Feynman rules for effective Regge action

April 29, 2005

Abstract

Starting from the gauge invariant effective action derived in the quasi-multi-Regge kinematics (QMRK), we obtain the effective reggeized gluon (R) – particle (P) vertices of the following types: RP, RPP, RRP, $RRPP$, $RPPP$, $RRPPP$, where *on*-mass-shell particles are the gluons, or sets of gluons with small invariant masses. The explicit expressions satisfying the Bose-symmetry and gauge invariance conditions are obtained. As a comment to Feynman rules for writing down the amplitudes in terms of effective vertices we present a kind of a "vocabulary" for practitioners.

1 Introduction

The behavior of total cross sections of QCD processes at high energy and, as a consequence, the problem of unitarization of the BFKL Pomeron are important directions of investigations in phenomenology of strong interactions. In the leading logarithmic approximation, the BFKL equation gives the power-law rise of cross sections with energy that obviously violates the Froissart bound [1]. The next-to-leading corrections to the BFKL kernel are large enough and must be taken into account even at presently accessible energies [2]. The straightforward calculation of corrections to the Pomeron intercept which have been performed in a series of the ten-years old papers by L.N. Lipatov and V.S. Fadin [3] is rather cumbersome and not convenient for numerical analysis.

The motivation of this work is necessity for developing a proper mathematical apparatus for calculation of the next-to-leading corrections to Feynman inelastic amplitudes of peripherical processes. It is convenient and physically well-motivated to perform the systematic self-consistent study of non-leading contributions to the BFKL Pomeron intercept within the framework of an effective theory describing the interactions of reggeized gluons with elementary particles (quarks and gluons). In the papers [4], such an effective action approach for inelastic high-energy scattering in QCD was proposed and developed. In the work by L.N. Lipatov of 1995, this approach was generalized to the processes with arbitrary number of produced particles separated into several groups by rapidity satisfying the multi-Regge kinematics conditions. As a result, the non-Abelian gauge invariant effective action for quasi-multi-Regge kinematics (QMRK) processes was derived [5]. This action allows to take into account arbitrary number of produced particles and contains the fields of two sorts: the reggeized gluons in the t-channel, and the ordinary gluons and quarks in the production channel. Several examples of the corresponding effective vertices were further calculated by L.N. Lipatov, V.S. Fadin *et al.* [6, 7], and A. Leonidov and D. Ostrovsky [8]. The string theory motivated approach to study of effective vertices was developed in Refs. [9].

In the present work, we reproduce these results within the unified framework of the gauge invariant effective action. There is a hope that building and application of the set of such rules derived form the effective action can provide significant simplification of evaluation of differential cross sections related to creation of several particle clusters separated by rapidity gaps. We concentrate our attempts on presentation of the results in the form most convenient for use in numerical simulations for phenomenological applications.

The processes of such a kind can be investigated, *e.g.*, in experiments at RHIC and LHC. Another important branch of possible applications—calculation of peripherical amplitudes with several reggeized gluon states in the scattering channel—is out of scope of the present paper and will not be considered here.

The structure of the paper is following: In the Section 2, we fix the notations and give an elementary "vocabulary" of terms and kinematic relations that are used in the work. In the next Section, a number of Reggeon-Particle vertices is derived systematically within the effective gauge invariant action approach and presented in an explicit analytical form.

2 Vocabulary

Let us give a "vocabulary" which is used throughout the present work. We consider the parton-parton collision at high energy in the center-of-mass system. The main contribution to the total cross section stems from the QMRK of final state particles (see Fig.1). In this regime, the final state particles compose several groups with arbitrary number of gluons, or quarks with fixed masses s_i, which are produced in the Regge kinematics to esch other:

$$s = 2P_A P_B = 4E^2 \gg s_i = 2p_{i-1}p_i \gg |t_i| = |q_i^2| \ . \tag{1}$$

Also, we introduced the light-cone vectors

$$n^+ = \frac{P_B}{E} \quad , \quad n^- = \frac{P_A}{E} \quad , \quad n^+ n^- = 2 \ , \ (n^\pm)^2 = 0 \tag{2}$$

and thus the light-cone projections of momenta and derivatives read, respectively

$$k^\pm = (n_\mu)^\pm \cdot k^\mu \ , \ \partial_\pm = (n_\mu)^\pm \cdot \partial^\mu \ . \tag{3}$$

We imply also that in the derivatives act on the particle $v(p)$ and reggeon $A(p)$ fields in the momentum representation as:

$$\partial_\pm v(p) = -ip_\pm v(p) \ , \ \frac{1}{\partial_\pm} v(p) = \frac{i}{p_\pm} v(p) \ , \ \partial_\sigma^2 A(q) = -q^2 A(q) \ . \tag{4}$$

Besides this, we define two transferred momentum 4-vectors:

$$q_1 = P_A - P_{A'} \quad , \quad q_2 = P_B - P_{B'} \tag{5}$$

and their Sudakov decompositions

$$q_1 = q_{1\perp} + \frac{q_1^+}{2}\, n^- \ , \ q_2 = q_{2\perp} + \frac{q_2^-}{2}\, n^+ \ , \ q_1^- = q_2^+ = 0 \ . \tag{6}$$

The Sudakov variables for produced particles are:

$$p_i = \frac{p_i^+}{2}\, n^- + \frac{p_i^-}{2}\, n^+ + \boldsymbol{p}_{i\perp} \ . \tag{7}$$

In the fragmentation region, one has

$$P_A \to p_1 + p_2 + ... + p_n + q_1 \ : \ P_A^+ = \sum_{i=1}^{n} p_i^+ \ , \ |q_1^+| \ll p_i^+ \ , \tag{8}$$

$$P_B \to p_1 + p_2 + ... + p_n + q_2 \ : \ P_B^- = \sum_{i=1}^{n} p_i^- \ , \ |q_2^+| \ll p_i^- \ . \tag{9}$$

Deriving the expressions for the vertices, we assume that all the 4-momenta of particles and reggeons in each vertex are incoming and obey the conservation law

$$\sum_i p_i = 0 \quad , \quad \sum_i p_i^{\pm} = 0 \ . \tag{10}$$

To every reggeized gluon line (existing only in the scattering channel), one should attribute the sign $(+)$ to its end attached to the initial parton with the momentum P_A, and the sign $(-)$ to the end attached to one with the momentum P_B. Throughout this paper, we use the notation $a^{\pm} = (a \cdot n^{\pm})$. Note, that the horizontal lines at the Fig. 1 represent the reggeons, or particles, which are *off*-mass-shell. Initial, final and produced particles (presented by no-horizontal lines at the Fig. 1) are the *on*-mass-shell particles, which we will suggest, as a rule, to be massless. Let us emphasize that the presence of reggeon lines within any production block is forbidden in the QMRK due to the rapidity gap, Eq. (1).

The effective action approach can be formulated in terms of the physical particles (quarks and gluons) and the reggeized gluons. It is convenient to write down the effective action in the following form:

$$S = \int dx \left[\mathcal{L}_{YM} + \mathcal{L}_{ind} \right] \quad , \tag{11}$$

where the standard Yang-Mills part consists of the gluon-gluon interactions

$$\mathcal{L}_{YM} = -\frac{1}{4} \left(\partial_\mu v_\nu^a - \partial_\nu v_\mu^a \right) + \frac{g}{2} f_{abc} \left(\partial_\mu v_\nu^a \right) v_\mu^b v_\nu^c - \frac{g^2}{4} f_{lbc} f_{lde} v_\mu^b v_\nu^c v_\mu^d v_\nu^e \ , \tag{12}$$

and the induced part contains gluon-reggeon couplings:

$$\mathcal{L}_{ind}(v_\pm, A_\pm) = - \int dx \ \mathrm{Tr} \Bigg\{ \Bigg[v_+ - g v_+ \frac{1}{\partial_+} v_+ + g^2 v_+ \frac{1}{\partial_+} v_+ \frac{1}{\partial_+} v_+ - ... - A_+ \Bigg] \cdot \partial_\sigma^2 A_- + \left[v_+, \partial_+, A_\pm \to v_-, \partial_-, A_\mp \right] \Bigg\} \ , \tag{13}$$

with $v_\pm \equiv T^a \left(n^\pm\right)^\mu v_\mu^a$ are the particle (ordinary gluon) fields, and $A_\pm \equiv T^a \left(n^\pm\right)^\mu A_\mu^a$—the reggeized gluon fields. For the hermitian color generators T^a, we use

$$\left[T^a, T^b\right] = i f^{abc} T^c \quad , \quad \mathrm{Tr}\left(T^a T^b\right) = \frac{1}{2}\delta^{ab} \ . \tag{14}$$

3 Vertices

3.1 Standard QCD Feynman rules

The Yang-Mills interaction part of the effective action yields the standard QCD Feynman rules (see Fig. 2a-c):

$$\begin{aligned}
3g\text{-vertex}: &\qquad g f^{abc}\left[(p_1-p_2)_\lambda g_{\mu\nu} + (p_2-p_3)_\mu g_{\nu\lambda}\right. \\
&\qquad\quad \left. +(p_3-p_1)_\nu g_{\lambda\mu}\right] \ , &(15)\\
4g\text{-vertex}: &\qquad i g^2 \left[f_{abl} f_{cdl}(g_{\mu\sigma}g_{\nu\lambda} - g_{\mu\lambda}g_{\nu\sigma})\right. \\
&\qquad\quad + f_{acl} f_{dbl}(g_{\mu\nu}g_{\lambda\sigma} - g_{\mu\sigma}g_{\lambda\nu}) \\
&\qquad\quad \left. + f_{adl} f_{bcl}(g_{\mu\lambda}g_{\sigma\nu} - g_{\mu\nu}g_{\sigma\lambda})\right] \ , &(16)\\
\text{gluon propagator}: &\qquad -i\delta^{ab}\,\frac{g_{\mu\nu}}{k^2} \ . &(17)
\end{aligned}$$

3.2 Induced vertices

Induced part of the action takes into account emission in the fragmentation region of initial particles. The simplest particle-reggeon (PR) vertex in the momentum space is defined as (see Fig. 3a)

$$\begin{aligned}
& i\langle 0|\ \mathcal{L}_{ind}^{PR}\ |v_\nu^a(q) A_\pm^b\rangle = V_{\pm\nu}^{\ ab}(q) \ , \\
& \mathcal{L}_{ind}^{PR} = -\mathrm{Tr}\left(v_+ \partial_\sigma^2 A_- + v_- \partial_\sigma^2 A_+\right) \ ,
\end{aligned} \tag{18}$$

and is given by

$$V_{\pm\nu}^{\ ab}(q) = i\delta^{ab} q^2 \left(n^\pm\right)_\nu \ . \tag{19}$$

One can easily check fulfillment of the gauge invariance condition for this vertex:

$$q_2^\nu \cdot V_{+\nu}^{\ ab}(q_2) = 0 \ , \ q_1^\nu \cdot V_{-\nu}^{\ ab}(q_1) = 0 \ . \tag{20}$$

The induced vertices of the PPR type read

$$V_{+\ \ \mu\nu}^{abd} = \left\langle 0 \right| \ -ig\mathrm{Tr}\left[v_+ \frac{1}{\partial_+} v_+ \ \partial_\sigma^2 A_-\right] \ \left| v_\mu^a(P_A) v_\nu^b(P_{A'}) A_+^d(q_1) \right\rangle$$
$$= g f^{abd} \ \frac{q_1^2}{P_A^+} \left(n^+\right)_\mu \left(n^+\right)_\nu \ , \quad (21)$$

$$V_{-\ \ \mu\nu}^{abd} = \left\langle 0 \right| \ -ig\mathrm{Tr}\left[v_- \frac{1}{\partial_-} v_- \partial_\sigma^2 A_+\right] \ \left| v_\mu^a(P_B) v_\nu^b(P_{B'}) A_-^d(q_1) \right\rangle$$
$$= g f^{abd} \ \frac{q_2^2}{P_B^-} \left(n^-\right)_\mu \left(n^-\right)_\nu \ , \quad (22)$$

where we imply that

$$P_A^+ = P_{A'}^+ \ , \ P_B^- = P_{B'}^- \ .$$

The induced vertices of higher orders will be specified below.

3.3 Effective PPR vertices

Knowledge of the induced and ordinary 3-vertices allows to build the effective 3-vertices which obey the Bose- and gauge-symmetries. We distinguish four types of the PPR vertices. First group consists of the margin type vertices: $q_1 = P_A - P_{A'}$, $q_2 = P_{B'} - P_B$.

- "Left" margin type (see Fig. 3b):

$$\gamma_{\parallel}^{\nu\nu'+}\left(P_A, a; P_{A'}, b; q_1, c\right) = g f^{abc} \ \Gamma_{\parallel}^{\nu\nu'+}\left(P_A, P_{A'}\right) \ , \quad (23)$$

$$\Gamma_{\parallel}^{\nu\nu'+}\left(P_A, P_{A'}\right) = 2P_A^+ g^{\nu\nu'} + \left(n^+\right)^\nu \left(-2P_A + P_{A'}\right)^{\nu'}$$
$$+ \left(n^+\right)^{\nu'} \left(-2P_{A'} + P_A\right)^\nu - \frac{q_1^2}{P_A^+} \left(n^+\right)^\nu \left(n^+\right)^{\nu'} \ . \quad (24)$$

One can check that the condition of gauge invariance is explicitly satisfied:

$$\Gamma_{\parallel}^{\nu\nu'+} \cdot \left(P_{A'}\right)_{\nu'} = P_A^\nu P_A^+ \ , \quad (25)$$

$$\Gamma_{\parallel}^{\nu\nu'+} \cdot \left(P_A\right)_\nu = P_{A'}^{\nu'} P_A^+ - P_{A'}^2 \left(n^+\right)^{\nu'} \ . \quad (26)$$

For *on*-mass-shell particle A: $P_A^2 = 0$ with the gauge condition $(e(P_A) \cdot P_A) = 0$ we have

$$\Gamma_{\parallel}^{\nu\nu'+} \cdot (P_{A'})_{\nu'} \cdot e_\nu \left(P_A\right) = 0 \ . \tag{27}$$

- "Right" margin type (see Fig. 3c):

$$\gamma_{\parallel}^{\nu\nu'-} \left(P_B, a; P_{B'}, b; q_2, c\right) = g f^{abc} \; \Gamma_{\parallel}^{\nu\nu'-} \left(P_B, P_{B'}\right) \ ,$$

$$\begin{aligned}\Gamma_{\parallel}^{\nu\nu'-} \left(P_B, P_{B'}\right) = 2P_B^- g^{\nu\nu'} - \left(n^-\right)^{\nu'} \left(2P_{B'} - P_B\right)^\nu \\ - \left(n^-\right)^\nu \left(2P_B - P_{B'}\right)^{\nu'} - \frac{q_2^2}{P_B^-} \left(n^-\right)^\nu \left(n^-\right)^{\nu'} \ .\end{aligned} \tag{28}$$

In this case, the gauge-invariance tests read:

$$\Gamma_{\parallel}^{\nu\nu'-} \cdot (P_{B'})_{\nu'} = -P_B^\nu P_B^- \ , \tag{29}$$

$$\Gamma_{\parallel}^{\nu\nu'-} \cdot (P_B)_\nu = -P_{B'}^{\nu'} P_B^- + P_{B'}^2 \left(n^-\right)^{\nu'} \ . \tag{30}$$

The other group includes the effective vertices of the central type: $k = q_1 + q_2$, $q_{1,2}^2 \neq 0$.

- "Left" central type (see Fig. 3d):

$$\gamma_{\perp}^{\nu\nu'+} \left(q_1, a; k, b; q_2, c\right) = -g f^{abc} \; \Gamma_{\perp}^{\nu\nu'+}(q_1, q_2) \ ,$$

$$\begin{aligned}\Gamma_{\perp}^{\nu\nu'+}(q_1, q_2) = 2q_1^+ g^{\nu\nu'} - \left(n^+\right)^\nu \left(q_1 - q_2\right)^{\nu'} \\ - \left(n^+\right)^{\nu'} \left(q_1 + 2q_2\right)^\nu - \frac{q_2^2}{q_1^+} \left(n^+\right)^\nu \left(n^+\right)^{\nu'} \ .\end{aligned} \tag{31}$$

The corresponding Ward identities read:

$$\Gamma_{\perp}^{\nu\nu'+} \cdot k_{\nu'} = q_1^+ q_1^\nu - q_1^2 \left(n^+\right)^\nu \ , \tag{32}$$

$$\Gamma_{\perp}^{\nu\nu'+} \cdot (q_1)_\nu = q_1^+ k^{\nu'} - k^2 \left(\nu^+\right)^{\nu'} \ . \tag{33}$$

- "Right" central type (see Fig. 3e):

$$\gamma_\perp^{\nu\nu'-}(q_1, a; k, b; q_2, c) = -g f^{abc}\, \Gamma_\perp^{\nu\nu'-}(q_1, q_2)\ ,$$

$$\Gamma_\perp^{\nu\nu'-}(q_1, q_2) = 2q_2^- g^{\nu\nu'} + \left(n^-\right)^\nu (q_1 - q_2)^{\nu'} \\ + \left(n^-\right)^{\nu'} (-q_2 - 2q_1)^\nu - \frac{q_1^2}{q_2^-} \left(n^-\right)^\nu \left(n^-\right)^{\nu'}\ , \quad (34)$$

for which one has

$$G_\perp^{\nu\nu'-} \cdot k_{\nu'} = q_2^- q_2^\nu - \left(n^-\right)^\nu q_2^2\ , \quad (35)$$

$$G_\perp^{\nu\nu'-} \cdot (q_2)_\nu = q_2^- k^{\nu'} - k^2 \left(n^-\right)^{\nu'}\ . \quad (36)$$

3.4 Effective PRR vertex

Production of a single gluon with momentum $k_\mu = (q_1 + q_2)_\mu$ and color index b in the "two reggeons collision" in color-momentum states, respectively, $(q_1,\ a;\ q_2\ ,\ c)$, is described by the PRR vertex (see Fig. 3f)

$$\Gamma^{\mu+-}(q_1, a; q_2, c; k, b) = g f^{abc}\, C^\mu(q_1, q_2)\ ,$$

$$C^\mu = 2\left[\left(n^-\right)^\mu \left(q_1^+ + \frac{q_1^2}{q_2^-}\right) \right. \\ \left. - \left(n^+\right)^\mu \left(q_2^- + \frac{q_2^2}{q_1^+}\right) + (q_2 - q_1)^\mu \right]\ . \quad (37)$$

The 4-vector C^μ obeys the gauge condition $k_\mu \cdot C^\mu = 0$.

3.5 Effective $RRPP$ vertices

We consider first the case when pair of gluons in color-momenta states $(p_1\ ,\ \mu\ ,\ a;\ p_2\ ,\ \nu\ ,\ b)$ are created in collision of two reggeons with color-momenta states $(q_1\ ,\ c;\ q_2\ ,\ d)$ with the momentum conservation relation $q_1 + q_2 = p_1 + p_2$. It can be build in terms of the effective 3-vertices given above (see Fig. 4) It has the form

$$
\begin{aligned}
&\frac{1}{ig^2} V_{abcd}^{\mu\nu-+}\left(q_1, q_2; p_1, p_2\right) = \\
&\qquad = \frac{T_1}{k^2} C^{\eta}\left(q_1, q_2\right) \gamma^{\mu\nu\eta}\left(-p_1, -p_2, k\right) \\
&\qquad - \frac{T_3}{\left(p_2 - q_2\right)^2} \Gamma^{\eta\mu-}\left(q_1, p_1 - q_1\right) \Gamma^{\eta\nu+}\left(p_2 - q_2, q_2\right) \\
&\qquad + \frac{T_2}{\left(p_1 - q_2\right)^2} \Gamma^{\eta\nu-}\left(q_1, p_2 - q_1\right) \Gamma^{\eta\mu+}\left(p_1 - q_2, q_2\right) \\
&\qquad - T_1\left[\left(n^-\right)^{\mu}\left(n^+\right)^{\nu} - \left(n^-\right)^{\nu}\left(n^+\right)^{\mu}\right] - T_2\left[2g_{\mu\nu} - \left(n^-\right)^{\mu}\left(n^+\right)^{\nu}\right] \\
&\qquad - T_3\left[\left(n^-\right)^{\nu}\left(n^+\right)^{\mu} - 2g_{\mu\nu}\right] - 2q_2^2\left(n^+\right)^{\mu}\left(n^+\right)^{\nu}\left(\frac{T_3}{p_2^+ q_1^+} - \frac{T_2}{p_1^+ q_1^+}\right) \\
&\qquad - 2q_1^2 (n^-)^{\mu} (n^-)^{\nu}\left(\frac{T_3}{p_1^- q_2^-} - \frac{T_2}{p_2^- q_2^-}\right) , \qquad (38)
\end{aligned}
$$

with $T_1 = f_{abk} f_{cdk}$, $T_2 = f_{bck} f_{adk}$, $T_3 = f_{cak} f_{bdk}$, related as $T_1 + T_2 + T_3 = 0$ by virtue of the Jacobi identity. The last two terms in the Eq. (38) are the induced ones derived directly from the corresponding part of the effective action, Eq. (13). The ordinary QCD 3-gluon vertex reads

$$
\gamma^{\mu\nu\eta}\left(-p_1, -p_2, k\right) = g_{\mu\nu}(p_2 - p_1)_{\eta} + g_{\nu\eta}(-2p_2 - p_1)_{\mu} + g_{\eta\mu}\left(2p_1 + p_2\right)_{\nu} \ .
$$

One can verify the gauge and Bose-symmetry fulfillment:

$$
V_{abcd}^{\mu\nu-+}\left(q_1, q_2; p_1, p_2\right) p_{1\mu} = 0 \ , \qquad (39)
$$

$$
V_{abcd}^{\mu\nu-+}\left(q_1, q_2; p_1, p_2\right) = V_{bacd}^{\nu\mu-+}\left(q_1, q_2; p_2, p_1\right) \ . \qquad (40)
$$

3.6 Effective $PPPR$ vertices

Let us consider now the margin $PPPR$ vertices. The effective $PPPR$ vertex can be constructed in terms of combination of previously calculated effective vertices, and ordinary 3-vertices (see Figs. 5,6).

For the vertex describing the A-particle fragmentation region to two gluons and a reggeon $P_A \to g_1(p_1) + g_2(p_2) + R(q_1)$ (see Fig. 3), one has

$$
\begin{aligned}
&\frac{1}{ig^2} V_{abcd}^{\nu_1\nu_2\rho+}(p_1, p_2, P_A, q_1) = \\
&\qquad = \frac{T_1}{k^2}\gamma^{\sigma\nu_1\nu_2}(k, -p_1, -p_2)\Gamma_{\parallel}^{\rho\sigma+}(P_A, k) - \\
&\qquad - \frac{T_3}{(p_2+q_1)^2}\gamma^{\rho\nu_1\sigma}(P_A, -p_1, -p_2 - q_1)\Gamma_{\perp}^{\sigma\nu_2+}(p_2+q_1, p_2) - \\
&\qquad - \frac{T_2}{(p_1+q_1)^2}\gamma^{\rho\nu_2\sigma}(P_A, -p_2, -p_1 - q_1)\Gamma_{\perp}^{\sigma\nu_1+}(p_1+q_1, p_1) + \\
&\qquad + T_3\left[p_1^{\nu_2}\left(n^+\right)^{\rho} - p_1^+ g^{\rho\nu_2}\right] - T_2\left[p_1^{\nu_2}\left(n^+\right)^{\rho} - p_1^{\rho}\left(n^+\right)^{\nu_2}\right] + \\
&\qquad + T_1\left[p_1^+ g^{\rho\nu_2} - \left(n^+\right)^{\nu_2} p_1^{\rho}\right] + \\
&\qquad + q_1^2 (n^+)^{\nu_1}\left(n^+\right)^{\nu_2}(n^+)^{\rho}\left(\frac{T_2}{p_2^+ p_1^+} + \frac{T_1}{P_A^+ p_2^+}\right) .
\end{aligned} \tag{41}
$$

One can be convinced in the fulfillment of a gauge condition (here we use the following relation $p_{1,2}^+ \gg q_1^+$, $p_1^+ + p_2^+ = P_A^+$):

$$
V_{abcd}^{\nu_1\nu_2\rho+}(p_1, p_2, P_A, q_1) \cdot (p_1)_{\nu_1} = 0 \ , \tag{42}
$$

as well as the Bose-symmetry condition.

Similarly, we find for the relevant vertex, describing the decay $P_B \to g_1(p_1) + g_2(p_2) + R(q_2)$:

$$
\begin{aligned}
&\frac{1}{g^2} V_{abcd}^{\nu_1\nu_2\rho-}(p_1, p_2, P_B, q_2) = \\
&\qquad = -\frac{T_1}{k^2}\gamma^{\sigma\nu_1\nu_2}(k, -p_1, -p_2)\Gamma_{\parallel}^{\sigma\rho-}(P_B, k) + \\
&\qquad + \frac{T_3}{(p_2+q_2)^2}\gamma^{\sigma\nu_1\rho}(-q_2 - p_2, -p_1, P_B)\Gamma_{\perp}^{\sigma\nu_2-}(p_2+q_2, p_2) - \\
&\qquad - \frac{T_2}{(p_1+q_2)^2}\gamma^{\nu_2\rho\sigma}(-p_2, P_B, -p_1 - q_2)\Gamma_{\perp}^{\nu_1\sigma-}(p_1, q_2+p_1) + \\
&\qquad + T_1[(n^-)^{\nu_1} g^{\rho\nu_2} - (n^-)^{\nu_2} g^{\rho\nu_1}] + \\
&\qquad + T_3\left[g^{\nu_1\nu_2}(n^-)^{\rho} - (n^-)^{\nu_1} g^{\rho\nu_2}\right] + T_2\left[\left(n^-\right)^{\nu_2} g^{\rho\nu_1} - \left(n^-\right)^{\rho} g^{\nu_1\nu_2}\right] + \\
&\qquad q_2^2\left(n^-\right)^{\nu_1}\left(n^-\right)^{\nu_2}\left(n^-\right)^{\rho}\left[\frac{T_2}{p_2^- p_1^-} + \frac{T_1}{P_B^- p_2^-}\right] .
\end{aligned} \tag{43}
$$

Again, the gauge condition is fulfilled (provided that $p^-_{1,2} \gg q^-_2$, $p^-_1 + p^-_2 = P^-_B$):

$$V^{\nu_1\nu_2\rho-}_{abcd}\,(p_1, p_2, P_B, q_2) \cdot (p_1)_{\nu_1} = 0 \ . \tag{44}$$

3.7 Effective $RRPPP$, and $RPPPP$ vertices

These vertices can be constructed by means of combinations of the already known effective vertices given above, Eqs. (15,16,17,19,31,34) . The result is graphically presented at the Figs. 7-9. Corresponding explicit analytical expressions appear to be rather cumbersome; moreover, a problem of verification of the gauge invariance arises. The accurate derivation of these vertices requires additional work and will be considered separately.

4 Discussions and Conclusions

The main formulas presented in this work were derived first in Refs. [3, 5, 6, 7], and thus our paper is mostly a compilation, on the one hand. Besides this, we did not consider here the possible fermion contributions and helicity amplitudes. On the other hand, the primary purpose of our investigation is to give a systematic self-consistent approach to derivation of the Feynman rules from the effective reggeon-particle action, and, what is essential, to present the results in a most convenient form for numerical simulations. We hope that our results will provide a practical help in writing relevant computer codes for phenomenological needs.

Indeed, we see that all the RRP^n and P^nR effective vertices can be constructed in terms of the effective vertices of lower orders PR, PR, RPP supplied with the ordinary QCD vertices. Unfortunately, the corresponding explicit expressions for them appear to be very complicated. For instance, the $RRPPP$ vertex contains 22 terms but for the induced ones. In the paper by one of us [5], the recurrence relation was derived for induced vertices that allows one, in principle, to build vertices of higher orders provided that the lower ones are known. For the margin RP^n case, this relation reads:

$$\Delta^{\nu_0\nu_1...\nu_r+}_{a_0a_1...a_r}\left(P^+_A, k^+_1, ..., k^+_r\right) =$$
$$= \frac{\left(n^+\right)^{\nu_r}}{k^+_r} \sum_{i=0}^{r-1} f^{aa_2a_i} \times$$
$$\times \, \Delta^{\nu_0\nu_1...\nu_{r-1}+}_{a_0a_1...a_{i-1}aa_{i+1}...a_{r-1}}\left(P^+_A, k^+_1, ..., k^+_{i-1}, k^+_i + k^+_r, k^+_{i+1}, ..., k^+_{r-1}\right) \ , \quad (45)$$
$$P^+_A + \sum k^+_i = 0 \ ,$$

with
$$\Delta^{\nu_0\nu_1+}_{a_0a_1c} = -f^{ca_1a_0}\,\frac{q_1^2}{k_1^+}\left(n^+\right)^{\nu_0}\left(n^+\right)^{\nu_1} \ , \ P^+_A + k^+_1 = 0 \ . \quad (46)$$

The similar expression holds for $\Delta^{\nu_0\nu_1...\nu_r-}_{a_0a_1...a_r}\left(P^-_B, k^-_1, ..., k^-_r\right)$. The Bose-symmetry is obviously satisfied.

For induced vertices in the central region—RRP^n—one has:

$$\Gamma^{\nu_1...\nu_n+-}_{d_1...d_nc_2c_1} = \Delta^{+\nu_1...\nu_n-}_{c_1d_1...d_nc_2}\left(k^+_0, ..., k^+_n\right) + \Delta^{+\nu_1...\nu_n-}_{c_2d_1...d_nc_1}\left(k^-_0, ..., k^-_n\right) \ , \quad (47)$$

where
$$\Delta^{+\nu_1...\nu_n-}_{c_1d_1...d_nc_2} = \left(n^+\right)_{\nu_0} \cdot \Delta^{\nu_0\nu_1...\nu_n-}_{c_1d_1...d_nc_2} \ ,$$

and
$$\sum_0^n k^+_i = \sum_0^n k^-_i = 0 \ .$$

5 Acknowledgements

The authors thank the Organizers of the International Seminar "QUARKS-2004" for hospitality and support. The work is partially supported by INTAS (Grant No. 00-00-366), RFBR (Grants Nos. 04-02-16445, 03-02-17077, 03-02-17291), and Russian Federation President's Grant 1450-2003-2.

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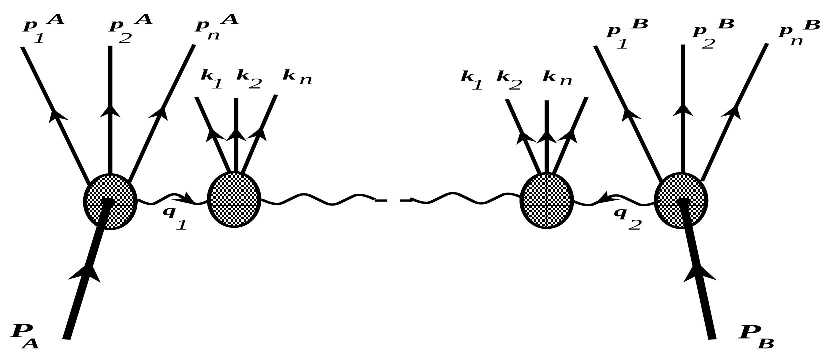

Figure 1: Quasi-multi-Regge kinematics: notations.

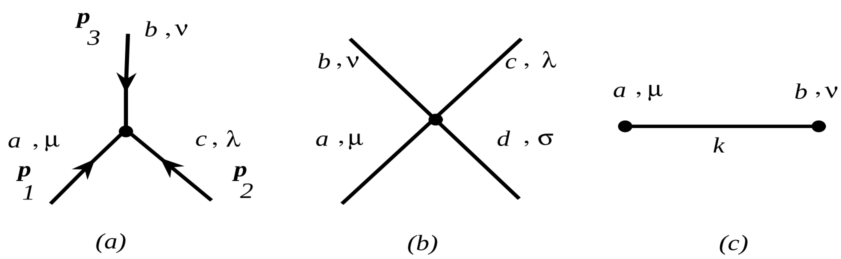

Figure 2: Standard QCD Feynman rules: vertices (a-c), and gluon propagator (d).

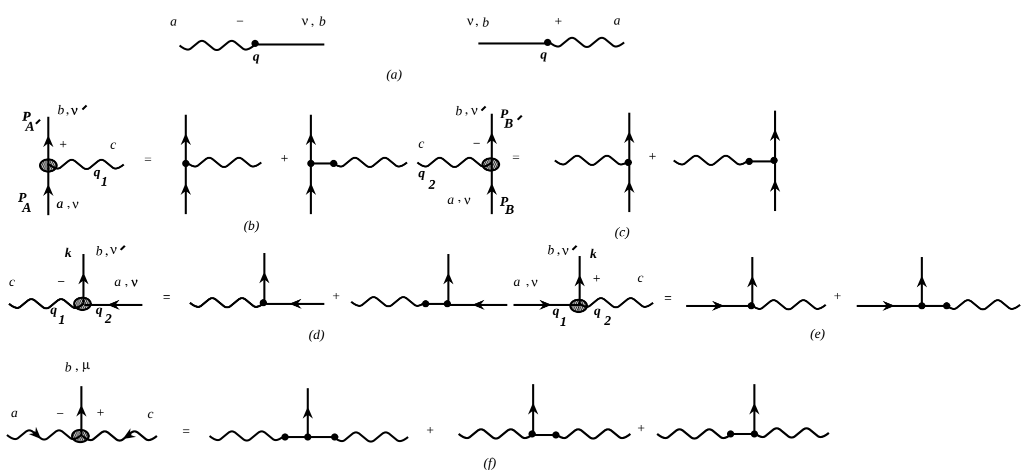

Figure 3: RP, PR (a), PPR (b-e) and RRP (f) effective vertices.