Chiral phase transition in hadronic matter at non-zero baryon density.

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Abstract

A qualitative analysis of the chiral phase transition in QCD at non–zero baryon density is performed. It is assumed that at zero baryonic density, $\rho = 0$, the temperature phase transition is of the second order and quark condesate $\eta =\mid \langle 0 \mid \bar{u}u \mid 0\rangle \mid=\mid \langle 0 \mid \bar{d}d \mid 0\rangle \mid$ may be taken as order parameter of phase transition. It is demonstrated, that the proportionality of baryon masses to quark condensate in the power 1/3, $m_B \sim\mid \langle 0 \mid \bar{q}q \mid 0\rangle \mid^{1/3}$ is valid in the wide interval of quark condensate values. By supposing, that such specific dependence of baryon masses on quark condensate takes place up to phase transition point, it is shown, that at finite baryon density ρ the phase transition becomes of the first order at the temperature $T = T_{\rm ph}(\rho)$ for $\rho > 0$. At temperatures $T_{\rm cont}(\rho) > T > T_{\rm ph}(\rho)$ there is a mixed phase consisting of the quark phase (stable) and the hadron phase (unstable). At the temperature $T = T_{\rm cont}(\rho)$ the system experiences a continuous transition to the pure chirally symmetric phase.

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1 Introduction

It is well known, that the chiral symmetry is valid in perturbative quantum chromodynamics (QCD) with massless quarks. It is expected also, that the chiral symmetry takes place in full–perturbative and nonperturbative QCD at high temperatures, ($T \gtrsim 200$ MeV), if heavy quarks (c, b, t) are ignored. The chiral symmetry is strongly violated, however, in hadronic matter, *i.e.* in QCD at $T = 0$ and low density. What is the order of phase transition between two phases of QCD with broken and restored chiral symmetry at variation of temperature and density is not completely clear now. There are different opinions about this subject (for a detailed review see Ref. [1, 2] and references therein).

In this talk I discuss the phase transitions in QCD with two massless quarks, u and d. Many lattice calculations [3, 4, 5, 6] indicate, that at zero chemical potential the phase transition is of the second order. It will be shown below, that the account of baryon density drastically changes the situation and the transition becomes of the first order, and, at high density, the matter is always in the chirally symmetric phase.

The masses of light u, d, s quarks which enter the QCD Lagrangian, especially the masses of u and d quarks, from which the usual (nonstrange) hadrons are built, are very small, $m_u, m_d < 10$ MeV as compared with characteristic mass scale $M \sim 1$ GeV. Since in QCD the quark interaction proceeds through the exchange of vector gluonic field, then, if light quark masses are neglected, QCD Lagrangian (its light quark part) is chirally symmetric, i.e. not only vector, but also axial currents are conserved and the left and right chirality quark fields are conserving separately. This chiral symmetry is not realized in hadronic matter, in the spectrum of hadrons and their low energy interactions. Indeed, in chirally symmetrical theory the fermion states must be either massless or degenerate in parity. It is evident, that the baryons (particularly, the nucleon) do not possess such properties. This means, that the chiral symmetry of QCD Lagrangian is not realized on the spectrum of physical states and is spontaneously broken. According to Goldstone theorem spontaneous breaking of symmetry leads to appearance of massless particles in the spectrum of physical states – the Goldstone bosons.

Let us first consider a case of the zero baryonic density and suppose that the phase transition from chirality violating phase to the chirality conserving one is of the second order. The second order phase transition is, generally, characterized by the order parameter η. The order parameter is a thermal average of some operator which may be chosen in various ways. The physical results are independent on the choice of the order parameter. In QCD the quark condensate, $\eta = |\langle 0|\bar{u}u|0\rangle| = |\langle 0|\bar{d}d|0\rangle| \geq 0$, may be taken as such parameter. In the phase of broken chiral symmetry (hadronic phase) the quark condensate is non-zero and has the normal hadronic scale, $\langle 0 \mid \bar{q}q \mid 0\rangle = (250MeV)^3$, in the phase of restored chiral symmetry it is vanishing.

The quark condensate has the desired properties: as it was demonstrated in the chiral effective theory [7, 8], η decreases with the temperature increasing and an extrapolation of the curve $\eta(T)$ to the higher temperatures indicates, that η vanishes at $T = T_c^{(0)} \approx 180$ MeV. Here the superscript "0" indicates that the critical temperature is taken at zero baryon density. The same conclusion follows from the lattice calculations [3, 6, 9], where it was also found that the chiral condensate η decreases with increase of the chemical potential [10, 11].

Apply the general theory of the second order phase transitions [12] and consider the thermodynamical potential $\Phi(\eta)$ at the temperature T near $T_c^{(0)}$. Since η is small in this domain, $\Phi(\eta)$ may be expanded in η:

$$\Phi(\eta) = \Phi_0 + \frac{1}{2}A\,\eta^2 + \frac{1}{4}B\,\eta^4\,, \qquad B > 0\,. \tag{1}$$

For a moment we neglect possible derivative terms in the potential.

The terms, proportional to η and η^3 vanish for general reasons [12]. In QCD with massless quarks the absence of η and η^3 terms can be proved for any perturbative Feynman diagrams. At small $t = T - T_c^{(0)}$ the function $A(t)$ is linear in t: $A(t) = at$, $a > 0$. If $t < 0$ the thermodynamical potential $\Phi(\eta)$ is minimal at $\eta \neq 0$, while at $t > 0$ the chiral condensate vanishes $\eta = 0$. At small t the t-dependence of the coefficient $B(t)$ is inessential and may be neglected. The minimum, $\bar{\eta}$, of the thermodynamical potential can be found from the condition, $\partial\Phi/\partial\eta = 0$:

$$\bar{\eta} = \begin{cases} \sqrt{-at/B}\,, & t < 0\,; \\ 0\,, & t > 0\,. \end{cases} \tag{2}$$

It corresponds to the second order phase transition since the potential is quartic in η and – if the derivative terms are included in the expansion – the correlation length becomes infinite at $T = T_c^{(0)}$.

2 Nucleon mass and quark condensate

I show now, that the existence of large baryon masses and the appearance of violating chiral symmetry quark condensate are deeply interconnected and even more, that baryon masses arise just due to quark condensate. I will use the QCD sum rule method invented by Shifman, Vainstein and Zakharov [13], in its applications to baryons [14-17]. (For a review and collection of relevant original papers see [18]). The idea of the method is that at virtualities of order $Q^2 \sim 1$ GeV^2 the operator product expansion (OPE) may be used in consideration of hadronic vacuum correlators. In OPE the nonoperturbative effects reduce to appearance of vacuum condensates and condensates of the lowest dimension play the most important role. The perturbative terms are moderate and do not change the results in essential way, especially in the cases of chiral symmetry violation, where they can appear as corrections only.

For definiteness consider the proton mass calculation [14,15]. Introduce the polarization operator

$$\Pi(p) = i \int d^4x e^{ipx} \langle 0|T\xi(x), \bar{\xi}(0)|0\rangle \tag{3}$$

where $\xi(x)$ is the quark current with proton quantum numbers and p^2 is chosen to be space-like, $p^2 < 0$, $|p^2| \sim 1\ GeV^2$. The current ξ is the colourless product of three quark fields, $\xi = \varepsilon^{abc}\ q^a q^b q^c$, $q = u, d$, the form of the current will be specialized below. The general structure of $\Pi(p)$ is

$$\Pi(p) = \hat{p} f_1(p) + f_2(p) \tag{4}$$

The first structure, proportional to $\hat{p}$ is conserving chirality, while the second is chirality violating.

For each of the functions $f_i(p^2)$, $i = 1, 2$ the OPE can be written as:

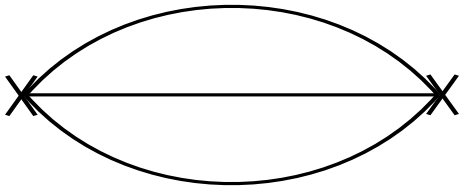

Figure 1: The bare loop diagram, contributing to chirality conserving function $f_1(p^2)$: solid lines correspond to quark propagators, crosses mean the interaction with external currents.

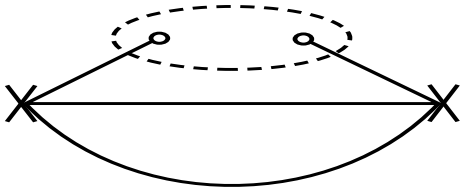

Figure 2: The diagram, corresponding to chirality violating dimension 3 operator (quark condensate). The dots, surrounded by circle mean quarks in the condensate phase. All other notation is the same as on Fig.1.

$$f_i(p^2) = \sum_n C_n^{(i)}(p^2)\langle 0|O_n^{(i)}|0\rangle \tag{5}$$

where $\langle 0|O_n^{(i)}|0\rangle$ are vacuum expectation values (v.e.v) of various operators (vacuum condensates), $C_n^{(i)}$ are coefficient functions calculated in QCD. For the first, conserving chirality structure function $f_i(p^2)$ OPE starts from dimension zero ($d = 0$) unit operator. Its contribution is described by the diagram of Fig.1 and

$$\hat{p}f_1(p^2) = C_0\hat{p}p^4 ln[\Lambda_u^2/(-p^2)] + polynomial, \tag{6}$$

where C_0 is a constant, Λ_u is the ultraviolet cutoff. The OPE for chirality violating structure $f_2(p^2)$ starts from $d = 3$ operator, and its contribution is represented by the diagram of Fig.2:

$$f_2(p^2) = C_1 p^2 \langle 0|0\bar{q}q|0\rangle ln\frac{\Lambda_u^2}{(-p^2)} + polynomial \tag{7}$$

Let us for a moment restrict ourselves to this first order terms of OPE and neglect higher order terms (as well the perturbative corrections).

On the other hand, the polarizaion operator (3) may be expressed via the characteristics of physical states using the dispersion relations

$$f_i(s) = \frac{1}{\pi}\int \frac{Imf_i(s')}{s'+s}\,ds' + polynomial, \quad s = -p^2 \tag{8}$$

The proton contribution to $Im\Pi(p)$ is equal to

$$Im\Pi(p) = \pi\langle 0|\xi|p\rangle\langle p|\overline{\xi}|0\rangle\delta(p^2-m^2) = \pi\lambda_N^2(\hat{p}+m)\delta(p^2-m^2), \tag{9}$$

where

$$\langle 0|\xi|p\rangle = \lambda_N v(p), \tag{10}$$

λ_N is a constant, $v(p)$ is the proton spinor and m is the proton mass. Still restricting ourselves to this rough approximation, we may take equal the calculated in QCD expression for $\Pi(p)$ (Eq.'s(6),(7)) to its phenomenological representation Eq.(9). The best way to get rid of unknown polynomial, is to apply to both sides of the equality the Borel(Laplace) transformation, defined as

$$\begin{aligned} \mathcal{B}_{M^2} f(s) &= \lim_{\substack{n\to\infty, s\to\infty, \\ s/n=M^2=Const}} \frac{s^{n+1}}{n!}\left(-\frac{d}{ds}\right)^n f(s) \\ &= \frac{1}{\pi}\int\limits_0^\infty ds Im f(s) e^{-s/M^2} \end{aligned} \tag{11}$$

if $f(s)$ is given by dispersion relation (8). Notice, that

$$\mathcal{B}_{M^2}\frac{1}{s^n} = \frac{1}{(n-1)!(M^2)^{n-1}} \tag{12}$$

Owing to the factor $1/(n-1)!$ in (12) the Borel transformation suppresses the contributions of high order terms in OPE.

Specify now the quark current $\xi(x)$. It is clear from (9) that proton contribution will dominate in some region of the Borel parameter $M^2 \sim m^2$ only in the case when both calculated in QCD functions f_1 and f_2 are of the same order. This requirement, together with the requirements of absence of derivatives and of renormcovariance fixes the form of current in unique way (for more details see [14,15]):

$$\xi(x) = \varepsilon^{abc}(u^a C\gamma_\mu u^b)\gamma_\mu\gamma_5 d^c \tag{13}$$

where C is the charge conjugation matrix. With the current $\eta(x)$ (13) the calculations of the diagrams Fig.2 can be easily performed, the constants C_0 and C_1 are determined. The terms of OPE up to dimension 8 were accounted in the sum rule for the conserving chirality structure function $f_1(p^2)$ and the terms up to dimension 7 were accounted in the violating chirality structure function $f_2(p^2)$. The perturbative corrections are neglected. The phenomenological parts of the sum rules were represented by contributions of proton and continuum, the latter were transferred to QCD parts. The resulting sum rules after Borel transformation have the form [16,17]

$$M^6 E_2(M)L^{-4/9} + \frac{4}{3}a^2 L^{4/9} + \frac{1}{4}bM^2 E_0(M)L^{-4/9} -$$

$$-\frac{1}{3}a^2 m_0^2 \frac{1}{M^2} = \bar{\lambda}_N^2 exp\left(-\frac{m^2}{M^2}\right) \tag{14}$$

$$a\left[2M^4 E_1(M) + \frac{272}{81}\frac{\alpha_s}{\pi}\frac{a^2}{M^2} - \frac{1}{12}b\right] = m\bar{\lambda}_N^2 exp\left(-\frac{m^2}{M^2}\right) \tag{15}$$

$$a = -(2\pi)^2 \langle 0 \mid \bar{q}q \mid 0\rangle \tag{16}$$

$$b = (2\pi)^2 \langle 0 \mid \frac{\alpha_s}{\pi} G_{\mu\nu}^2 \mid 0\rangle \tag{17}$$

$$g\langle 0 \mid \bar{q}\sigma_{\mu\nu}\frac{\lambda^n}{2} G_{\mu\nu}^2 q \mid 0\rangle = m_0^2 \langle 0 \mid \bar{q}q \mid 0\rangle \tag{18}$$

$$E_n\left(\frac{s_0}{M^2}\right) = \frac{1}{n!}\int\limits_0^{s_0/M^2} z^n e^{-z} dz \tag{19}$$

$$L = \frac{\alpha_s(\mu)}{\alpha_s(M)} \tag{20}$$

$$\bar{\lambda}_N^2 = 32\pi^2 \lambda_N^2 \tag{21}$$

$L(M,\mu)$ in (14),(15) account the anomalous dimensions of the current $\xi(x)$ and various operators, $E_n(\frac{s_0}{M^2})$ accounts the continuum contribution, which starts at $s = s_0$. The ratio of (15) to (14) gives for proton mass

$$m = 2a\frac{1}{M^2} f(M^2) \tag{22}$$

The inspection of the r.h.s. of (21) at the values of QCD parameters $a = 0.63$ GeV^3, $b = 0.24$ GeV^4, $m_0^2 = 0.8$ GeV^2, $s_0 = 2.3$ GeV^2 shows, that at $0.8 < M^2 < 1.4$ GeV^2 with a good accuracy $f(M^2)$ is equal to 1 and the value $M^2 = m^2$ is close to the middle in the plateau. So we get a simple formula for proton mass [14]

$$m = [-2(2\pi)^2 \langle 0|\bar{q}q|0\rangle]^{1/3} \tag{23}$$

Let us now check, if eq.(23) is valid at variation of quark condensate value. Perform the scale variation $\langle 0 \mid qq \mid 0\rangle \to \gamma\langle 0 \mid \bar{q}q \mid 0\rangle$, $m \to \gamma^{1/3}m, M \to \gamma^{1/3}m, s_0 \to \gamma^{2/3}s_0$. Then as can be seen from the (14,15) the only violations of eq.(23), come from terms, proportional to b and m_0^2 in (14),(15). But they are very small. Therefore, even at $\gamma = 1/3$, the relation (23) changes no more, than by 7%. To go to lower values of γ is not possible, because of strongly increasing perturbative corrections.

This formula demonstrates the fundamental fact, that the appearance of the proton mass is caused by spontaneous violation of chiral invariance: the

presence of quark condensate. (Numerically, (23) gives the experimental value of proton mass with an accuracy better than 10%).

In the same way, the hyperons, isobar and some resonances masses were calculated, all in a good agreement with experiment [14,16]. I will not dwell on these results. The main conclusion is: the origin of baryon masses is in spontaneous violation of chiral invariance – the existence of quark condensate in QCD.

3 The chiral phase transition in the presense of finite baryon density

I would like to consider here the influence of baryon density on the chiral phase transition in hadronic matter. Kogan, Kovner and Tekin [19] have suggested the idea, that baryons may initiate the restoration of chiral symmetry, if their density is high – when roughly half of the volume is occupied by baryons. The physical argument in favour of this idea comes from the hypothesis (supported by calculation in chiral soliton model of nucleon [20]), that inside the baryon the chiral condensate has the sign opposite to that in vacuum. This hypothesis is not proved. Even more, it is doubtful, that the concept of quark condensate inside the nucleon can be formulated in a correct way in quantum theory. But the idea on the strong influence of baryon density on the chiral phase transition looks very attractive. For this reason no assumption on the driving mechanism of chiral phase transition at zero baryon density will be done here. The problem, under consideration is: how the phase transition changes in the presence of baryons. The content of my talk closely follows ref.21.

For completeness it must be mentioned, that the variation of quark codensate with temperature is not the only source of baryon effective mass shift. At low T the effective baryon mass shift arises also due to interaction with pions in thermal bath [22,23]. However, this mass shift, which may be called external (unlike the internal, arising from variation of quark condensate) is related to effective mass, i.e. to propagation of baryon in the matter and has nothing to do with the properties of the matter as a whole and the phase transition. (A similar phenomenon takes place in case of vector mesons, where because of interaction with pions in the thermal bath the mixing with axial mesons arises [24].)

Consider the case of the finite, but small baryon density ρ (by ρ we mean here the sum of baryon and anti–baryon densities). For a moment, consider only one type of baryons, *i.e.* the nucleon. The temperature of the phase transition, $T_{\rm ph}$, is, in general, dependent on the baryon density, $T_{\rm ph} = T_{\rm ph}(\rho)$, with $T_{\rm ph}(\rho = 0) \equiv T_c^{(0)}$ At $T < T_{\rm ph}(\rho)$ the term, proportional to $E\rho$, where $E = \sqrt{p^2 + m^2}$ is the baryon energy, must be added to the thermodynamical potential (1). As was shown above the nucleon mass m (as well as the masses of other baryons) arises due to the spontaneous violation of the chiral symmetry

and is approximately proportional to the cubic root of the quark condensate: $m = c\eta^{1/3}$, with $c = (8\pi^2)^{1/3}$ for a nucleon. At small temperatures T the baryon contribution to Φ is strongly suppressed by the Boltzmann factor $e^{-E/T}$ and is negligible. Below we assume that the proportionality $m \sim \eta^{1/3}$ is valid in a broad temperature interval up to phase transition point. Arguments in favor of such an assumption are based on the expectation that the baryon masses vanish at $T = T_{\rm ph}(\rho)$ and on the dimensional grounds. Near the phase transition point $E = \sqrt{p^2 + m^2} \approx p + c^2\,\eta^{2/3}/(2p)$. At $\eta \to 0$ all baryons are accumulating near zero mass and a summation over all baryons gives us – instead of eq. (1) – the following:

$$\Phi(\eta, \rho) = \Phi_0 + \frac{1}{2} at\,\eta^2 + \frac{1}{4} B\,\eta^4 + C\eta^{2/3}\rho\,, \tag{24}$$

where $C = \sum_i c_i^2/(2p_i)$. The term $\rho \sum_i p_i$ is absorbed into Φ_0 since it is independent on the chiral condensate η. The typical momenta are of the order of the temperature, $p_i \sim T$. Thus, Eq. (24) is valid in the region $\eta \ll T^3$. In the leading approximation the coefficient C can be considered as independent on the temperature at $T \sim T_c^{(0)}$.

Due to the last term in Eq. (24) the thermodynamical potential *always* have a local minimum at $\eta = 0$ since the condensate η is always non–negative. At small $t < 0$ there also exists a local minimum at $\eta > 0$, which is a solution of the equation:

$$\frac{\partial \Phi}{\partial \eta} \equiv (at + B\,\eta^2)\eta + \frac{2}{3} C\rho\,\eta^{-1/3} = 0\,. \tag{25}$$

At small enough baryon density ρ, Eq. (25) [visualized in Figure 3(a)] has, in general, two roots, $\eta_1 < \eta_0$ and $\eta_2 > \eta_0$, where $\eta_0 = (-at/3B)^{1/2}$ is the minimum of the first term in the right-hand side of Eq. (25). The calculation of the second derivative $\partial^2\Phi/\partial\eta^2$ shows that the second root η_2 (if exists) corresponds to minimum of $\Phi(\eta)$ and, therefore, is a local minimum of Φ. The point $\eta = \eta_1$ corresponds to a local maximum of the thermodynamical potential since at this point the second derivative is always non–positive.

The thermodynamical potential $\Phi(\eta, \rho)$ at (fixed) non–zero baryon density ρ has the form plotted in Figure 3(b). At low enough temperatures (curve T_1) the potential has a global minimum at $\eta > 0$ and system resides in the chirally broken (hadron) phase. As temperature increases the minima at $\eta = 0$ and at $\eta = \bar{\eta}_2 > 0$ becomes of equal height (curve $T_2 \equiv T_{\rm ph}$). At this point the first order phase transition to the quark phase takes place. At somewhat higher temperatures, $T = T_3 > T_{\rm ph}$, the $\eta > 0$ minimum of the potential still exist but $\Phi(\eta = 0) < \Phi(\bar{\eta}_2)$. This is a mixed phase, in which the bubbles of the hadron phase may still exist. However, as temperature increases further, the second minimum disappears (curve $T_4 \equiv T_{\rm cont}$). This temperature corresponds to a continuous transition to the pure quark phase, in which the thermodynamic potential has the form T_5.

Let us calculate the temperature of the phase transition, $T_{\rm ph}(\rho)$, at non–zero baryon density ρ. The transition corresponds to the curve T_2 in Figure 3(b),

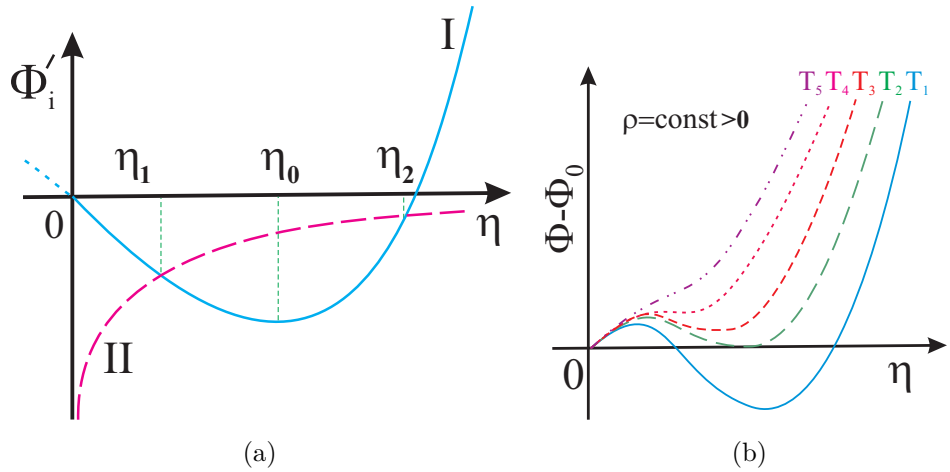

Figure 3: (a) Graphical representation of Eq. (25): "I" is the first term and "II" the second term (with the opposite sign) in the *r.h.s.* of the equation. (b) The thermodynamic potential (24) *vs.* the chiral condensate at a fixed baryon density $\rho > 0$. At low enough temperatures, $T = T_1$, the system resides in the chirally broken (hadron) phase. The first order phase transition to the quark phase takes place at $T_{\rm ph} = T_2 > T_1$. At somewhat higher temperatures, $T_3 > T_{\rm ph}$ the system is in a mixed state. The temperature $T_4 \equiv T_{\rm cont}$ corresponds to a continuous transition to the pure quark phase, in which the thermodynamic potential has the form T_5.

which is defined by the equation $\Phi(\bar{\eta}_2, \rho) = \Phi(\eta = 0, \rho)$, where $\bar{\eta}_2$ is the second root of Eq. (25) as discussed above. The solution is

$$T_{\rm ph}(\rho) = T_c^{(0)} - \frac{5}{a}\left(\frac{2\,C\,\rho}{3}\right)^{3/5}\left(\frac{B}{4}\right)^{2/5}, \tag{26}$$

and the second minimum of the thermodynamic potential is at
$\bar{\eta}_2 = \left[4a\,(T^{(0)} - T_{\rm ph}(\rho))/(5\,B)\right]^{1/2}$.

At a temperature slightly higher than $T_{\rm ph}(\rho)$ the potential is minimal at $\eta = 0$, but it has also an unstable minimum at some $\eta > 0$. The existence of metastable state is also a common feature of the first order phase transition (*e.g.*, the overheated liquid in case of liquid–gas system). With a further increase of the density ρ (at a given temperature) the intersection of the two curves in Figure 3(a) disappears and the two curves only touch one another at one point $\eta = \bar{\eta}_4$. At this temperature a continuous transition (crossover) takes place. The corresponding potential has the characteristic form denoted as T_4 in Figure 3(a). The temperature $T_4 \equiv T_{\rm cont}$ is defined by the condition that the first (25)

and the second derivatives of Eq. (24) vanish:

$$T_{\rm cont}(\rho) = T_c^{(0)} - \frac{5}{a}\left(\frac{2\,C\,\rho}{9}\right)^{3/5}\left(\frac{B}{2}\right)^{2/5}, \tag{27}$$

and the value of the chiral condensate, where the second local minimum of the potential disappears is given by $\bar{\eta}_4 = \left[2a(T_{\rm cont}(\rho) - T_c^{(0)})/(5\,B)\right]^{1/2}$. At temperatures $T > T_{\rm cont}(\rho)$ the potential has only one minimum and the matter is in the state with the restored chiral symmetry. Thus, in QCD with massless quarks the type of phase transition with the restoration of the chiral symmetry strongly depends on the value of baryonic density ρ. At a fixed temperature, $T < T_c^{(0)}$, the phase transition happens at a certain critical density, $\rho_{\rm ph}$. According to Eq. (26) the critical density has a kind of a "universal" dependence on the temperature, $\rho_{\rm ph}(T) \propto [T_c^{(0)} - T]^{5/3}$, the power of which does not depend on the parameters of the thermodynamic potential, a and B.

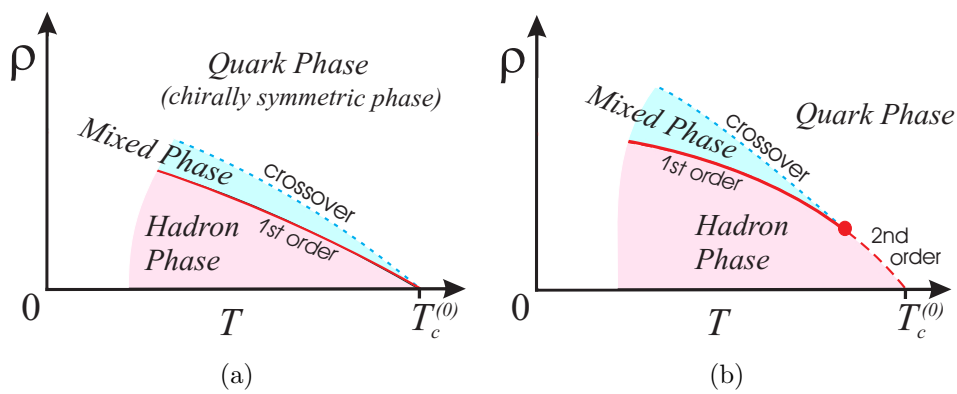

Figure 4: The qualitative phase diagram at finite baryon density and temperature based on the analysis (a) without and (b) with indication of the approximate 2-nd order transition domain.

The expected phase diagram is shown qualitatively in Figure 4(a). This diagram does not contain an end-point which was found in lattice simulations of the QCD with a finite chemical potential [25,26]. One may expect that this happens because in our approach a possible influence of the confinement on the order of the chiral restoration transition was ignored. Intuitively, it seems that at low baryon densities such influence is absent indeed: the deconfinement phenomenon refers to the large quark–anti-quark separations while the restoration of the of the chiral symmetry appears due to fluctuations of the gluonic fields in the vicinity of the quark. However, the confinement phenomenon dictates the value of the baryon size which can not be ignored at high baryon densities, when the baryons are overlapping. If the melting of the baryons happens in the

hadron phase depicted in Figure 4(a), then at high enough density the nature of the transition could be changed. This may give rise in appearance of the end-point observed in Ref.[25,26].

The domain where the inequality $|at| \gg C\rho\eta^{2/3}$, $\rho \neq 0$ is fulfilled, has specific features. In this domain the phase transition looks like a smeared second order phase transition: the specific heat has (approximately) a discontinuity at the phase transition point, $\Delta C_p = a^2 T_c/2B$. This statment follows from general theory [12]. At $\mid at \mid \gg C\rho\eta^{2/3}$ the last term in (17) may be neglected and we find for the entropy

$$S = -\frac{\partial \Phi}{\partial T} = S_0 - \frac{1}{2}\frac{\partial A}{\partial T}\eta^2 \tag{28}$$

In the phase above phase transition, $\eta = 0, S = S_0$. Below phase transition

$$S = S_0 + \frac{a^2}{2B}(T - T_c) \tag{29}$$

The specific heat $C_p = T(\partial S/\partial T)p$ below the phase transition in the limit $T \to T_c$ is equal

$$C_p = C_{p_0} + \frac{a^2}{2B}T_c \tag{30}$$

The correlation length increases as $(T - T_c^{(0)})^{-1/2}$ at $T - T_c^{(0)} \to 0$. The latter arises if we include the derivative terms in the effective thermodynamical potential. The phase diagram with this domain indicated may look as it is shown in Figure 4(b). Note that the applicability of our considerations is limited to the region $|T - T_c^{(0)}|/T_c^{(0)} \ll 1$ and low baryon densities.

In the real QCD the massive heavy quarks (the quarks c, b, t) do not influence on this conclusion, since their concentration in the vicinity of $T \approx T_c^{(0)} \sim$ 200 MeV is small. However, the strange quarks, the mass of which $m_s \approx$ 150 MeV is just of order of expected $T_c^{(0)}$, may change the situation. This problem deserves further investigation.

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