar and pseudoscalar fields and using the nonlinear parametrization one can write for the operator $\hat{M}$ in (10)

$$\hat{M}(x,y) = M_S(x,y)\hat{U}(x,y), \hat{U} = exp(i\gamma_5\hat{\phi}), \hat{\phi}(x,y) = \phi^f(x,y)t^f. \quad (12)$$

After integrating out the quark fields one obtains the ECL in the form

$$L^{(2)}_{ECL}(M_S,\hat{\phi}) = -2n_f(\tilde{J}(x,y))^{-1}M^2_S(x,y) \\ + N_c tr\log[(i\hat{\partial} + im)\hat{1} + iM_S\hat{U}]. \quad (13)$$

The stationary point equations $\frac{\delta L^{(2)}_{ECL}}{\delta M_s} = \frac{\delta L^{(2)}_{ECL}}{\delta\hat{\phi}} = 0$ at $\hat{\phi} = \hat{\phi}_0$, $M_s = M^{(0)}_s$ immediately show that $\hat{\phi}_0 = 0$ and $M^{(0)}_s$ satisfies nonlinear equation

$$iM^{(0)}_S(x,y) = \frac{N_c}{4}trS\tilde{J}(x,y) = N_c(\gamma_4 S\gamma_4)\tilde{J}(x,y), \\ S(x,y) = -[i\hat{\partial} + im + iM_S\hat{U}]^{-1}_{x,y}. \quad (14)$$

The solution of (14) was studied in [17] and it was shown that in the limit of small T_g one obtains for $M_S(x,y)$ a localized expression

$$M_S(x,y) \approx \sigma|\mathbf{x}|\delta^{(4)}(x-y), |\mathbf{x}| \gg T_g. \quad (15)$$

The ECL (13) with the operator M_s in (15) signals both confinement and CSB which create the Nambu-Goldstone meson spectrum, discussed in the next section.

3 Nambu-Goldstone spectrum from ECL

Expanding ECL (13) in the powers of field $\hat{\phi}$ one arrives at the expression

$$W^{(2)}(\phi) = \frac{N_c}{2}\int \phi_a(k)\phi_a(-k)\bar{N}(k)\frac{d^4k}{(2\pi)^4} \quad (16)$$

with

$$\bar{N}(k) = \frac{1}{2} tr\{(\Lambda_+ M_S)_0 + \int d^{(4)} z e^{ikz} \Lambda_+(0,z) M_S(z) \Lambda_-(z,0) M_S(0)\} \quad (17)$$

and $\Lambda_\pm = (\hat{\partial} \pm m \pm M_s)^{-1}$.

The pion mass is proportional to $\bar{N}(0)$, which can be written as

$$\bar{N}(0) = \frac{1}{2} tr(\Lambda_+ M_s \Lambda_-(\hat{\partial} - m)) = \frac{m}{2} tr\Lambda_+ = -\frac{m}{4N_c}\langle\bar{\psi}\psi\rangle. \quad (18)$$

Taking into account that $\phi_a = \frac{2\pi_a}{f_\pi}$, $f_\pi = 93$ MeV, one obtains GOR relations

$$2m_\pi^2 f_\pi^2 = (m_u + m_d)|\bar{\psi}\psi|, \quad |\bar{\psi}\psi| = |\bar{u}u| + |\bar{d}d|, \quad (19)$$

and similar relations for m_k^2, m_η^2 [19]. An interesting question now is what happens with the pion radial excitations, and this will bring us to the point of connection between quark-model poles and Nambu-Goldstone poles in the total PS Green's function G_{ab} – correlator of the currents $J_a^{(5)}(x) = \bar{\psi}(x)\gamma_5 t_a \psi(x)$. At large N_c one can expand in powers of $\hat{\phi} = \frac{2\varphi_a t_a}{f_\pi} = O\left(\frac{1}{\sqrt{N_c}}\right)$ and keep the terms up to $O(\hat{\phi}^2)$ which allows to express G_{ab} in terms of the function $G_{ab}^{(0)} = tr[S(x,y)\gamma_5 t_b S(y,x)\gamma_5 t_a]$ and $G_{ab}^{(M)}$ and $G_{ab}^{(MM)}$ which contain one or two operators M_S as factors of $\gamma_5 t_a, \gamma_5 t_b$. Now each of the function $G^{(0)}, G^{(M)}, G^{(MM)}$ can be computed using spectral representation in the quark model without chiral degrees of freedom, e.g.

$$G^{(0}(k) = \sum_{n=0}^{\infty} \frac{c_n^2}{k^2 + m_n^2}, \quad G^{\{(M),(MM)\}} = \sum_{n=0}^{\infty} \frac{\{c_n c_n^{(M)}, (c_n^{(M)})^2\}}{k^2 + m_n^2} \quad (20)$$

Finally the total $G_{ab}(k) = \delta_{ab} G(k)$ describing the $q\bar{q}$ PS system with chiral field to the second order is written as

$$G(k) = -\frac{N_c}{2} \frac{\Psi(k)}{(k^2 + m_\pi^2)\Phi(k)},$$
$$\Phi(k) = \sum_{n=0}^{\infty} \frac{(c_n^{(M)})^2}{(k^2 + m_n^2)(m_n^2 - m_\pi^2)}. \quad (21)$$

One can see in (21) the pion pole given by GOR relation (19) and separated from the quark model dynamics, while the $q\bar{q}$ (quark model) poles at $k^2 = -m_n^2$ are shifted into a new position, e.g. the first pion radial excitation in shifted down

$$k^2 = -m_1^2(1 + \delta_1), m_1^2 \delta_1 = -\frac{c_1^2(m_1^2 - m_0^2)(m_0^2 - m_\pi^2)}{c_1^2(m_0^2 - m_\pi^2) + c_0^2(m_1^2 - m_\pi^2)}. \quad (22)$$

Resultingly using string Hamiltonian in [19] it was found that shifts are

$$\pi(1S),\ \ m_0 = 0.51\ \text{GeV}\ \ \to m_0' = 0.14\ \text{GeV}\ \ \text{(exact)}$$

$$\pi(2S),\ \ m_1 = 1.51\ \text{GeV}\ \ \to m_1' = 1.25\ \text{GeV}\ (\exp: 1.3\ \text{GeV})$$

$$\pi(3S),\ \ m_2 = 2.18\ \text{GeV}\ \ \to m_2' = 1.98\ \text{GeV}\ (\exp: 1.8\ \text{GeV})$$

and similar results for kaon radial excitations [19].

4 Using QML for decay and channel coupling

The QML, Eq. (10) with the chiral degrees of freedom to the lowest order can be rewritten as

$$\Delta L^{(1)} = \int \overline{\psi}(x)\sigma|\mathbf{x}|\gamma_5 \frac{\pi^A \lambda^A}{F_\pi}\psi(x) dt d^3x \tag{23}$$

Using Dirac equation for $\psi(x), \bar{\psi}(x)$ one can connect (23) with the standard Weinberg Lagrangian [2], also expanded to the lowest order

$$\Delta L^{ch} = g_A^q tr(\overline{\psi}\gamma_\mu\gamma_5\omega_\mu\psi), \omega = \frac{i}{2F_\pi}(u\partial_\mu u^+ - u^+\partial_\mu u),\ \ u = \sqrt{\hat{U}} \tag{24}$$

where the constant g_A^q should be taken as $g_A^q = 1$. This Lagrangian was tested for pionic transitions successfully in [21]. On the other hand the Lagrangian (23) was suggested in [22] as the basic Lagrangian for chiral decays of mesons and baryons. This decay Lagrangian couples only quark and chiral degrees of freedom and physically corresponds to the case when pion (kaon) is emitted or absorbed by the quark at the end of the string. Note that QML (23) does not contain any fitting parameters and therefore produces unambigious predictions for decay amplitudes. Recently [23] it was used to calculate the shift of the mass of $D_s(J^P = 0^+)$ due to the coupling to the channel DK. The resulting shift is expressed in terms of overlap integral of the s-quark wave-functions and constant $f_k \cong 1.2 f_\pi = 1.2 \cdot 94$ MeV, and was found to be equal to 100 MeV, in good agreement with experimental mass of 2317 MeV.

5 Conclusions

It is shown that the same bilinear field correlator $\langle F(x)F(y)\rangle \sim D(x-y)$, which is responsible for confinement produces also CSB with the chiral fields entering simply as a factor $\exp(i\hat{\phi}\gamma_5)$ multiplying the scalar confining potential $\sigma|\mathbf{r}|$. Therefore CSB and confinement should disappear at the same temperature T_c, in agreement with lattice data [24]. This combined chiral-confining dynamics naturally explains pattern of meson spectra, numerical values of f_π and quark condensate and decay transitions.

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