Problem of Light Scalar Mesons

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April 29, 2005

Abstract

The following topics are considered.

1. Confinement, chiral dynamics and light scalar mesons.
2. ϕ-meson radiative decays about nature of light scalar resonances.

Arguments in favor of the four-quark model of the $a_0(980)$ and $f_0(980)$ mesons are given.

1 Kategorischer Imperativ

To discuss actually the nature of the putative nonet of the light scalar mesons: the red putative $f_0(600)$ (or $\sigma(600)$) and $\kappa(700-900)$ mesons and the red well established $f_0(980)$ and $a_0(980)$ mesons, one should explain not only their mass spectrum, particularly the mass degeneracy of the $f_0(980)$ and $a_0(980)$ states, but answer the next real challenges.

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1. The copious $\phi \to \gamma f_0(980)$ decay and especially the copious $\phi \to \gamma a_0(980)$ decay, which looks as the decay plainly forbidden by the Okubo-Zweig-Izuka (OZI) rule in the quark-antiquark model $a_0(980) = (u\bar{u} - d\bar{d})/\sqrt{2}$.

2. Absence of $J/\psi \to a_0(980)\rho$ and $J/\psi \to f_0(980)\omega$ with copious $J/\psi \to a_2(1320)\rho$, $J/\psi \to f_2(1270)\omega$, $J/\psi \to f_2'(1525)\phi$ if $a_0(980)$ and $f_0(980)$ are P-wave states of $q\bar{q}$ like $a_2(1320)$ and $f_2(1270)$ respectively.

3. Absence of $J/\psi \to \gamma f_0(980)$ with copious $J/\psi \to \gamma f_2(1270)$ and $J/\psi \to \gamma f_2'(1525)\phi$ if $f_0(980)$ is P-wave state of $q\bar{q}$ like $f_2(1270)$ or $f_2'(1525)$.

4. Suppression of $a_0(980) \to \gamma\gamma$ and $f_0(980) \to \gamma\gamma$ with copious $a_2(1320) \to \gamma\gamma$, $f_2(1270) \to \gamma\gamma$, $f_2'(1525) \to \gamma\gamma$ if $a_0(980)$ and $f_0(980)$ are P-wave state of $q\bar{q}$ like $a_2(1320)$ and $f_2(1270)$ respectively.

Unfortunately, I have opportunity to dwell on the ϕ radiative decays only in this talk.

2 Place in QCD, Chiral Limit,Confinement, σ-Models, History, Chiral Shielding, Troubles and Expectancies

Study of the nature of light scalar resonances has become a central problem of non-perturbative of QCD. The point is that the elucidation of their nature is important for understanding both the confinement physics and the chiral symmetry realization way in the low energy region, i.e., the main consequences of QCD in the hadron world.

$$L = -(1/2)Tr\,(G_{\mu\nu}G^{\mu\nu}) + \bar{q}(\imath\hat{D} - M)q,$$

$$M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}, \quad q^\alpha \equiv q^\alpha(x) = \begin{pmatrix} u^\alpha(x) \\ d^\alpha(x) \\ s^\alpha(x) \end{pmatrix}$$

M mixes Left and Right Spaces $q_L(x) = (1/2)(1+\gamma_5)q(x)$ and $q_R(x) = (1/2)(1-\gamma_5)q(x)$. But in chiral limit $M_{ij} \to 0$ these spaces separate realizing $U_L(3) \times U_R(3)$ flavour symmetry, $q'_{L,R}(x) = \mathrm{U}_{L,R} \cdot q_{L,R}(x)$, which, however, is broken by the gluonic anomaly up to $U_{vec}(1) \times SU_L(3) \times SU_R(3)$. As

Experiment suggests, Confinement forms colourless observable hadronic fields and spontaneous breaking of chiral symmetry with massless pseudoscalar fields. There are two possible scenarios for OCD at low energy.

1. Non-linear σ-model.

$$L = \left(F_\pi^2/2\right) Tr\left(\partial_\mu V(x)\partial^\mu V^+(x)\right) + \dots ,$$
$$V(x) = \exp\left\{2\imath\phi(x)/F_\pi\right\}, \quad V'(x) = \mathrm{U}_L V(x)\mathrm{U}_R .$$

2. Linear σ-model.

$$L = (1/2)\, Tr\left(\partial_\mu \mathrm{V}(x)\partial^\mu \mathrm{V}^+(x)\right) - W\left(\mathrm{V}(x)\mathrm{V}^+(x)\right) ,$$
$$\mathrm{V}(x) = \left(\sigma(x) + \imath\pi(x)\right), \quad \mathrm{V}'(x) = \mathrm{U}_L \mathrm{V}(x)\mathrm{U}_R .$$

The experimental nonet of the light scalar mesons, the putative $f_0(600)$ (or $\sigma(600)$) and $\kappa(700-900)$ mesons and the well established $f_0(980)$ and $a_0(980)$ mesons as if suggests the $U_L(3)\times U_R(3)$ linear σ-model.

Hunting the light σ and κ mesons had begun in the sixties already and a preliminary information on the light scalar mesons in Particle Data Group (PDG) Reviews had appeared at that time. But long-standing unsuccessful attempts to prove their existence in a conclusive way entailed general disappointment and an information on these states disappeared from PDG Reviews. One of principal reasons against the σ and κ mesons was the fact that both $\pi\pi$ and $\pi\kappa$ scattering phase shifts do not pass over 90^0 at putative resonance masses.

Situation changes when we showed (1994) that in the linear σ-model

$$\begin{aligned} L &= (1/2)\left[(\partial_\mu\sigma)^2 + (\partial_\mu\vec{\pi})^2\right] + \left(\mu^2/2\right)\left[(\sigma)^2 + (\vec{\pi})^2\right] \\ &\quad - (\lambda/4)\left[(\sigma)^2 + (\vec{\pi})^2\right]^2 \end{aligned}$$

there is a negative background phase which hides the σ meson. It has been made clear that shielding wide lightest scalar mesons in chiral dynamics is very natural.

This idea was picked up and triggered new wave of theoretical and experimental searches for the σ and κ mesons.

We considered the simplest Dyson equation for the $\pi\pi$ scattering amplitude with real intermediate $\pi\pi$ states only (1994).

$$
\begin{aligned}
T_0^{0(tree)} &= \frac{m_\pi^2 - m_\sigma^2}{16\pi F_\pi^2}\left[5 - 3\frac{m_\sigma^2 - m_\pi^2}{m_\sigma^2 - s}\right. \\
&\qquad \left. -2\frac{m_\sigma^2 - m_\pi^2}{s - 4m_\pi^2}\ln\left(1 + \frac{s - 4m_\pi^2}{m_\sigma^2}\right)\right], \\
T_0^0 &= \frac{T_0^{0(tree)}}{1 - \imath\rho_{\pi\pi}T_0^{0(tree)}} = \frac{e^{2\imath\left(\delta_{bg}+\delta_{res}\right)} - 1}{2\imath\rho_{\pi\pi}} \\
&= \frac{1}{\rho_{\pi\pi}}\left[\left(\frac{e^{2\imath\delta_{bg}} - 1}{2\imath}\right) + e^{2\imath\delta_{bg}}T_{res}\right], \\
T_{res} &= \frac{\sqrt{s}\Gamma_{res}(s)}{M_{res}^2 - s + \Re(\tilde{\Pi}_{res}(M_{res}^2)) - \tilde{\Pi}_{res}(s)} = \frac{e^{2\imath\delta_{res}} - 1}{2\imath\rho_{\pi\pi}}\,.
\end{aligned}
$$

In theory the principal problem is impossibility to use the linear σ-model in the tree level approximation inserting widths into σ meson propagators because such an approach breaks the both unitarity and Adler self-consistency conditions, as we showed (1994). Strictly speaking, the comparison with the experiment requires the non-perturbative calculation of the process amplitudes. Nevertheless, now there are the possibilities to estimate odds of the $U_L(3) \times U_R(3)$ linear σ-model to underlie physics of light scalar mesons in phenomenology. Really, even now there is a huge body of information about the S- waves of different two-particle pseudoscalar states and what is more the relevant information go to press almost continuously from BES, BNL, CERN, CESR, DAΦNE, FNAL, KEK, SLAC and others. As for theory, we know quite a lot about the scenario under discussion: the nine scalar mesons, the putative chiral shielding of the $\sigma(600)$ and $\kappa(700-900)$ mesons, the unitarity and Adler self-consistency conditions. In addition, there is the light scalar meson treatment motivated by field theory. The foundations of this approach were formulated in our papers (1979-1984).

3 Four-quark Model

The nontrivial nature of the well established light scalar resonances $f_0(980)$ and $a_0(980)$ is no longer denied practically anybody. In particular, there exist numerous evidences in favour of the $q^2\bar{q}^2$ structure of these states. As for the

nonet as a whole, even a dope's look at PDG Review gives an idea of the four-quark structure of the light scalar meson nonet, $\sigma(600)$, $\kappa(700-900)$, $f_0(980)$, and $a_0(980)$, inverted in comparison with the classical P-wave $q\bar{q}$ tensor meson nonet, $f_2(1270)$, $a_2(1320)$, $K_2^*(1420)$, $\phi_2'(1525)$. Really, while the scalar nonet cannot be treated as the P-wave $q\bar{q}$ in the naive quark model, it can be easy understood as the $q^2\bar{q}^2$ nonet, where $\sigma(600)$ has no strange quarks, $\kappa(700-900)$ has the s quark, $f_0(980)$ and $a_0(980)$ have the $s\bar{s}$ pair, R.L. Jaffe, J. Schechter et al..

The scalar mesons $a_0(980)$ and $f_0(980)$, discovered more than thirty years ago, became the hard problem for the naive $q\bar{q}$ model from the outset. Really, on the one hand the almost exact degeneration of the masses of the isovector $a_0(980)$ and isoscalar $f_0(980)$ states revealed seemingly the structure similar to the structure of the vector ρ and ω mesons, and on the other hand the strong coupling of $f_0(980)$ with the $K\bar{K}$ channel as if suggested a considerable part of the strange pair $s\bar{s}$ in the wave function of $f_0(980)$.

In 1977 R.L. Jaffe noted that in the MIT bag model, which incorporates confinement phenomenologically, there are light four-quark scalar states. He suggested also that $a_0(980)$ and $f_0(980)$ might be these states with symbolic structures

$$a_0^0(980) = (us\bar{u}\bar{s} - ds\bar{d}\bar{s})/\sqrt{2} \quad \text{and} f_0(980) = (us\bar{u}\bar{s} + ds\bar{d}\bar{s})/\sqrt{2}.$$

From that time $a_0(980)$ and $f_0(980)$ resonances came into beloved children of the light quark spectroscopy.

4 Radiative Decays of ϕ-Meson

Ten years later we showed that the study of the radiative decays $\phi \to \gamma a_0 \to \gamma\pi\eta$ and $\phi \to \gamma f_0 \to \gamma\pi\pi$ can shed light on the problem of $a_0(980)$ and $f_0(980)$ mesons. Over the next ten years before experiments (1998) the question was considered from different points of view. Now these decays have been studied not only theoretically but also experimentally. The first measurements have been reported by the SND and CMD-2 Collaborations which

obtain the following branching ratios

$$
\begin{aligned}
&BR(\phi \to \gamma\pi^0\eta) = (0.88 \pm 0.14 \pm 0.09) \cdot 10^{-4} \quad \text{SND}, \\
&BR(\phi \to \gamma\pi^0\pi^0) = (1.221 \pm 0.098 \pm 0.061) \cdot 10^{-4} \quad \text{SND}, \\
&BR(\phi \to \gamma\pi^0\eta) = (0.9 \pm 0.24 \pm 0.1) \cdot 10^{-4} \quad \text{CMD-2}, \\
&BR(\phi \to \gamma\pi^0\pi^0) = (0.92 \pm 0.08 \pm 0.06) \cdot 10^{-4} \quad \text{CMD-2}.
\end{aligned}
$$

More recently the KLOE Collaboration has measured

$$
\begin{aligned}
&BR(\phi \to \gamma\pi^0\eta(\to \gamma\gamma)) = (0.851 \pm 0.051 \pm 0.057) \cdot 10^{-4} \\
&BR(\phi \to \gamma\pi^0\eta(\to \pi^+\pi^-\pi^0)) = (0.796 \pm 0.060 \pm 0.040) \cdot 10^{-4} \\
&BR(\phi \to \gamma\pi^0\pi^0) = (1.09 \pm 0.03 \pm 0.05) \cdot 10^{-4}
\end{aligned}
$$

in agreement with the Novosibirsk data but with a considerably smaller error.

Note that $a_0(980)$ is produced in the radiative ϕ meson decay as intensively as $\eta'(958)$ containing $\approx 66\%$ of $s\bar{s}$, responsible for $\phi \approx s\bar{s} \to \gamma s\bar{s} \to \gamma\eta'(958)$. It is a clear qualitative argument for the presence of the $s\bar{s}$ pair in the isovector $a_0(980)$ state, i.e., for its four-quark nature.

4.1 K^+K^--Loop Model

When basing the experimental investigations, we suggested one-loop model $\phi \to K^+K^- \to \gamma a_0(980)$ (or $f_0(980)$). This model is used in the data treatment and is ratified by experiment.

Below we argue on gauge invariance grounds that the present data give the conclusive arguments in favor of the K^+K^--loop transition as the principal mechanism of $a_0(980)$ and $f_0(980)$ meson production in the ϕ radiative decays. This enables to conclude that production of the lightest scalar mesons $a_0(980)$ and $f_0(980)$ in these decays is caused by the four-quark transitions, resulting in strong restrictions on the large N_C expansions of the decay amplitudes. The analysis shows that these constraints give new evidences in favor of the four-quark nature of $a_0(980)$ and $f_0(980)$ mesons.

4.2 Spectra and Gauge Invariance

To describe the experimental spectra

$$\frac{dBR(\phi\to\gamma R\to\gamma ab\,,\,m)}{dm}=\frac{2}{\pi}\frac{m^2\Gamma(\phi\to\gamma R\,,\,m)\Gamma(R\to ab\,,\,m)}{\Gamma_\phi|D_R(m)|^2}$$
$$=\frac{4|g_R(m)|^2\omega(m)p_{ab}(m)}{\Gamma_\phi\,3(4\pi)^3m_\phi^2}\left|\frac{g_{Rab}}{D_R(m)}\right|^2,$$
$$R=a_0,\ f_0,\ ab=\pi^0\eta,\ \pi^0\pi^0,$$

the function $|g_R(m)|^2$ should be smooth (almost constant) in the range $m\leq$ 0.99 GeV. But the problem issues from gauge invariance which requires that

$$A\left[\phi(p)\to\gamma(k)R(q)\right]$$
$$=G_R(m)\left[p_\mu e_\nu(\phi)-p_\nu e_\mu(\phi)\right]\left[k_\mu e_\nu(\gamma)-k_\nu e_\mu(\gamma)\right].$$

Consequently, the function

$$g_R(m)=-2(pk)G_R(m)=-2\omega(m)m_\phi G_R(m)$$

is proportional to the photon energy $\omega(m)=(m_\phi^2-m^2)/2m_\phi$ (at least!) in the soft photon region.

Stopping the function $(\omega(m))^2$ at $\omega(990\,\mathrm{MeV})=29$ MeV with the help of the form-factor $1/\left[1+(R\omega(m))^2\right]$ requires $R\approx 100$ GeV^{-1}. It seems to be incredible to explain such a huge radius in hadron physics. Based on rather great $R\approx 10$ GeV^{-1}, one can obtain an effective maximum of the mass spectrum only near 900 MeV.

To exemplify this trouble let us consider the contribution of the isolated R resonance: $g_R(m)=-2\omega(m)m_\phi G_R\left(m_R\right)$. Let also the mass and the width of the R resonance equal 980 MeV and 60 MeV, then $S_R(920\,\mathrm{MeV}):S_R(950\,\mathrm{MeV}):S_R(970\,\mathrm{MeV}):S_R(980\,\mathrm{MeV})=3:2.7:1.8:1$.

So stopping the $g_R(m)$ function is the crucial point in understanding the mechanism of the production of $a_0(980)$ and $f_0(980)$ resonances in the ϕ radiative decays.

The K^+K^--loop model $\phi\to K^+K^-\to\gamma R$ solves this problem in the elegant way: fine threshold phenomenon is discovered, see Fig. 1.

To demonstrate the threshold character of this effect we present Fig. 2 and Fig. 3 in which the function $|g(m)|^2$ is shown in the case of K^+ meson mass is 25 MeV and 50 MeV less than in reality.

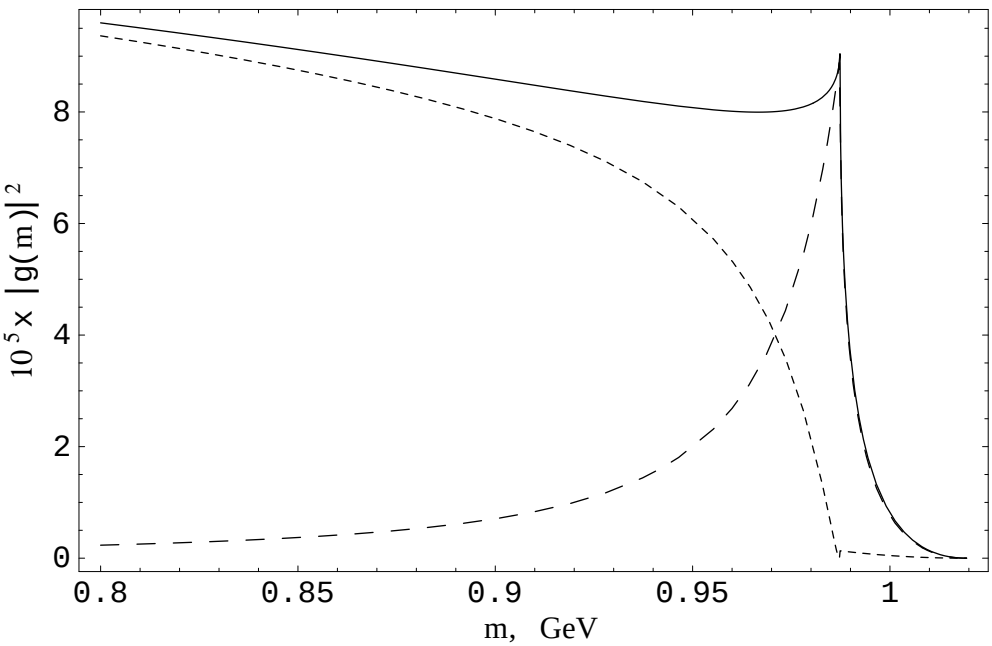

Figure 1: The universal in the K^+K^- loop model function $|g(m)|^2 = |g_R(m)/g_{RK^+K^-}|^2$ is drawn with the solid line. The contribution of the imaginary part is drawn with the dashed line. The contribution of the real part is drawn with the dotted line.

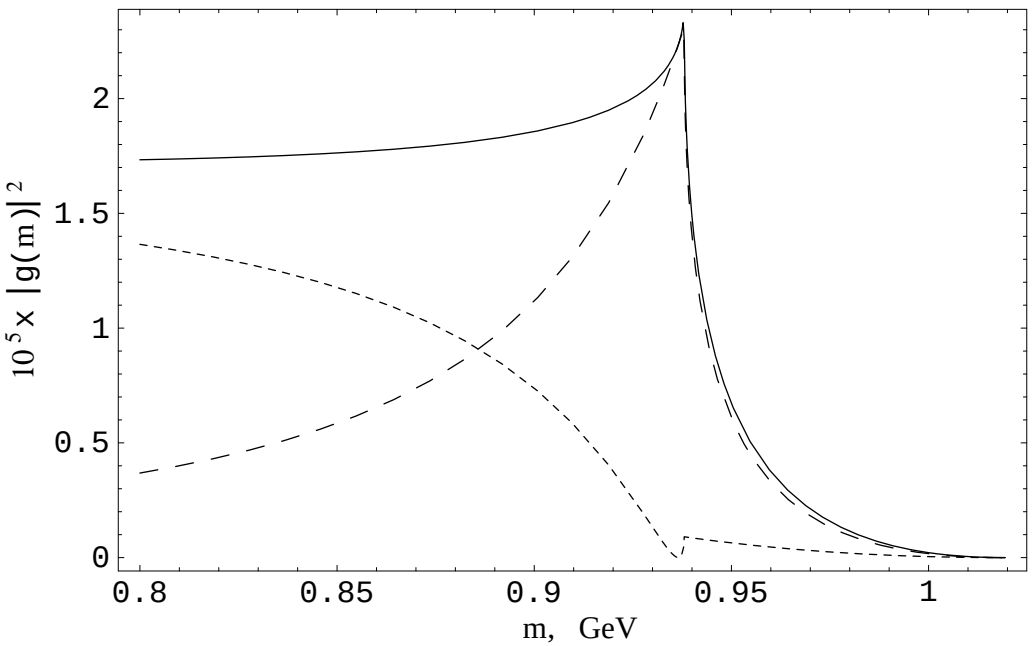

Figure 2: The function $|g(m)|^2$ for $m_{K^+} = 469$ MeV is drawn with the solid line. The contribution of the imaginary part is drawn with the dashed line. The contribution of the real part is drawn with the dotted line.

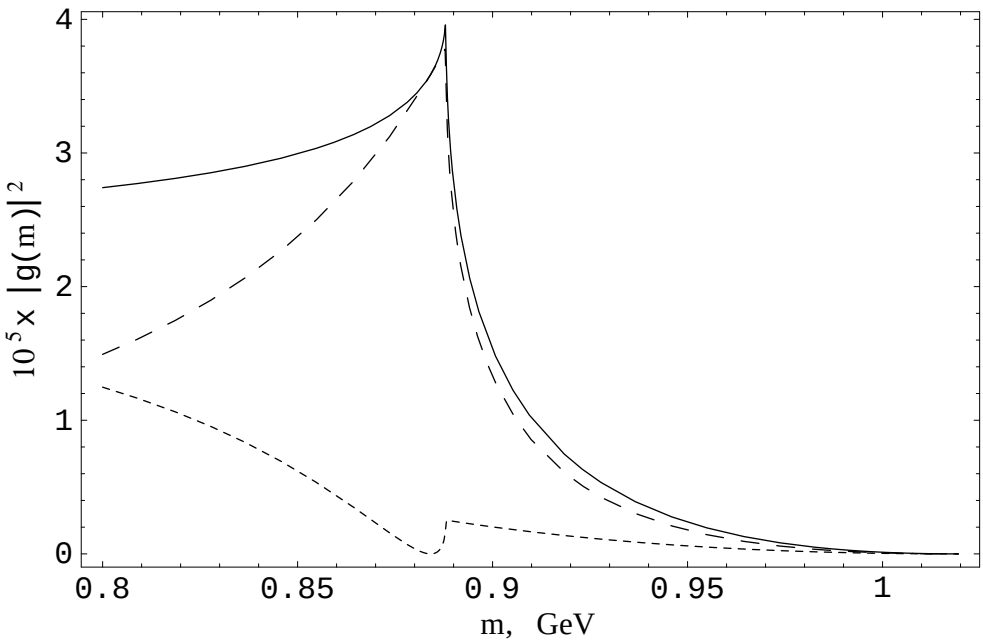

Figure 3: The function $|g(m)|^2$ for $m_{K^+} = 444$ MeV is drawn with the solid line. The contribution of the imaginary part is drawn with the dashed line. The contribution of the real part is drawn with the dotted line.

As seen from Figs. 2 and 3, the function $|g(m)|^2$ is suppressed by the $(\omega(m))^2$ law in the region 950-1020 MeV and 900-1020 Mev respectively. In the mass spectrum this suppression is increased by one more power of $\omega(m)$, so that we cannot see the resonance in the region 980-995 MeV . The spectrum maximum is effectively shifted to the region 935-950 MeV and 880-900 MeV respectively.

In truth this means that $a_0(980)$ and $f_0(980)$ resonances are seen in the radiative decays of ϕ meson owing to the K^+K^- intermediate state, otherwise the maxima in the spectra would be shifted to 900 MeV.

So the mechanism of production of $a_0(980)$ and $f_0(980)$ mesons in the ϕ radiative decays is established.

4.3 Four-quark Transition

Both real and imaginary parts of the $\phi \to \gamma R$ amplitude are caused by the K^+K^- intermediate state. The imaginary part is caused by the real K^+K^- intermediate state while the real part is caused by the virtual compact K^+K^- intermediate state, i.e., we are dealing here with the four-quark transition. [1] Needless to say, radiative four-quark transitions can happen between two $q\bar{q}$ states as well as between $q\bar{q}$ and $q^2\bar{q}^2$ states but their intensities depend strongly on a type of the transitions.

4.4 Four-quark Transition, OZI rule, and large N_C expansion

A radiative four-quark transition between two $q\bar{q}$ states requires creation and annihilation of an additional $q\bar{q}$ pair, i.e., such a transition is forbidden according to the Okuba-Zweig-Izuka (OZI) rule, while a radiative four-quark transition between $q\bar{q}$ and $q^2\bar{q}^2$ states requires only creation of an additional $q\bar{q}$ pair, i.e., such a transition is allowed according to the OZI rule.

Let us consider this problem from the large N_C expansion standpoint, using the G.'t Hooft rules : $g_s^2 N_C \to const$ at $N_C \to \infty$ and a gluon is equivalent to a quark-antiquark pair ($A^i_j \sim q^i \bar{q}_j$).

The point is that the large N_C expansion is the most clear heuristic understanding of the OZI rule because the OZI forbidden branching ratio as a

[1] It will be recalled that the imaginary part of every hadronic amplitude describes a multi-quark transition.

rule is suppressed by the factor $N_C^2 = 9$, but not $N_C = 3$, in comparison with the OZI allowed decay. In our case the results of the analysis are even more interesting.

4.5 Summary on ϕ radiative decays from the large N_C expansion standpoint

The analysis shows that the $\phi \to K^+K^- \to \gamma a_0$ decay intensity in the two-quark model and the $\phi \to K^+K^- \to \gamma f_0$ decay intensity in the $f_0 = (u\bar{u} + d\bar{d})/\sqrt{2}$ model are suppressed by the factor $1/N_c^3$ in comparison with the ones in the four-quark model, but in the $f_0 = s\bar{s}$ model the $\phi \to K^+K^- \to \gamma f_0$ decay intensity is suppressed only by the factor $1/N_c$ in comparison with the one in the four-quark model. In this model, in addition to the serious trouble with the $a_0 - f_0$ mass degeneration, the $(N_C)^0$ order transition without creation of an additional $q\bar{q}$ pair $\phi \approx s\bar{s} \to \gamma s\bar{s} \to \gamma f_0(980)$ (similar to the principal mechanism of the $\phi \approx s\bar{s} \to \gamma s\bar{s} \to \gamma\eta'(958)$ decay) is bound to have a small weight in the large N_C expansion of the ϕ*becausethistermdoesnotcontainthe*K$^+K^-$ intermediate state, which emerges only in the next to leading term of the $1/N_C$ order, i.e., in the OZI forbidden transition. So, in this model the $\phi \approx s\bar{s} \to \gamma f_0(980)$ amplitude has the $1/N_C$ order like the $\phi \approx s\bar{s} \to \gamma\pi^0$ one. Emphasize that the mechanism without creation and annihilation of the additional $u\bar{u}$ pair cannot explain the $f_0(980)$ spectrum because it does not contain the K^+K^- intermediate state!

4.6 Conclusion of ϕ-radiative decays

So, the fine threshold phenomenon is discovered, which is to say that the K^+K^- loop mechanism of the $a_0(980)$ and $f_0(980)$ scalar meson production in the ϕ radiative decays is established at a physical level of proof. The case is rarest in hadron physics. This production mechanism is the four-quark transition what constrains the large N_C expansion of the $\phi \to \gamma a_0(980)$ and $\phi \to \gamma f_0(980)$ amplitudes and gives the new strong (if not crucial) evidences in favor of the four-quark nature of $a_0(980)$ and $f_0(980)$ mesons.

Acknowledgments

I thank Organizers of QUARKS-2004 very much for the kind invitation, the generous hospitality, and the financial support.

This work was supported in part by the RFBR Grant No. 02-02-16061 and the Presidential Grant No. 2339.2003.2 for support of Leading Scientific Schools.

NB **References** contain only author's articles on basis of which the present talk has been written. Detailed references to other authors are in these articles.

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