Random Lattice QCD and chiral effective theories

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Abstract

Resent developments in the Random Matrix and Random Lattice Theories give a possibility to find low-energy theorems for many physical models in the Born-Infeld form [1]. In our approach that based on the Random Lattice regularization of QCD we try to used the similar ideas in the low-energy baryon physics for finding of the low-energy theory for the chiral fields in the strong-coupling regime.

PACS number(s): 11.15.Ha, 12.38.Gc, 12.39.Fe, 11.15.Me, 11.30.Cp, 11.30.Rd, 12.39.Dc

1 Introduction and motivation

The derivation of effective chiral theories from QCD has a long history. This task has the strong motivation which come from the necessity of the constructing of the QCD motivated baryon state model. From the DIS experiments we know, that the baryon consists of the charged constituents (so called

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constituent quarks), but the experiments with the spin of proton showed that only twenty percents of the baryon spin can be associated with the charged constituents [2]! Therefore, the realistic model for the low-energy baryon state should describe the quark constituents as well as the chiral degrees of freedom of the meson cloud around them. But what is a theory that describe this chiral degrees of freedom and how such theory can be derived from the origin theory - from QCD?

There are many conceptions and methods were proposed for finding the answer on these questions so far. The first essential contribution in this theme was done in [3]. Using the method of the large N expansion, a effective chiral theory was suggested in the form of the series of the chiral invariants. The second order theory of such type is the well-known Skyrme model [4] of the low-energy baryon state - the phenomenological unified theory for mesons and baryons where the baryon treads as a topological soliton of non-linear chiral fields. Another interesting approach was proposed in [5] where the Skyrme model was derived from the integration of the chiral anomaly.

All such methods play the very essential role in the particle physics and give a appropriate description of the chiral field behavior with the low tensity. However for studying of the chiral field near the confinement boundary, all series of chiral perturbations must be analyzed. The Chiral Bag Model [6] is a very good illustration of this problem. In this model a boundary between chiral fields and color fields is specified by hand. In such approach the agreement with the experiment dates is quit good but in spite of this fact it is not clear now what is a physical mechanism of formation of such chiral bag.

Fortunately today Lattice QCD numerical experiments give us a many interesting information about the behavior of the color fields (quarks and gluons) in the strong coupling regime. These dates are very essential for understanding of the low-energy baryon's physics and this approach is the empirical (and theoretical) basis of the the baryon string model [7]. Against a background of these facts it would looks very astonishing that Skyrme model gives a good agreement with the experiment dates although this model does not describe the color degrees of freedom at all. So the question about unification of these paradigms looks very essential now and the seeking of the way of such unification naturally to begin with an analyzing of the chiral limit of QCD on the lattice.

2 Why we need in the Random Lattice QCD

The attempts to obtain a chiral effective lagrangian from lattice QCD had been performed many times a long ago. Using of the well-known Brezin&Gross trick [8] it could be possible to perform the link's matrix integration in strong coupling regime and obtain the various first order chiral effective theories [9].

In spite of first great success this approach had not been very popular and origin of this stems from the fact that the approaches from [9] does not take the possible to obtain any corrections to the first order results. The lattice regularization breaks a rotational symmetry of the initial theory from the continues rotation group to a discrete group of rotations on fixed angles. And the lattice regularization approach gives the correct results only such tensors which are invariant by respect to this discrete group. In particular using the ordinary Hyper-Cubical (HC) lattice one can obtains the only first order effective theory and for corrections this method generates non-rotational (non-lorentz) invariant terms. For generating of the high-order effective field theories more symmetrical lattice must be considered.

The problem of the rotational symmetry broking on a lattice has attracted a principal attention for a long time. It was shown [10] that in 4 dimension so-called Body Centered Hyper-Cubical (BCHC) or F4 lattice has the largest discrete symmetry group. (BCHC consists from the all sites of the HC lattice together with centers of its elementary cells.) This property of the BCHC lattice gives a possibility to obtain the next-to-leading (NL) correction to the first order of the chiral perturbation theory [11]. Results of the works [11] are very essential for our analysis and its point out on the effectiveness of this method. Moreover, this results are very interesting from phenomenological point of view because, as is well known [4], the NL corrections violate the scale invariance of the prototype (first order) chiral theory that lead to generation of the chiral topological solitons (Skyrmions). the Next-Leading order chiral effective theory that was done in the [11] is in agreement with our phenomenological propositions about this [12] and in our work we will used the methodological ideas from [11] in order to define the behavior of chiral field near the confinement surface.

As one could see, in order to solve our problem, the Next-Leading order corrections are not enough. This theory has no solutions that look like chiral "bag". Moreover, as would be shown later, near the confinement surface (near the source of chiral field), the influence of high order corrections became large

and large. But the BCHC lattice method gives the NL corrections only, the further use of this method for the defining of the high order terms leads to generation of non-relativistic (non-rotational) invariant terms. It means that we need more a symmetric lattice than the BCHC lattice.

Unfortunately, the lattice which would be more symmetric than BCHC lattice can not be constructed in 4 dimension. Moreover any method based on a lattice with the fixed positions of sets has artifacts concerned with priority directions that correspond with the basis vectors of the lattice. Finally just these artifacts lead to the problems with the rotational (relativistic) invariance and on this evidence the using of the BCHC lattice is only half measure. For solving of our problem the modification of the initial conception of the lattice regularization must be performed. We need to find the conception of the lattice regularization that has no any priority directions. Fortunately this conception is known for a long time and is called the Random Lattice Approach [13].

The ideas of the Random Lattice was proposed by Voronoi and Deloune and today this method is very widely used in the modern science and technology. For the quantum field theory method was modified by Christ, Friedberg and Lee [13]. In these articles have been shown that in order to obtain the restoration of the Lorentz (rotational) invariance, it is necessary to perform an average over an ensemble of random lattices. As result one get the averaging over all possible directions and it is intuitively clear that this procedure leads to the disappearance of the artifacts connected this violation of the group of the space rotation.

But how to perform such random discretization? This procedure has the tree steps:

1) Draw N sites x_i at random in the volume V.

2) Associate with each x_i a so-called Voronoi cell c_i

$$c_i = \{x | d(x, x_i) \leq d(x, x_j), \forall j \neq i\}$$

where $d(x, y)$ is a distance between points x and y. It is means that Voronoi cell c_i consists of all points x that are closer to the center site x_i than any other site.

3) Constrict the dual Delaune lattice by linking the center sites of all Voronoi cells which share a common face.

After this if one consider the the big ensemble of such Voronoi-Deloune random lattices based on various distributions of sites x_i, it possible to prove

the origin rotational symmetry will restored [13]. In our work we use this procedure for obtaining of an effective chiral lagrangian from lattice QCD. This methodological point of view this is a modification of the method proposed in [11] on the case of the Random Lattice approach.

3 From Lattice QCD to chiral lagrangians: step by step

Now let me briefly remand a general steps of the algorithm of the chiral lagrangian derivation from the lattice QCD that was proposed in [11]. The starting point of our analysis is a standard lattice action with Willson fermions

$$Z = \int [DG][D\bar{\psi}][D\psi] \exp\{-S_{\rm pl}(G) - S_q(G,\bar{\psi},\psi) - S_J\}$$

where:

1) plaquette gauge field term is

$$\begin{aligned} S_{\rm pl} &= \frac{2N_c}{g^2}\sum_{pl}\left[1 - \frac{1}{N_c} ReG_{x,\mu}G_{x-\mu,\nu}G^{=}_{x+\nu,\mu}G^{+}_{x,\nu}\right],\ G_{x,\mu} \\ &= \exp\{ig\int_{\rm link} dx'_\mu \mathcal{A}_\mu(x')\}; \end{aligned}$$

2)link fermions term is

$$S_q = \sum_{x,\mu} Tr(\bar{A}_\mu(x)G_\mu(x) + G^+_\mu(x)A_\mu(x))$$

$$A_\mu(x)^a_b = \bar{\psi}_b(x+\mu)P^+_\mu\psi^a(x),\ \ \bar{A}_\mu(x)^a_b = \bar{\psi}_b(x)P^-_\mu\psi^a(x+\mu)$$

and $P^\pm_\mu = \frac{1}{2}(r \pm \gamma_\mu)$;

3) source term is

$$S_J = \sum_x J^\alpha_\beta(x)M^\beta_\alpha(x),\ \ M^\beta_\alpha = \frac{1}{N_c}\psi^{a,\beta}(x)\bar{\psi}_{a,\alpha}(x).$$

In order to realize the strong-coupling regime on the lattice let us consider the limit of the large coupling constant g ($g \to \infty$). This limit was very wide

studied [9] and main result is that in such limit integral over the gauge field can be performed. (Of course, the direct integration is difficult since the exists the plaquette term $S_{\rm pl}$, but due to the strong-coupling limit on the first step it possible to neglect such plaquette contributions by respect to the contribution from link integral S_q. The plaquette contributions could be considered in the systematic manner as perturbations by $1/g$ [9].)

Let us consider the leading order contribution in this strong-coupling expansion. The integrals over the gauge degrees of freedom can be calculated into the large N limit by using of the standard procedure [9] and result of these calculations is following

$$Z = \int [D\bar{\psi}][D\psi] \exp\{-N \sum_{x,\nu} {\rm Tr}[F(\lambda(x,\nu))] - S_J\} \qquad (1)$$

where $\lambda_\nu = -M(x)P^-_\nu M(x+\nu)P^+_\nu$ and

$$F(\lambda) = {\rm Tr}[(1-\sqrt{1-\lambda})] - {\rm Tr}[\log(1-\frac{1}{2}\sqrt{1-\lambda})]$$

Now it would be very interesting to point out that the function $F(\lambda)$ has the typical form of the Born-Infeld action with first logarithmic correction. It is no casual fact. In [1], it was shown by means of the very similar technics that the low-energy theory of the IIB superstring has a Born-Infeld form. From the methodological point of view we perform a similar analysis for QCD on the lattice and it would significant to note ahead of the process of our proving that our result would has a Born-Infeld form too.

Our next step is the integration over the fermion degrees of freedom in (1). Using the source technics it was shown [11] that integral (1) can be re-written into the form of the integral over the unitary bosons matrix M_x

$$Z = \int DM \exp S_{\rm eff}(M). \qquad (2)$$

As a matter of principle, we already perform the transformation from the color lattice degrees of freedom (G and ψ) to the boson lattice degrees of freedom (M). Now our task is to realize the continuum limit of expression (2).

The nest step of our analysis correspond with the studying of the stationary points of the lattice action $S_{\rm eff}$. Fortunately this is very well studied

task [14]. This problem is connected with well-known investigations of the critical behavior of the chiral field on the lattice and with the problem of the phase transformation on the lattice (for references see the issue [15]). In [11], it was shown that for our task the stationary point is

$$\hat{M}_0 = u_0\hat{I},\ u_0(m_q = 0, r = 1) = 1/4.$$

Now one can expressed $M(x)$ in terms of the pseudoscalar Goldstone bosons

$$M = u_0 \exp(i\pi_i\tau_i\gamma_5/f_\pi) = u_0[U(x)\frac{1+\gamma_5}{2} + U^+(x)\frac{1-\gamma_5}{2}]$$

and the effective action is given in the form of the Taylor expansion around this stationary point

$$S_{\text{eff}}(U) = -N\sum_{k=1}^{\infty}\frac{F^{(k)}(\lambda_0)}{k!}\sum_{x,\nu}\text{Tr}[(\lambda_\nu(x) - \lambda_0)^k] \tag{3}$$

Let us consider the expansion of the chiral field $U = \exp(i\pi_i\tau_i/f_\pi)$ on the lattice around point x (by respect to the small step of the lattice a)

$$U(x+\nu) = U(x) + a(\partial_\nu U(x)) + \frac{a^2}{2}(\partial_\nu^2 U(x)) + \cdots$$

And for components of the Taylor expansion (3) one obtain

$$\begin{array}{rcccc}
\text{Tr}[(\lambda_\nu(x) - \lambda_0)] &=& -2\lambda_0\text{Tr}(\alpha) & & \\
\text{Tr}[(\lambda_\nu(x) - \lambda_0)^2] &=& 2\lambda_0^2\text{Tr}(\alpha^2) & -4\lambda_0^2\text{Tr}(\alpha) & \\
\text{Tr}[(\lambda_\nu(x) - \lambda_0)^3] &=& -2\lambda_0^3\text{Tr}(\alpha^3) & +6\lambda_0^3\text{Tr}(\alpha^2) & \\
\text{Tr}[(\lambda_\nu(x) - \lambda_0)^4] &=& 2\lambda_0^4\text{Tr}(\alpha^4) & -8\lambda_0^4\text{Tr}(\alpha^3) & +4\lambda_0^4\text{Tr}(\alpha^2) \\
\text{Tr}[(\lambda_\nu(x) - \lambda_0)^5] &=& -2\lambda_0^5\text{Tr}(\alpha^5) & \cdots\cdots & \cdots\cdots
\end{array} \tag{4}$$

where $\alpha = a^2\partial_\nu U\partial_\nu U^+ + O(a^4)$.

Expressions (4) are very essential because these are a simplest illustration of all aspects of the violation of the rotational symmetry on the lattice. For this moment we specially say nothing about the structure of our lattice. We try to formulate our result as general as possible and all information about the lattice contains into the vectors ν that correspond with the basic vectors of the lattice (for example, the vectors ν for the Hyper-Cubical lattice are the Cartesian basic vectors $\vec{i}$, $\vec{j}$, $\vec{k}$ and $\vec{t}$). The leading order part can be

calculated trivially. Indeed, using the simple Hyper-Cubical lattice where $\nu = i, j \ : i = (1 \ldots 4)$ it is easy to show that the leading order contribution is the prototype chiral lagrangian

$$P_{O(p^2)} \sim \mathrm{tr}[\partial_\mu U \partial^\mu U^+] \tag{5}$$

As I said before the rotational symmetry violation arguments does not admit to use the HC lattice calculation for the next-leading order contributions. For obtaining of these contributions a more symmetrical lattice must be used. In [10] it was shown that this lattice is a Body Centered Hyper-Cubical (BCHC) or F4 lattice. The basis vectors of this lattice are

$$\mathrm{BCHL} \rightarrow \nu = \nu_{ij}^{\alpha} = \frac{1}{\sqrt{2}}(e_i + \alpha e_j), (1 \leq i < j \leq 4, \alpha \pm 1) \tag{6}$$

and one can show that the next-leading order contribution (6) in this basis is following

$$P_{O(p^4)} \sim \mathrm{tr}[(\partial_\mu U \partial^\mu U^+)^2] + \frac{1}{2}\mathrm{tr}[\partial_\mu U \partial^\nu U^+ \partial_\mu U \partial^\nu U^+]$$

It is easy to see that this is just we expect to receive because this contribution was obtained from many another approaches [3, 5]. But unfortunately, this method can not directly used for finding next contributions and the origin of this fact is again the violation of the rotation symmetry but now on the F4 lattice. Moreover, there are no any more symmetrical lattice with fixed position of sites in 4-dimensional [10]. It means that we are need in the absolutely different lattice conception that guaranteed the restoration of the initial symmetries. Fortunately this conception is known now. This is a Random Lattice conception (RL) [13].

The basic idea of the RL is the averaging over the big ensemble of various lattices with random distributions of sites and it possible to show that such averaging leads to the restoration of the rotational invariance. There are two methods of the realization of such scheme. A first one based on the Christ, Friedberg and Lee (CFL) technics. The basis of vectors in the CFL method is following

$$\mathrm{RL} \rightarrow \nu = \nu_{ij}^{\mu} = e_{ij}^{\mu} s_{ij} / l_{ij}$$

where s_{ij} is a volume of the corresponding 3-dimensional boundary surface of the Voronoi cells and $l_{ij} = |\vec{r}_i - \vec{r}_j|$ is the length of link. Using the summation

formulas from [13] one get that after the averaging only pairs are survive

$$< \prod_a (e^a)_\nu >= \sum_{\text{pairings}} \prod_{\text{pairs}} < (e^a)_i (e^b)_i > \tag{7}$$

At other hand, the result (7) could be obtained by means of the following trick [16, 17]. For beginning let us consider a lattice with fixed position (for simplicity it possible to use the trivial HC lattice) in a flat space. Now let us consider small deformations of the geometry of this space ($\gamma_{ij} \to g_{ij}$). Using of this idea one can rewrite the problem of the random lattice averaging in the terms of the random surface [16]. This is the standard quantum gravity task and using the methods of the Matrix Theory one can show that our result (7) is just the direct consequence of well-known Wick's Theorem about the pairings [18].

The expression (7) gives us possibility to calculate all terms in expansion (4). Let us consider just the first column in the expression (4). It is easy to show that either of these is proportional to some power of the leading order contribution (5)

$$\begin{array}{ccccc} \mathrm{Tr}[\alpha] & \sim & \mathrm{tr}[\partial_\mu U \partial^\mu U^+] & + & \cdots \\ \mathrm{Tr}[\alpha^2] & \sim & \mathrm{tr}[(\partial_\mu U \partial^\mu U^+)^2] & + & \cdots \\ \mathrm{Tr}[\alpha^3] & \sim & \mathrm{tr}[(\partial_\mu U \partial^\mu U^+)^3] & + & \cdots \\ \cdots\cdots & \cdots & \cdots\cdots & \cdots & \cdots\cdots \\ \mathrm{Tr}[\alpha^n] & \sim & \mathrm{tr}[(\partial_\mu U \partial^\mu U^+)^n] & + & \cdots \\ \cdots\cdots & \cdots & \cdots\cdots & \cdots & \cdots\cdots \end{array} \tag{8}$$

Substituting (8) into the (3) and collecting of all terms which depend on the power of the prototype lagrangian one obtain following expression for the effective chiral lagrangian

$$\begin{aligned} \mathcal{L}_{\text{eff}} \;\sim\; & -\mathrm{tr}\left[1 - \sqrt{1 - 1/\beta^2 \partial_\mu U \partial^\mu U^+}\right] \\ & -\mathrm{tr}\left[\log(1 - \frac{1}{2}(1 - \sqrt{1 - 1/\beta^2 \partial_\mu U \partial^\mu U^+}))\right] + \cdots \end{aligned} \tag{9}$$

where $\cdots$ are all another terms (in particular the Skyrme term) and β is a effective coupling constant that depend on the value of our stationary point u_0.

Now let discussed the result (9). It was obtained that the some part of chiral effective action has a Born-Infeld form plus first logarithmic correction to it. In [19], it was shown that such form of the effective action play a very essential role in the problem of the chiral bag formation because just these square-root terms generate the step-like distribution solutions that can be interpreted as internal phase into the two-phase model of the low-energy baryon states. Another terms play essential role only on large distance from the confinement surface and can be considered as corrections.

4 Conclusions

The aim of this paper is to derive the chiral effective lagrangian from QCD on the lattice at the strong coupling limit. We find that this theory looks like a Born-Infeld theory for the prototype chiral lagrangian. Such form of the effective lagrangian is expected. From the methodological point of view our consideration is very similar with the low-energy theorem in string theory that lead to the Born-Infeld action [1]. Moreover, in [19], it was shown that Chiral Born-Infeld Theory (without logarithmic corrections) has very interesting "bag"-like solution for chiral fields. It was additional motivation of our work.

The Chiral Born-Infeld theory is a good candidate on the role of the effective chiral theory and the model for the chiral cloud of the baryons. In this model one can find not only spherical "bags", it is possible to also study the "string"-like, toroidal or "**Y**-Sign"-like solutions, or some another geometry. The geometry of the confinement surface depends directly on the model of color confinement and it would be very interesting to use, for example, the Lattice QCD simulations for the color degrees of freedom in combination with our model for the external chiral field.

This work is partially supported by the Russian Federation President's Grant 1450-2003-2. The hospitality and financial support of the ICT* in Trento is gratefully acknowledged.

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