Current correlators in QCD: OPE versus large distance dynamics*

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Abstract

We analyze the structure of current-current correlators in coordinate space in large N_c limit when the corresponding spectral density takes the form of an infinite sum over hadron poles. The latter are computed in the QCD string model with quarks at the ends, including the lowest states, for all channels. The corresponding correlators demonstrate reasonable qualitative agreement with the lattice data without any additional fits. Different issues concerning the structure of the short distance OPE are discussed.

1 Introduction

Correlators of hadron currents (denoted as CC in what follows)

$$\mathcal{P}^{(c)}(x) = \langle j^{(c)}(x) j^{(c)}(0)^\dagger \rangle \tag{1}$$

are the basic elements in QCD which have been studied in the framework of perturbation theory [1] and also using the Operator Product Expansion

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(OPE) [2]. More recently CC have been the subject of exciting lattice calculations [3, 4] as a tool to study nonperturbative vacuum configurations. Since then it has been realized that CC can provide information of two sorts: a) one is encoded in the hadron mass poles m_n^2 and quark coupling constants c_n and can be associated with the long distance dynamics (LDD), b) another kind of information can be extracted from the behavior of $\mathcal{P}^{(c)}(x)$ at small x (or at large Q^2 from the Fourier transform $\mathcal{P}^{(c)}(Q)$) with the help of the OPE (see, e.g. [5]) where the coefficients represent vacuum matrix elements of local field operators.

We do not address general issues of the OPE structure here (see [8] and references therein in this respect). Instead we shall compare below the explicit expression of CC in LDD at large N_c with the lattice data [3, 4]. These data provide an essential information on CC in the range $0 \leq x \leq 1.5$ fm where both large distance dynamics (LDD) and small distance dynamics (SDD) are present and therefore one may check in principle i) the region of validity of the standard OPE, ii) the transition region from SDD to LDD and the quark-hadron duality (QHD), iii) the validity of specific vacuum models and finally iv) the validity of LDD.

In the latter case one has large N_c limit where there is a solid knowledge of the spectrum properties [9]. Some aspects of OPE at large N_c have been discussed recently in [10]. Let us remind that the masses and residues (properly normalized) of CC, m_n and c_n, are stable at large N_c limit, and recent lattice data (see [11] and references therein) confirm that typically corrections to stable quantities for $N_c = 4, .., 8$ are small.[1] Moreover at large N_c one can in principle compute the hadron spectrum (i.e. the set of m_n, c_n) quasiclassically in analytic form for all n and thus calculate CC explicitly. This was done for the masses (see [12] and references therein) and the residues ([13], see also [14]) and compared in [15] with OPE, SVZ sum rules and experiment. In doing so one encounters and solves several important problems, which we briefly comment on below.

First, the knowledge of m_n, c_n at large N_c allows to find the limit of the CC at large Q (small x) and compare it with the quark-partonic expression, thus checking the quark-hadron duality which was suggested long ago and studied since then in numerous works (see reviews [16] containing extensive lists of references to the original papers).

[1] In what follows the flavour-singlet channels where corrections can be large, are not considered.

Secondly, the large N_c LDD provides the small x OPE with coefficients depending only on the string tension σ, and this LDD OPE is to be distinguished from the standard, or SDD OPE. These two types of OPE are in principle distinct, however in the vector channel both agree with the experimental e^+e^- data, as was shown in [15]. Many efforts have been undertaken to match these OPE's (see, e.g. [18, 19, 20]), but here we do not follow this line.

Since theoretical CC in LDD contain no adjustable parameters at all, and are computed in terms of fixed values of σ and α_s only (and final forms of CC contain only two fixed masses, expressed through σ, α_s) this comparison has a form of a fixed prediction.

The data from [3] we are going to compare with were obtained on $16^3 \times 24$ lattice with the spacing 0.17 Fm and $\beta = 5.7$, while the author of [4] used the lattice of the total size 1.5 Fm and the spacing 0.13 Fm at $\beta = 5.9$ Both simulations were performed in quenched approximation, however the current fermions were taken into account differently. An interested reader is encouraged to consult the original papers [3, 4] for all technical details concerning the simulations.

2 The model

The model of zero-width equidistant resonances [23, 24, 25] has been analyzed in different respects in connection with OPE and large N_c QCD, where the main interest has been concentrated on the momentum-space representation [18, 19]. We are following a different path, keeping ourselves in coordinate space.

The main object of our interest is the current-current correlator in coordinate space given by (1), where the current $j^{(c)}(x) = \bar{\psi}(x)\Gamma_c\psi(x)$ is defined for the given channel c. We consider only flavor nonsinglet charged currents (of the type $\bar{u}\Gamma d$) in this paper. The matrix Γ_c carries Lorentz, flavor and color indices (with the latter structure being always $\delta_{\alpha\beta}$), corresponding to quantum numbers of the channel c. So we have for vector (ρ - channel), axial (a_1 - channel), pseudoscalar (π - channel) and scalar (a_0 - channel) the following expressions for the (charged) currents

$$j_\mu^{(v)}(x) = \bar{u}\gamma_\mu d\ ; \;\; j_\mu^{(a)}(x) = \bar{u}\gamma_\mu\gamma_5 d\ ; \;\; j^{(p)}(x) = \bar{u}i\gamma_5 d\ ;$$
$$j^{(s)}(x) = \bar{u}d \tag{2}$$

We adopt the standard normalization of [1, 2], which is different for neutral currents by a factor $1/\sqrt{2}$ from the one used in [5].

Needless to say that each channel has its own typical features and generally speaking should be analyzed separately. Our procedure, on the contrary, is rather uniform. Due to the lack of space we demonstrate here the results only for the vector channel. The situation in the other channels as well as the corresponding lattice data are discussed in details in the paper [32] this talk is based on.

It is convenient to factor out the free part of the correlator (1) and to define the ratio $R^{(c)}(x)$ as

$$R^{(c)}(x) = \frac{\mathcal{P}^{(c)}(x)}{\mathcal{P}^{(c)}_{free}(x)} \tag{3}$$

where by definition for the vector and axial channels the sum over Lorentz indices is always taken:

$$\mathcal{P}^{(v,a)}(x) = g^{\mu\nu}\,\mathcal{P}^{(v,a)}_{\mu\nu}(x) \tag{4}$$

The free part $\mathcal{P}^{(c)}_{free}(x)$ is given by the following expression in the chiral limit:

$$\mathcal{P}^{(v,a,p,s)}_{free}(x) = \frac{N_c}{\pi^4 x^6}\cdot(2,\ 2,\ -1,\ -1) \tag{5}$$

The important relation $R^{(c)}(x)$ has to obey is given by (see discussion in [32])

$$\lim_{x\to 0} R^{(c)}(x) = 1 \tag{6}$$

We find it instructive to explain our attitude, which in some respects is different from that of the cited papers [18, 19, 20]. Our basic assumption is that QCD exhibits confinement in the form of minimal area law in large N_c limit. Since we always have in mind large N_c, the picture of zero-width states is justified and the confinement property guarantees the spectrum to be discrete. We get for the imaginary part of polarization operator

$$\frac{1}{\pi}\,\mathrm{Im}\,\Pi(s) = \sum_{n=0}^{\infty} c_n\cdot\delta(s-m_n^2) \tag{7}$$

The mass spectrum is obtained quasiclassically to have the following form [15, 17, 29]

$$m_n^2 = m_0^2 + m^2 n \tag{8}$$

where the quantity m^2 is defined universally for all channels to be $m^2 = 4\pi\sigma$ as it was found by the quasiclassical analysis (see also [31] and references therein). Here σ is physical string tension. The residues for the ground state and the excited states are treated differently. As for the latter, they are fixed by the requirement of quark-hadron duality. The lowest state residue is chosen in different physically motivated ways for different channels (see below). Our approach is phenomenological in this respect, since we have not computed all c_n starting from some consistent theoretical scheme (it will be done elsewhere). It is worth stressing however, that we do not perform any kind of fitting procedure. In some sense, we have no fitting parameters at all in our formulas since all quantities like m_0, σ take their physical values. We do not try to fit the lattice data, instead, we use them for comparison with the results of quasiclassical spectrum of large N_c QCD.

2.1 Vector channel

For conserved vector current one can write the following expression in momentum space

$$\mathcal{P}^{(v)}_{\mu\nu}(q) = i \int d^4x\, \mathcal{P}^{(v)}_{\mu\nu}(x) \exp(iqx) = (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi^{(v)}(q^2) \tag{9}$$

and the determination of $\Pi^{(v)}(q^2)$ for $-q^2 = Q^2 > 0$ is of interest.

It was shown in [13, 15, 17] that for a system made of relativistic quarks connected by the straight-line string with tension σ one has

$$m_n^2 = 2\pi\sigma(2n_r + L) + m_0^2 \quad ; \quad c_n = \frac{N_c}{12\pi^2} \cdot 4\pi\sigma \tag{10}$$

which leads to

$$\Pi^{(v)}(-Q^2) = \frac{\lambda_\rho^2}{Q^2 + m_0^2} + \frac{N_c}{12\pi^2} \sum_{n=1}^{\infty} \frac{a_n}{Q^2 + m_n^2} \tag{11}$$

where

$$m_n^2 = m^2 n + m_0^2 \quad ; \quad a_n = m^2 = 4\pi\sigma \tag{12}$$

Despite we work in the large N_c framework, we have kept the factor N_c in front of the second term in (11) in order to make contact with the asymptotic expression (13). The residue of the first (ρ-meson) pole should in principle be calculated in the same dynamical framework, which is used for the spectrum calculation.[2] The condition (10) provides the quark-hadron duality in this channel:

$$\Pi^{(v)}(-Q^2) \stackrel{Q^2\to\infty}{\longrightarrow} -\frac{N_c}{12\pi^2}\log\left(\frac{Q^2}{\mu^2}\right) \tag{13}$$

Numerically we use values $m_0^2 = M_\rho^2 = 0.6$ GeV^2, $\lambda_\rho^2 = 0.047$ GeV^2 and $m^2 = 4\pi\sigma = 2.1$ GeV^2, corresponding to $\sigma = 0.17$ GeV^2. The value of λ_ρ^2 is consistent with the one used in [5, 20] and also the one computed in [30]. Notice that in the latter case we are not to take into account the large $O(\alpha_s)$ correction which is not seen in lattice simulations. It is interesting that the value of the lowest state residue λ_ρ^2 is different from the asymptotic value c_n by less than 15% in our case.

Since we want to compare our model with the lattice results, the Wick rotation has to be performed. The reader is referred to the paper [32] where all relevant formulas are collected. The resulting expression in coordinate space, corresponding to (11) is given by:

$$R^{(v)}(z) = \xi z^5 K_1(z) + \frac{bz^6}{2^7}\,\bar{p}_b(z) \tag{14}$$

where dimensionless distance $z = x_E m_0$ and mass ratio $b = m^2/m_0^2$ have been introduced and

$$\xi = \frac{\pi^2}{8}\left(\frac{\lambda_\rho^2}{m_0^2} - \frac{b}{4\pi^2}\right) \tag{15}$$

The universal function $\bar{p}_b(z)$ is given by

$$\bar{p}_b(z) = \int_0^\infty \frac{du}{u^2}\exp\left(-\frac{z^2}{4u} - u\right)\frac{1+\exp(-bu)(b-1)}{(1-\exp(-bu))^2} \tag{16}$$

As it has already been mentioned, we have three parameters in the expression (14): m_0, which fixes the overall scale of distance, λ_ρ^2 and b characterizing the spectrum, and all of them are fixed by their physical values. The resulting plot is shown on Figs. 1.1 and 1.2, to be compared with the lattice data of [3] and [4], respectively. One sees reasonable qualitative agreement with the lattice data. In fact, the results from [3] and from [4] were obtained

[2] This is also true for higher resonances, however, we choose another way and fix those residues by quark-hadron duality for those channels where c_n have not yet been calculated dynamically. Notice that in some channels like the vector one, dynamically calculated c_n indeed provide quark-hadron duality (see [31]).

for different lattice parameters (see the cited papers for details) and strictly speaking they do not agree with each other at large distances. However the qualitative agreement takes the place, and both sets of data reasonably correspond to our simple expression (14).

Of separate interest is the short distance limit of our results. It is straightforward to expand (14) near $z = 0$, the answer is (see [32])

$$R^{(v)}(z) = \left[\xi z^4 + \frac{\xi}{4}\, z^6 \log z^2 + ...\right] +$$

$$+ \left[1 + \frac{1}{3 \cdot 2^7}\left(6b - 6 - b^2\right)\, z^4 + \frac{3b - 2 - b^2}{3 \cdot 2^8}\, z^6 \log z^2 + ...\right] \qquad (17)$$

The first and the second brackets in the r.h.s. of (17) correspond to the first and the second terms in the r.h.s. of (14), respectively. The dots stay for higher order terms.

For the sake of completeness let us also cite the standard OPE answer for the discussed correlator (for the physical value $N_c = 3$) [5]:

$$R^{(v)}(\tau) = 1 + \frac{\alpha_s(\tau)}{\pi} - \frac{\langle (gG^a_{\mu\nu})^2\rangle}{3 \cdot 2^7}\, \tau^4 - \frac{7\pi^3}{81}\alpha_s \langle \bar{q}q\rangle^2\, \tau^6 \log \tau^2 \mu_v^2 + ... \qquad (18)$$

where τ is Euclidean time coinciding with x_E in our case. We use the standard values

$$\langle (gG^a_{\mu\nu})^2\rangle = 0.5\mathrm{GeV}^4 \;\; ; \;\; |\langle \bar{q}q\rangle| = (250\mathrm{MeV})^3 \qquad (19)$$

It is worth noticing[3] that the expression (18) contains a contact term $\langle j^\mu j_\nu\rangle$ which is not directly seen in the momentum space. This term's contribution is proportional to the sixth power of τ in (18), on the other hand, there is no α_s in front of it [22]. Numerically according to [22]

$$\langle \bar{u}\gamma_\mu d \bar{d}\gamma^\mu u\rangle / \langle \bar{q}q\rangle^2 = -\frac{1}{3}(0.90 \pm 0.15)$$

We take this circumstance into account by means of numerical redefining μ_v^2 in (18) accordingly, while still keeping the expression in the form (18). In fact, our main concern is to compare (14) with the lattice, while we need (18) mostly for illustrative purposes.

On Fig. 1.3 we compare the exact expression (14) with its own short distance expansion (17) and also with the conventional OPE result (18) where

[3] The authors are indebted to Prof. K.G.Chetyrkin for discussion of this point.

the three power terms have been kept in both cases. The Figure 1.3 shows striking difference from the Figs. 1.1, 1.2. Sizeable deviations of the standard OPE from (14) start as early as at $x \approx 0.4$ Fm. We have also plotted the first resonance contribution the answer should converge to at large distances. The sum over resonances smoothly interpolates between small and large distance regions.

3 Discussion of the results

We find the results of comparison of the lattice data and the model to be quite remarkable. We present here the pattern in the vector channel only, the other channels however demonstrate qualitatively similar situation (see [32] for all details). It is worth stressing that we have worked with the model of two relativistic quarks connected by the string with the tension σ, which is the only dimensionful parameter of the model. Using this picture, first, the lowest resonance masses were computed in all channels [12] and found to be in reasonable agreement with their experimental values. Second, the quasiclassical asymptotic for the mass spectrum was calculated [17] which has the same pattern for all channels. The latter must provide exact quark-hadron duality, which was explicitly shown in the vector case in [15, 30]. We have plugged these two ingredients into the corresponding spectral density and compared the results with the available lattice simulations [3, 4]. Another comparison is made for the standard SVZ short distance OPE expansion and the corresponding expansion provided by our spectral density. It is shown that they strongly deviate from the full curve (and from the lattice data) for the distances larger than $0.35 - 0.45$ Fm, as one could expect.

The main lesson is that one needs quite a few inputs (correct lowest resonance mass, correct lowest resonance residue and correct asymptotic behavior dictated by quark-hadron duality) to reproduce lattice data in a reasonable way. We have taken these inputs from the large N_c model of QCD string with quarks at the ends [17]. The absence of precise quantitative agreement between our results and the lattice should not disappoint since many effects have been ignored ($1/N_c$ effects, perturbative exchanges etc), notice also that the two sets of lattice data we have used do not agree with each other on quantitative level. On the other hand, qualitative agreement is rather good and certainly better than that of the standard short distance OPE in coordinate space. Notice that we have not performed any fitting of the lattice data, the latter was compared with the curves (14) and analogous expressions for the other channels computed independently. If we had fitted the data with these expressions, the agreement would have been much better.

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Figure captions

Fig. 1.1 Lattice data from [3] for the vector channel correlator vs expression (14).

Fig. 1.2 Lattice data from [4] for the vector channel correlator vs expression (14).

Fig. 1.3 Expression (14) (solid curve) together with its own short distance expansion (17) (dashed-dotted curve) and short distance OPE expansion (20) (dashed curve). The lightest resonance contribution is also shown (double dotted curve).

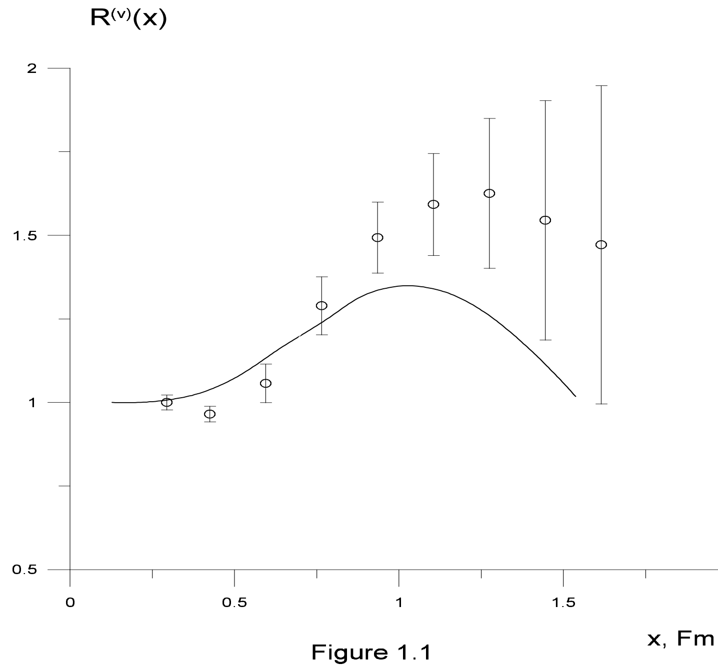

Figure 1.1

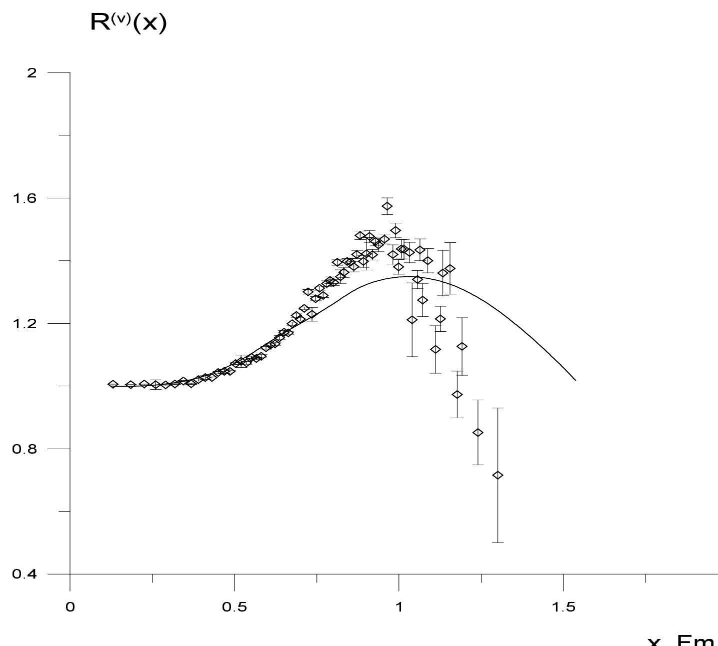

Figure 1.2

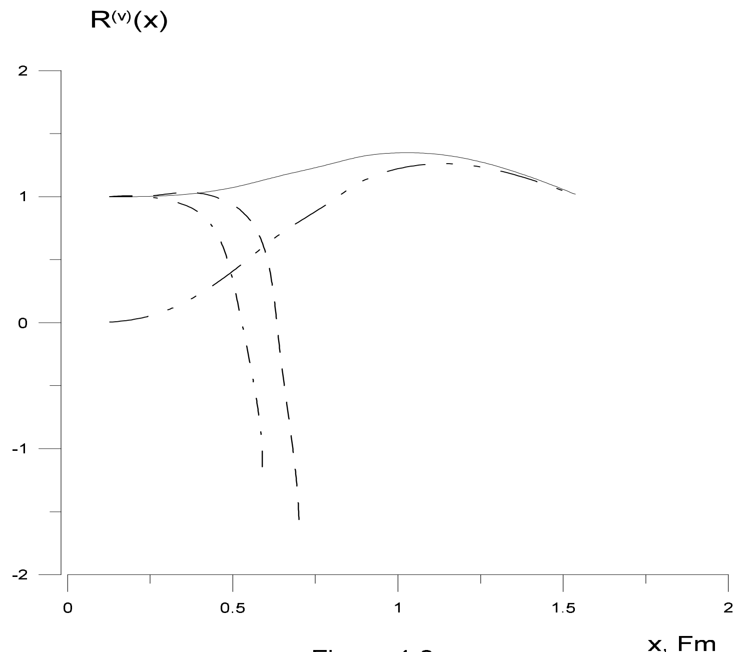

Figure 1.3