The symmetry, connecting the processes in 2- and 4-dimensional space-times, and the value $\alpha_0 = 1/4\pi$ for the bare fine structure constant

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Abstract

The symmetry between the creation of pairs of massless bosons or fermions by accelerated mirror in 1+1-space and the emission of single photons or scalar quanta by electric or scalar charge in 3+1-space embraces not only the processes of real quanta radiation. The general relation of Bogoliubov coefficients, describing the processes induced by the mirror, to Fourier components of current or charge density means that the spin of any disturbances bilinear in scalar or spinor field (i.e. pairs) coincides with the spin of quanta emitted by the electric or scalar charge. The mass and invariant momentum transfer of these pairs are essential for the integral connections between propagarors of a pair in 1+1-space and a single particle in 3+1-space. This allows to extend the symmetry to the processes of the mirror and the charge interactions with the fields carrying spacelike momenta. These fields accompany their sources and define the Bogoliubov matrix coefficients $\alpha^{B,F}_{\omega'\omega}$. It is shown that the Lorentz-invariant traces $\mathrm{tr}\,\alpha^{B,F}$ describe the vector and scalar interactions of accelerated mirror with a uniformly moving detector. This interpretation rests essentially on the relation between the propagators of the waves with spacelike momenta in 2- and 4-dimensional spaces. The $\mathrm{tr}\,\alpha^{B,F}$ for two important mirror's trajectories with subluminal velocities of the ends are found in explicitly analytical form and are in accordance with general consideration. The symmetry predicts one and the same value $e_0 = \sqrt{\hbar c}$ for electric and scalar charge in 3+1-space. The arguments are adduced in favour of that this value and the corresponding value $\alpha_0 = 1/4\pi$ for fine structure constant are the bare, nonrenormalized values.

1 Introduction

The Hawking's mechanism for particle production at the black hole formation is analogous to the emission from an ideal mirror accelerated in vacuum [1]. In its turn there is a close analogy between the radiation of pairs of scalar (spinor) quanta from accelerated mirror in 1+1 space and the radiation of photons (scalar quanta) by an accelerated electric (scalar) charge in 3+1 space [2,3]. Thus all these processes turn out to be mutually related. In problems with moving mirrors the *in*-set $\phi_{in\,\omega'}$, $\phi^*_{in\,\omega'}$ and *out*-set $\phi_{out\,\omega}$, $\phi^*_{out\,\omega}$ of the wave equation solutions are usually used. For massless scalar field they look as follows:

$$\begin{aligned}\phi_{in\,\omega'}(u,v) &= \frac{1}{\sqrt{2\omega'}}[e^{-i\omega' v} - e^{-i\omega' f(u)}],\\ \phi_{out\,\omega}(u,v) &= \frac{1}{\sqrt{2\omega}}[e^{-i\omega g(v)} - e^{-i\omega u}],\end{aligned} \tag{1}$$

with zero boundary condition $\phi|_{traj} = 0$ on the mirror's trajectory. Here the variables $u = t - x$, $v = t + x$ are used and the mirror's (or charge's) trajectory on the u, v plane is given by any of the two mutually inverse functions $v^{mir} = f(u)$, $u^{mir} = g(v)$.

For the *in*- and *out*-sets of massless Dirac equation solutions see [3]. Dirac solutions differ from (1) by the presence of bispinor coefficients at u- and v-plane waves. The current densities corresponding to these solutions have only tangential components on the boundary. So, the boundary condition both for scalar and spinor field is purely geometrical, it does not contain any dimentional parameters.

The Bogoliubov coefficients $\alpha_{\omega'\omega}$, $\beta_{\omega'\,\omega}$ appear as the coefficients of the expansion of the *out*-set solutions in the *in*-set solutions; the coefficients $\alpha^*_{\omega'\omega}$, $\mp\beta_{\omega'\omega}$ arise as the coefficients of the inverse expansion. The upper and lower signs correspond to scalar (Bose) and spinor (Fermi) field. The explicit form of Bogoliubov coefficients is very simple:

$$\begin{aligned}\alpha^B_{\omega'\omega},\ \beta^{B*}_{\omega'\omega} &= \sqrt{\frac{\omega'}{\omega}}\int_{-\infty}^{\infty} dv\, e^{i\omega' v \mp i\omega g(v)} \\ &= \pm\sqrt{\frac{\omega}{\omega'}}\int_{-\infty}^{\infty} du\, e^{\mp i\omega u + i\omega' f(u)}. \end{aligned} \tag{2}$$

The $\alpha^F_{\omega'\omega}$, $\beta^{F*}_{\omega'\omega}$ differ from these representations by the changes $\sqrt{\omega'/\omega} \to \sqrt{g'(v)}$, $\pm\sqrt{\omega/\omega'} \to \sqrt{f'(u)}$ under the integral signs.

Then the mean number $d\bar{n}_\omega$ of quanta radiated by accelerated mirror to the right semi-space with frequency ω and wave vector $\omega > 0$, and the total mean number $\bar{N}$ of quanta are given by the integrals

$$d\bar{n}^{B,F}_\omega = \frac{d\omega}{2\pi}\int_0^\infty \frac{d\omega'}{2\pi}|\beta^{B,F}_{\omega'\omega}|^2, \qquad \bar{N}^{B,F} = \iint\limits_0^{\infty} \frac{d\omega d\omega'}{(2\pi)^2}|\beta^{B,F}_{\omega'\omega}|^2. \tag{3}$$

These expressions do not contain $\hbar$, but their interpretation as mean numbers of quanta follows from the second-quantized theory.

At the same time the spectra of photons and scalar quanta emitted by electric and scalar charges moving along the trajectory $x_\alpha(\tau)$ in 3+1 space are defined by the Fourier transforms of the electric current density 4-vector $j_\alpha(x)$ and the scalar charge density $\rho(x)$,

$$\begin{aligned} j_\alpha(k),\ \rho(k) &= e\int d\tau\,\{\dot{x}_\alpha(\tau),\ 1\}e^{-ik^\alpha x_\alpha(\tau)}, \\ j_\alpha(x),\ \rho(x) &= e\int d\tau\,\{\dot{x}_\alpha(\tau),\ 1\}\delta_4(x - x(\tau)), \end{aligned} \tag{4}$$

and are given by the formulae

$$\begin{aligned} d\bar{n}^{(1,0)}_k &= \frac{1}{\hbar c}\{|j_\alpha(k)|^2,\ |\rho(k)|^2\}\frac{dk_+dk_-}{(4\pi)^2}, \\ \bar{N}^{(1,0)} &= \frac{1}{\hbar c}\iint\limits_0^{\infty}\frac{dk_+dk_-}{(4\pi)^2}\{|j_\alpha(k)|^2,\ |\rho(k)|^2\}, \end{aligned} \tag{5}$$

where the upper index in $d\bar{n}^{(s)}_k$, $\bar{N}^{(s)}$, and k^α denote the spin and 4-momentum of quanta,

$$k^2 = k_1^2 + k_\perp^2 - k_0^2 = 0, \quad k_\perp^2 = k_0^2 - k_1^2 = k_+k_-, \quad k_\pm = k^0 \pm k^1,$$

and in (5) it is supposed that the trajectory $x^\alpha(\tau)$ has only x^0 and x^1 nontrivial components, as the mirror's one.

In contrast to quantities in (3), the $d\bar{n}_k^{(s)}$ and $\bar{N}^{(s)}$ contain $\hbar$ since the charge entering into current and charge densities is considered as classical quantity.

The symmetry between the creation of Bose or Fermi pairs by accelerated mirror in 1+1 space and the emission of single photons or scalar quanta by electric or scalar charge in 3+1 space consists, first of all, in the coincidence of the spectra. If one puts $2\omega = k_+,\ 2\omega' = k_-$, then

$$|\beta^B_{\omega'\omega}|^2 = \frac{1}{e^2}|j_\alpha(k_+, k_-)|^2, \qquad |\beta^F_{\omega'\omega}|^2 = \frac{1}{e^2}|\rho(k_+, k_-)|^2. \tag{6}$$

So, the spectra coincide as a functions of two variables and a functionals of common trajectory of a mirror and a charge. The distinction in multiplier e^2 can be removed if one puts $e^2 = \hbar c$.

2 Symmetry and physical distinction of $\beta^*_{\omega'\omega}$ and $\alpha_{\omega'\omega}$

It follows from the second-quantized theory that the absolute pair production amplitude and the single-particle scattering amplitude are connected by the relation

$$\langle \mathrm{out}\omega''\omega|\mathrm{in}\rangle = -\sum_{\omega'}\langle \mathrm{out}\omega''|\omega'\mathrm{in}\rangle\, \beta^*_{\omega'\omega}. \tag{7}$$

It enables to interpret $\beta^*_{\omega'\omega}$ as the amplitude of a source of a pair of the massless particles potentially emitted to the right and to the left with frequences ω and ω' respectively [4]. While the particle with frequency ω actually escapes to the right, the particle with frequency ω' propagates some time then is reflected by the mirror and is actually emitted to the right with altered frequency ω''. Then, in the time interval between pair creation and reflection of the left particle, we have the virtual pair with energy k^0, momentum k^1, and mass m:

$$k^0 = \omega + \omega', \quad k^1 = \omega - \omega', \quad m = \sqrt{-k^2} = 2\sqrt{\omega\omega'}. \tag{8}$$

Apart from this polar timelike 2-vector k^α, very important is the axial spacelike 2-vector q^α,

$$q_\alpha = \varepsilon_{\alpha\beta}k^\beta, \quad q^0 = -k^1 = -\omega + \omega', \quad q^1 = -k^0 = -\omega - \omega' < 0. \tag{9}$$

With the help of k^α and q^α the symmetry between α and β coefficients becomes clearly expressed:

$$s=1, \qquad e\beta^{B*}_{\omega'\omega} = -\frac{q_\alpha j^\alpha(k)}{\sqrt{k_+k_-}}, \qquad e\alpha^{B}_{\omega'\omega} = -\frac{k_\alpha j^\alpha(q)}{\sqrt{k_+k_-}}, \tag{10}$$

$$s=0, \qquad e\beta^{F*}_{\omega'\omega} = \rho(k), \qquad e\alpha^{F}_{\omega'\omega} = \rho(q). \tag{11}$$

Note that the equations (4) define the current density $j^\alpha(k)$ and the charge density $\rho(k)$ as the functionals of the trajectory $x^\alpha(\tau)$ and the functions of any 2- or 4-vector k^α. It can be shown that in 1+1-space $j^\alpha(k)$ and $j^\alpha(q)$ are the spacelike and timelike polar vectors if k^α and q^α are the timelike and spacelike vectors correspondingly.

The boundary condition on the mirror evokes in the vacuum of massless scalar or spinor field the appearance of vector or scalar disturbance waves bilinear in massless fields. There are two types of these waves:

1) The waves with amplitude $\alpha_{\omega'\omega}$ ($\alpha^*_{\omega'\omega}$) which carry the spacelike momentum directed to the left (right), and

2) The waves with amplitude $\beta^*_{\omega'\omega}$ ($\beta_{\omega'\omega}$) which carry the timelike momentum with positive (negative) frequency.

The waves with the spacelike momenta appear even if the mirror is in rest or moves uniformly (Casimir effect), while the waves with the timelike momenta appear only in the case of accelerated mirror.

The pair of Bose (Fermi) particles has spin 1 (0) because its source is the current density vector (charge density scalar), see [5] or the problem 12.15 in [6].

3 Vacuum-vacuum amplitude $\langle\text{out}|\text{in}\rangle = e^{iW}$

It follows from the second quantized theory that in vacuum-vacuum amplitude $\langle\text{out}|\text{in}\rangle = e^{iW}$ the $\operatorname{Im} W^{B,F}$ is well-defined. According to DeWitt [7], Wald [8] and others (including myself [4])

$$2\operatorname{Im} W^{B,F} = \pm\frac{1}{2}\mathrm{tr}\,\ln(1\pm\beta^+\beta) \quad \text{or} \quad \pm\,\mathrm{tr}\,\ln(1\pm\beta^+\beta) \tag{12}$$

correspondingly to the cases when particle is identical or nonidentical to antiparticle. We confine ourselves by the last case and by the smallness of the tr $\beta^{+}\beta \ll 1$. Then

$$2\,\mathrm{Im}\,W^{B,F} \approx \mathrm{tr}\,(\beta^{+}\beta)^{B,F} \equiv \iint\limits_{0}^{\infty} \frac{d\omega d\omega'}{(2\pi)^2}\,|\beta^{B,F}_{\omega'\omega}|^2 = \bar{N}^{B,F}. \tag{13}$$

By using in the integrand of $\bar{N}^{B,F}$ the representations (2) for $\beta^{B,F}$, the variables $x_{\mp}(\tau)$ and $x_{\pm}(\tau')$ instead of u, $f(u)$ and v, $g(v)$, and hyperbolic variables ρ, θ instead of ω, ω',

$$d\omega d\omega' = \frac{1}{2}\rho d\rho d\theta, \quad \omega = \frac{1}{2}\rho e^{\theta}, \quad \omega' = \frac{1}{2}\rho e^{-\theta},$$
$$\rho = 2\sqrt{\omega\omega'}, \quad \theta = \ln\sqrt{\frac{\omega}{\omega'}}, \tag{14}$$

one obtains the imaginery part of the causal function in 1+1-space, $\mathrm{Im}\,\Delta_2^f(z,\rho)$, after integration over θ, and then the imaginery part of the causal function in 3+1-space, $\mathrm{Im}\,\Delta_4^f(z,\mu)$, after integration over $\rho = m$, the variable which coincides with the mass of virtual pair according to (8). This result is a special case of the very important integral relation between the causal functions of wave equations for d- and $d+2$-dimensional space-times [9],

$$\Delta^f_{d+2}(z,\mu) = \frac{1}{4\pi}\int_{\mu^2}^{\infty} dm^2\,\Delta^f_d(z,m), \tag{15}$$

The small mass parameter $\mu = 2\sqrt{\omega\omega'}|_{min} \neq 0$ is introduced instead of zero to avoid the infrared divergency in the following. Thus we obtain

$$\begin{aligned} 2\,\mathrm{Im}\,W^{B,F} &= \mathrm{Im}\iint d\tau d\tau' \left\{ \begin{array}{c} \dot{x}_\alpha(\tau)\dot{x}^\alpha(\tau') \\ 1 \end{array} \right\} \Delta_4^f(z,\mu), \\ & \quad\; z_\alpha = x_\alpha(\tau) - x_\alpha(\tau'). \end{aligned} \tag{16}$$

We may omit the Im-signs from both of sides of this equation and define the actions for bose- and fermi-mirrors in 1+1-space as

$$W^{B,F} = \frac{1}{2}\iint d\tau d\tau' \left\{ \begin{array}{c} \dot{x}_\alpha(\tau)\dot{x}^\alpha(\tau') \\ 1 \end{array} \right\} \Delta_4^f(z,\mu). \tag{17}$$

Compare this with the well known actions for electric and scalar charges in 3+1-space:

$$W^{1,0} = \frac{1}{2}e^2\iint d\tau d\tau' \left\{ \begin{array}{c} \dot{x}_\alpha(\tau)\dot{x}^\alpha(\tau') \\ 1 \end{array} \right\} \Delta_4^f(z,\mu). \tag{18}$$

The symmetry would be complete if $e^2 = 1$, i.e. if the fine structure constant were $\alpha = 1/4\pi$. This "ideal" value of fine structure constant for the charges would correspond to the ideal, geometrical boundary condition on the mirror.

The appearance in action the causal function $\Delta_4^f(z,\mu)$ has a lucid physical grounds.

1. The action must represent not only the radiation of real quanta but also the self-energy and polarization effects. While the first effects are described by the solutions of homogeneous wave equation the second ones require the inhomogeneous wave equation solutions which contain information about proper field of a source. Namely such solutions of homogeneous and inhomogeneous wave equations are the functions $(1/2)\Delta^1 = \mathrm{Im}\,\Delta^f$ and $\bar{\Delta} = \mathrm{Re}\,\Delta^f$.

2. While the appearance of $\mathrm{Im}\,\Delta^f$ in the imaginary part of the action (16) is a consequence of mathematical transformation of the integral $\bar{N}^{B,F}$ (similar to the Plancherel theorem), the function $\bar{\Delta} \equiv \mathrm{Re}\,\Delta^f$ in the real part of the action is unique if it appears as the real part of the analytical continuation of the function $i\,\mathrm{Im}\,\Delta^f(z,\mu)$ to negative z^2 that is even in z as $\mathrm{Im}\,\Delta^f$ itself.

Both the propagator $\Delta_2^f(z,m)$ of a virtual pair with mass $m = \rho = 2\sqrt{\omega\omega'}$ in two-dimensional space-time and the mass spectrum of these pairs arise owing to the transition from the variables ω, ω' to the hyperbolic variables ρ, θ, which reflect the Lorentz symmetry of the problem. Further integration over the mass leads to the propagator $\Delta_4^f(z,\mu)$ of a particle moving in four-dimensional space-time with the mass μ equal to the least mass of virtual pairs. Thus, the relation (15) is immanent to the Lorentz symmetry and the symmetry, connecting the processes in two- and four-dimensional space-times.

We exemplify here the selfaction changes $\Delta W_{1,0}$ of electric and scalar charges due to accelerated motion along the quasihyperbolic trajectory

$$x(t) = \frac{\beta_1^2}{w_0} - \beta_1\sqrt{\frac{\beta_1^2}{w_0^2} + t^2}, \beta_{1,2} = \pm\mathrm{th}\,\frac{\theta}{2},$$
$$\beta_{12} = \frac{\beta_1 - \beta_2}{1 - \beta_1\beta_2} = \mathrm{th}\,\theta. \qquad (19)$$

with initial β_1 and final β_2 velocities at $t = \mp\infty$ and proper acceleration $-w_0$ at $t = 0$. The selfaction changes $\Delta W_{1,0}(\theta,\lambda)$ are the Lorentz invariant functions of two variables $\theta = \mathrm{Arth}\,\beta_{12}$ and $\lambda = \mu^2/w_0^2$ with singularities at $\lambda = 0$ and $\theta = \pm\infty$.

At $\lambda \to 0,\ \theta$ arbitrary,

$$\Delta W_1 = \frac{e^2}{8\pi^2}\{\pi(\frac{\theta}{\mathrm{th}\,\theta} - 1) \\ + i[(\frac{\theta}{\mathrm{th}\,\theta} - 1)\ln\frac{4(\mathrm{ch}\,\theta + 1)^2}{\gamma^2\lambda(\mathrm{ch}\,\theta - 1)} + 2 - \ln 2 - \mathrm{ch}\,\theta\, R(\theta)]\}, \quad (20)$$

$$\Delta W_0 = \frac{e^2}{8\pi^2}\{\pi(1 - \frac{\theta}{\mathrm{sh}\,\theta}) \\ + i[(1 - \frac{\theta}{\mathrm{sh}\,\theta})\ln\frac{4(\mathrm{ch}\,\theta + 1)^2}{\gamma^2\lambda(\mathrm{ch}\,\theta - 1)} - 2 + \ln 2 + R(\theta)]\}, \quad (21)$$

where $R(\theta)$ is even function of θ connected with the Euler's dilogarithm $L_2(z)$ [10],

$$R(\theta) = \int_0^\infty d\alpha\,\frac{\ln(\mathrm{ch}\,\theta + \mathrm{ch}\,\alpha)}{\mathrm{ch}\,\theta + \mathrm{ch}\,\alpha} = \frac{L_2(1 - e^{-2\theta}) + \theta^2 - \ln 2\cdot\theta}{\mathrm{sh}\,\theta}. \quad (22)$$

For the case $\theta \to \pm\infty,\ \lambda$ arbitrary, considered in [11,12],

$$\Delta W_{1,0} = -|\,\theta|\frac{e^2}{8\pi^2}\,S_{1,0}(\lambda), \\ S_n(\lambda) = (-1)^{n+1}\int_0^\infty dz\, e^{-i\lambda/2z}[e^{iz}\,K_n(iz) - \sqrt{\frac{\pi}{2iz}}], \quad (23)$$

$K_n(iz)$ is the Mcdonald function. At $\lambda \to 0$

$$S_1(\lambda) = -\pi - i(\ln\frac{4}{\gamma^2\lambda} - 1), \quad S_0(\lambda) = -i. \quad (24)$$

For the trajectory with relative velocity β_{12} of the ends the $\mathrm{Re}\,\Delta W_{1,0}$ are given by the formulae

$$\mathrm{Re}\,\Delta W_1 = \frac{e^2}{8\pi}(\frac{\theta}{\mathrm{th}\,\theta} - 1), \qquad \mathrm{Re}\,\Delta W_0 = \frac{e^2}{8\pi}(1 - \frac{\theta}{\mathrm{sh}\,\theta}). \quad (25)$$

When $\beta_{12} \to 1$ the trajectory becomes actually hyperbolic one with charge's velocity $\beta(\tau) = -\mathrm{th}\,w_0\tau$ at proper time τ, and $\theta = w_0(\tau_2 - \tau_1) \to \infty$. Then

$$\mathrm{Re}\,\Delta W_1 = \frac{e^2 w_0}{8\pi}\,(\tau_2 - \tau_1), \qquad \mathrm{Re}\,\Delta W_0 = \frac{e^2}{8\pi}, \quad (26)$$

while the mass shifts of uniformly accelerated charges are

$$\Delta m = -\frac{\partial \Delta W}{\partial \tau_2} = \frac{e^2 w_0}{8\pi^2} S(\lambda), \quad \mathrm{Re}\,\Delta m_1 = -\frac{e^2 w_0}{8\pi}, \\ \mathrm{Re}\,\Delta m_0 = 0. \tag{27}$$

Due to the symmetry the quantities $\Delta W^{B,F}$, $\Delta m^{B,F}$ for the mirror interacting with massless bose- or fermi-field can be obtained from $\Delta W_{1,0}$, $\Delta m_{1,0}$ by the change $e^2 \to \hbar c$.

4 Arguments in favour of the value $\alpha_0 = 1/4\pi$ for the bare fine structure constant

The symmetry predicts one and the same value $e_0 = \sqrt{\hbar c}$ for electric and scalar charges in 3+1-space. Since the radiative corrections are not taken into account by the theory, this value for electric and scalar charges and the corresponding value $\alpha_0 = 1/4\pi$ for the fine structure constant should be considered as the bare, nonrenormalized values.

As the like electric charges are repulsed and the scalar ones are attracted, the vacuum polarization leads to the screening of electric charge and to the antiscreening of scalar charge. This is confirmed by the effective, renormalized value $\alpha = 1/137.036$ of the fine structure constant in QED, which is essentually less than $\alpha_0 = 1/4\pi$. The situation discussed corresponds to the variant (b), considered by Gell-Mann and Low [13], according to which if the value α_0 is finite, then

1) it does not depend on effective value of fine structure constant α,

2) the α must be less than α_0, and

3) the charge density at very small distances reduces to the delta-function $e_0\delta(\mathbf{x})$.

This means that α is determined by a such vacuum polarization mechanism with such mass spectrum of charged particles that the screening of a point bare charge begins at finite, though very small, distances from it and ends at the distances of the order of Compton length of electron, the charge with the smallest mass.

If now one uses the well-known connection [14,15] between the bare and renormalized charges in QED,

$$\alpha^{-1} = \alpha_0^{-1} + \frac{N}{3\pi} \ln \frac{\Lambda^2}{m^2} = 137.036, \tag{28}$$

then it is possible to evaluate the approximate number N of different charged particles screening the bare charge and having masses in interval (m, Λ), where Λ is the upper limit of particle energy up to which QED is correct.

Thus, for $\Lambda = 1/\sqrt{G} = 1.22 \cdot 10^{19}$ GeV defined by gravitational constant one has $N_G = 11.4$, and for $\Lambda_{GU} \approx 10^{15}$ GeV of grand unification theory one has $N_{GU} = 13.9$. It is well known now that the N is greater than 8,

$$N = n_l + 3(2/3)^2\, n_u + 3(1/3)^2\, n_d + \ldots = 8 + \ldots\ ,$$

where n_l, n_u, n_d are the numbers of charged leptons, quarks with charge $(2/3)e$, quarks with charge $-(1/3)e$, and $\ldots$ must include the contributions of the loops with $W^{\pm}$ particles etc.

An additional argument in favour of the assertion that the symmetry for the processes in 3+1-space appears in all its completeness at large energies and momentum transfers of incident particles is the space one-dimensionality of these processes.

So, for the collision of electron with electron or positron the elastic scattering cross-section depends on two invariants s and t, which in the center of mass system equal to

$$s = -4E^2, \quad t = 2p^2(1 - \cos\theta),$$

where $E = \sqrt{p^2 + m^2}$, p and θ are the energy, momentum and scattering angle of electron in c.m.s. At fixed energy E the smallest distance between the charges is attained at the largest momentum transfer, i.e. at $\theta = \pi$, when charges move along the same straight line. Namely in this case each of them most deeply penetrates under the screening coat of other.

About 45 years ago E.P. Wigner remarked that the special relativity is the physics of Lorentz transformation, and the quantum mechanics is the physics of Fourier transformation. Processes induced by a point mirror in 1+1-space are described by the symplest relativistic quantum theory, which is incarnated in Bogoliubov coefficients. They are Lorentz-invariant scalar products reduced to Fourier transforms of massless scalar and spinor wave equation solutions. They can be considered as concentrate of genetic information about processes in 3+1- space.

5 Self-action changes $\Delta W_{1,0}$ and traces $\mathrm{tr}\,\alpha^{B,F}$

The basis for the symmetry between the processes induced by the mirror in two-dimensional and by the charge in four-dimensional space-time is the relation (10), (11) between the Bogoliubov's coefficients $\beta^{B,F}_{\omega'\omega}$ and the current density $j^{\alpha}(k)$ or charge density $\rho(k)$ depending on the timelike momentum k^{α}. The squares of these quantities represent the spectra of real pairs and particles radiated by accelerated mirror and charge.

The symmetry is extended to the selfactions of the mirror and the charge and to the corresponding vacuum-vacuum amplitudes, cf. (17) and (18). In essence, it is embodied in the integral relation (15) between propagators of a massive pair in two-dimensional space and of a single particle in four-dimensional space.

The formula (17) for $W^{B,F}$ was obtained provided that the mean number $\bar{N}^{B,F}$ of pairs created is small and the interference of two or more pairs is negligible. In the general case the $W^{B,F}$ is given by the formula (12), which can be written also in the form

$$2\,\mathrm{Im}\,W^{B,F} = \pm\mathrm{tr}\;\ln(\alpha^{+}\alpha)^{B,F}, \tag{29}$$

since $\alpha^{+}\alpha \mp \beta^{+}\beta = 1$, see [7], [4]. As is seen from (12), the imaginery part of the action differs from zero and then is positive only if $\beta \neq 0$, i.e. if the radiation of real particles is happened indeed.

Formula (29) allows to choose for $W^{B,F}$ the expression

$$W^{B,F} = \pm i\,\mathrm{tr}\;\ln\;\alpha^{B,F}, \tag{30}$$

that was called natural by DeWitt [7]. However, this expression is by no means unique, the expressions with $\alpha e^{i\gamma}$ or α^{+} have the same imaginery part. Nevertheless, the formula (30) is interesting as the definition both the real and imaginery parts of the selfactions $W^{B,F}$ by means of the Bogoliubov's coefficients $\alpha^{B,F}_{\omega'\omega}$ only, which, according to the formulae (10), (11), reduce to the current density $j^{\alpha}(q)$ or to the charge density $\rho(q)$ dependent on the spacelike momentum q^{α}. This means that the field of the corresponding perturbations propagates in vacuum together with the mirror, comoves it, and, at the same time, it containes the information about the radiation of the real quanta.

Unfortunately, the author failed to find a simple integral representation for the matrix ln α. Nevertheless, if one again assumes that the mean number of emitted particles is small, then one may consider α, or $i\alpha$, or $\pm i\alpha^{B,F}$ close to 1. Namely the last phase factor is most acceptable as will be seen below. Then, expanding the $\ln(\pm i\alpha^{B,F})$ near $\pm i\alpha^{B,F} = 1$ and confine ourselves by the first term we obtain

$$W^{B,F} = \pm i\,\mathrm{tr}\;\ln(\pm i\alpha^{B,F}) \approx \pm i\mathrm{tr}\,(\pm i\alpha^{B,F} - 1) = -\mathrm{tr}\,\alpha^{B,F} + \ldots\,. \tag{31}$$

These qualitative arguments allow to state that the functionals $\operatorname{tr}\alpha^{B,F}$ are similar to the corresponding selfactions with opposite sign and therefore must have the negative imaginery parts. This is confirmed by the general examples considered below in which at least the initial or the final velocity of the mirror is subluminal.

The Lorentz-invariant $\operatorname{tr}\alpha$ was defined [16] by the formula

$$\operatorname{tr}\alpha = \iint\limits_0^\infty \frac{d\omega d\omega'}{(2\pi)^2}\,\alpha_{\omega'\omega}\,2\pi\,\delta\left(\sqrt{\frac{\varkappa'}{\varkappa}}\omega - \sqrt{\frac{\varkappa}{\varkappa'}}\omega'\right),$$
$$\Omega = \sqrt{\frac{\varkappa'}{\varkappa}}\omega, \qquad \Omega' = \sqrt{\frac{\varkappa}{\varkappa'}}\omega', \tag{32}$$

in which the Lorentz-invariant argument of δ-function is the difference of the frequences Ω and Ω' of reflected and incident waves in the proper system of the mirror at zero point $u = v = 0$ where the mirror has velocity β_0 and acceleration $a_0 = -b\sqrt{\varkappa\varkappa'}$. The multipliers $\sqrt{\varkappa'/\varkappa}$, $\sqrt{\varkappa/\varkappa'}$ are the Doppler factors connecting the frequences in the laboratory system and zero point proper system. In proper system of the mirror $\Omega = \Omega' = \sqrt{\omega\,\omega'}$.

For the trajectories in the Minkowsky plane on the left from their tangent line $X^\alpha(\tau')$ at zero point the coordinate $z^1 = X^1(\tau') - x^1(\tau) \geqslant 0$. For these trajectories the $\operatorname{tr}\alpha$ can be transformed to the form

$$\operatorname{tr}\alpha^{B,F} = \pm i \iint d\tau d\tau' \left\{ \begin{array}{c} \dot x_\alpha(\tau)\dot X^\alpha(\tau') \\ 1 \end{array} \right\} \Delta_4^{LR}(z,\nu),$$
$$z^\alpha = X^\alpha(\tau') - x^\alpha(\tau), \tag{33}$$

where the singular function $\Delta_4^{LR}(z,\nu)$ differs from the causal function $\Delta_4^f(z,\mu)$ by complex conjugation and the replacement $\mu \to i\nu$ (or by the replacement $z^2 \to -z^2$, $\mu \to \nu$) [16]:

$$\Delta_4^{LR}(z,\nu) = \frac{1}{4\pi}\delta(z^2) - \frac{\nu}{8\pi\sqrt{z^2}}\theta(z^2)H_1^{(2)}(\nu\sqrt{z^2}) + i\frac{\nu}{4\pi^2\sqrt{-z^2}}\theta(-z^2)K_1(\nu\sqrt{-z^2}). \tag{34}$$

The expression obtained allows to interpret $\operatorname{tr}\alpha^{B,F}$ as a functional describing the interaction of two vector or scalar sources by means of exchange by vector or scalar quanta with spacelike momenta. At the same time one of the sources moves along the mirror's trajectory while another one moves along the tangent line to it at zero point. The last source can be considered as a probe or detector of exitation created by the accelerated mirror in vacuum.

As the detector moves with constant velocity β_0, its 2-velocity $\dot{X}^\alpha(\tau')$ does not depend on τ'. Consequently, $\dot{x}_\alpha(\tau)\dot{X}^\alpha(\tau') = -\gamma_*(\tau)$ is the relative Lorentz-factor defined by the relative velocity $\beta_*(\tau)$ of the mirror and detector:

$$\begin{aligned} \gamma_*(\tau) &= \frac{1-\beta(\tau)\,\beta_0}{\sqrt{1-\beta^2(\tau)}\,\sqrt{1-\beta_0^2}} = \frac{1}{\sqrt{1-\beta_*^2(\tau)}}, \\ \beta_*(\tau) &= \frac{\beta(\tau)-\beta_0}{1-\beta(\tau)\,\beta_0}, \end{aligned} \tag{35}$$

and is the Lorentz-invariant quantity for each τ. Then

$$\operatorname{tr}\alpha^{B,F} = -i\int d\tau \left\{ \begin{array}{c} \gamma_*(\tau) \\ 1 \end{array} \right\} J(\tau,\nu), \tag{36}$$

$$J(\tau,\nu) = \int d\tau'\, \Delta_4^{LR}(z(\tau,\tau'),\nu). \tag{37}$$

It is seen from this representation that at $\theta \neq \infty$, when Lorentz- factor $\gamma_*(\tau)$ is confined on the whole trajectory, the both traces have the same qualitative behaviour when parameter $\nu \to 0$. It is clear that their infrared (logarithmic) singularities in this parameter are indebted to the behaviour of the integral $J(\tau,\nu)$ at $\tau \to \pm\infty$. For the trajectories with subluminal relative velocities β_{10}, β_{20} of the ends both $\operatorname{tr}\alpha^{B,F}$ have infrared singularities at $\nu = 0$. Bedides, the singularities of $\operatorname{tr}\alpha^B$ differ from those of $\operatorname{tr}\alpha^F$ only by the values of the relative Lorentz-factor $\gamma_*(\tau)$ for initial and final ends of the trajectoty, i.e. by the factors $1/\sqrt{1-\beta_{10}^2}$ and $1/\sqrt{1-\beta_{20}^2}$. Since the infrared singularities from the initial and final ends appear in $\operatorname{tr}\alpha^F$ with the factors

$$\frac{\sqrt{1-\beta_{10}^2}}{2\beta_{10}}, \qquad \frac{\sqrt{1-\beta_{20}^2}}{2|\beta_{20}|}, \tag{38}$$

they disappear in $\operatorname{tr}\alpha^F$ for the trajectories with luminal velocities of the ends, $\beta_{10} = 1$, $\beta_{20} = -1$, but remain in $\operatorname{tr}\alpha^B$. The disappearance of singularities in $\operatorname{tr}\alpha^F$ for the such trajectories means that the function $J(\tau,\,\nu)$ is integrable in τ at $\tau \to \pm\infty$ even if $\nu = 0$. At the same time the function $\gamma_*(\tau)\,J(\tau,\nu)$ is integrable in this region only at $\nu \neq 0$.

The weakening of interaction of scalar charges with increasing their relative velocity, contrary to the constancy of interaction of electric charges, is connected with different geometrical structure of scalar and vector field sources $\rho(x)$ and $j^\alpha(x)$. They are given by (4) for pointlike charges moving along the trajectory $x^\alpha(\tau)$.

The charges of the scalar and vector field sources are defined by the space integrals of their charge densities $\rho(\mathbf{x},t)$ and $j^0(\mathbf{x},t)$:

$$Q_0,\ Q_1 = \int d^3x\,\{\rho(\mathbf{x},t),\ j^0(\mathbf{x},t)\} = e\int d\tau\{1,\ \dot{x}^0(\tau)\}\,\delta(t-x^0(\tau))$$
$$= e\{\gamma^{-1}(t),\ 1\}, \quad (39)$$

since $d\tau/dt' = \gamma^{-1}(t')$ if $t' = x^0(\tau)$. As is obvious, the charge for the source $T^{\alpha\beta}(x)$ of a tensor field with spin 2 increases as the particle's energy, $Q_2 = e\gamma(t)$.

The removal of ultraviolet divergences in the selfactions $W_{1,0}|^F$ of accelerated charges (force $F \neq 0$) consists in the subtraction of corresponding selfactions $W_{1,0}|^{F=0}$ of uniformly moving charges as a result of which the changes $\Delta W_{1,0} = W_{1,0}|_0^F$ of selfactions owing to acceleration do not contain ultraviolet singularities, have the positive imaginery part, $\mathrm{Im}\,\Delta W_{1,0} > 0$, and vanish together with acceleration.

The following representations for the selfactions of uniformly moving electric and scalar charges are very instructive

$$W_{1,0}|^{F=0} = \frac{1}{2}\,e^2 \iint d\tau\, d\tau'\{\dot{x}_\alpha(\tau)\dot{x}^\alpha(\tau'),\ 1\}\,\Delta_4^f(z,\mu)|^{F=0}$$
$$= \mp\frac{e^2}{4\pi}\cdot\frac{1-i}{2\sqrt{2\varepsilon}}\cdot\tau. \quad (40)$$

They arise if one introduces the integration variable $x = \tau' - \tau$ instead of τ', so that $z^2 = -x^2$, puts $\mu = 0$, and makes use of representation

$$\Delta_4^f(z,\mu)|_{\mu=0} = -\frac{1}{4\pi^2}\cdot\frac{i}{x^2 - i\varepsilon} = \frac{1}{4\pi^2}(\frac{\varepsilon}{x^4+\varepsilon^2} - i\frac{x^2}{x^4+\varepsilon^2}).$$

The opposite signs of the selfactions are due to repulsion of like electric charges and to attraction of scalar ones. The coefficients before τ are the classical proper energies $-\delta m_{1,0}$ of the charges taken with minus sign, and $\sqrt{2\varepsilon}$ characterizes the charge dimension. Different signs of $\mathrm{Im}\,W_{1,0}|^{F=0}$ lead, according to amplitudes $\exp(iW_{1,0}|^{F=0})$, to disappearance (screaning) of electric charge and to unlimited growing (antiscreaning) of scalar charge.

As is seen from regularized representation

$$\operatorname{tr}\alpha^{B,F}|_{reg} = \frac{1}{2\pi}\int_0^\infty ds[\int_{-\infty}^\infty dx\,\{1,\ \sqrt{G'(x)}\}\,e^{-is(G(x)-x)} - \sqrt{\frac{\pi}{ibs}}],$$
$$s = \frac{\omega}{\varkappa}, \tag{41}$$

obtained in [16], the ultraviolet divergences in $\operatorname{tr}\alpha^{B,F}$ are removed by subtraction from the integrand of the first term its asymptotical expansion in s, as $s \to \infty$. The invariant variable $s = \omega/\varkappa = \sqrt{\omega\omega'/\varkappa\varkappa'} = b\rho/2w_0$ is proportional to momentum transfer ρ in units of proper acceleration w_0 of the mirror at the point of its tangency with detector. The subtracted term, being integrated over ρ up to large but finite ρ_{max},

$$\frac{1}{2\pi}\int_0^{s_{max}} ds\sqrt{\frac{\pi}{ibs}} = \frac{1}{2\pi}\sqrt{\frac{\pi\rho_{max}}{w_0}}(1-i), \tag{42}$$

is one and the same for Bose and Fermi cases and explicitly depends on acceleration.

When the space interval Δx between the mirror and detector becomes less than $\hbar/2\Delta p$, the uncontrolled momentum transfer between them becomes greater than Δp and leads to ultraviolet divergency in nonregularized $\operatorname{tr}\alpha^{B,F}$. As the mirror coordinate near the point of tangency with detector changes in time according to the law $x(t) = -w_0\,t^2/2$, the time interval τ necessary for the momentum transfer Δp is of the order of $\tau \sim 2\sqrt{\hbar/\Delta p\,w_0} = 2/\sqrt{w_0\rho_{max}}$ if one sets $\Delta p = \hbar\rho_{max}$. Then the subtracted term which regularizes the $\operatorname{tr}\alpha^{B,F}$ acquires the form

$$\frac{1}{2\pi}\sqrt{\frac{\pi\rho_{max}}{w_0}}(1-i) = \frac{1}{4\pi}\sqrt{\pi}\rho_{max}\,(1-i)\cdot\tau, \qquad \tau \sim 2/\sqrt{w_0\rho_{max}}. \tag{43}$$

As distinct from (39), this term has one and the same sign for Bose and Fermi cases. This can be understood as a consequence of positive momentum transfer from detector to mirror in both cases. The differences in meanings of $\rho_{max} \sim 1/\sqrt{2\varepsilon}$ and τ are more understandable.

Unlike $\Delta W_{1,0}$, describing the change of selfaction of a charges due to acceleration, the functionals $\operatorname{tr}\alpha^{B,F}$ describe the interaction of accelerated mirror with the probe executing uniform motion along the tangent to the mirror's trajectory at the point where mirror has acceleration w_0. This interaction is transmitted by the vector or scalar perturbations created by the mirror in the vacuum of Bose- or Fermi-field and carring the spacelike momentum of the order of w_0. According to (34), at distances of the order of w_0^{-1} from the mirror, the field of these perturbations decreases exponentially in time-like directions and oscillates with damped amplitude in spacelike directions. It can be said that such a field moves together with the mirror and is its "proper field". Hence, the probe interacts with the mirror for a time of the order of w_0^{-1} while the charge all the time interacts with itself and feels the change of interaction over the all time of acceleration. Therefore, it is not surprising that the $-\mathrm{tr}\,\alpha^{B,F}$ coincide in essence with $\Delta W_{1,0}$ if in these latter one puts $\tau_2-\tau_1=2\pi/w_0,\ e^2=1$. In other words, the $\mathrm{tr}\,\alpha^{B,F}$ are the mass shifts of the mirror's proper field multiplied by characteristic proper time of their formation.

The $\mathrm{tr}\,\alpha$ for the trajectory with subluminal velocities of the ends is an invariant function of the relative velocities β_{12}, β_{10}, β_{20} connected by the relation $\beta_{12}=(\beta_{10}-\beta_{20})/(1-\beta_{10}\beta_{20})$. Let us consider the regularized $\mathrm{tr}\,\alpha^{B,F}$ for two important trajectories.

1. Quasihyperbolic trajectory, given by the formula (19), is time-reversed to itself.

$$\alpha^B_{\omega'\omega}=2ia\,\mathrm{sh}\,\theta\,\sqrt{\frac{\omega\omega'}{Q}}\,K_1(a\sqrt{Q}),\quad -\pi a\,\mathrm{sh}\,\theta\,\sqrt{\frac{\omega\omega'}{-Q}}\,H_1^{(2)}(a\sqrt{-Q}),\tag{44}$$

for $Q=\omega^2+\omega'^2-2\,\omega\omega'\,\mathrm{ch}\,\theta\gtrless 0$. Here $a=\beta_{10}\,\sqrt{1-\beta_{10}^2}/w_0,\ \beta_{10}=\mathrm{th}\,\theta/2$; as usual, $\theta=\mathrm{Arth}\,\beta_{12}$ is the Lorentz-invariant parameter defined by the relative velocity of the ends.

$$\alpha^F_{\omega'\omega}=a\,e^{i\frac{\omega+\omega'}{w_0}\beta_1^2}\int_{-\infty}^{\infty}dt\,\sqrt{\mathrm{sh}^2t+\mathrm{ch}^2\frac{\theta}{2}}\times$$
$$\times\exp[ia((\omega'-\omega)\mathrm{ch}\frac{\theta}{2}\,\mathrm{sh}t-(\omega'+\omega)\mathrm{sh}\frac{\theta}{2}\,\mathrm{ch}t)].\tag{45}$$

$$\mathrm{tr}\,\alpha^B=\frac{\mathrm{cth}\,\theta/2}{2\pi}\,[-\frac{\pi}{2}-i(\ln\frac{2}{\gamma\varepsilon}-1)],\qquad \varepsilon=\nu/w_0,\tag{46}$$

$$\mathrm{tr}\,\alpha^F = \frac{1}{2\pi}\{\frac{1}{\mathrm{sh}\,\theta/2}[-\frac{\pi}{2} - i(\ln\frac{2}{\gamma\varepsilon} - 1)] + i[\mathrm{th}\,\frac{\theta}{2}\,\mathbf{B}(k) + \frac{\ln\,\mathrm{ch}\,\theta/2}{\mathrm{sh}\,\theta/2}]\}, \quad (47)$$

$$\mathbf{B}(k) = \int_0^{\pi/2} \frac{\cos^2\varphi\, d\varphi}{\sqrt{1-k^2\sin^2\,\varphi}}, \quad k = \mathrm{th}\,\frac{\theta}{2}.$$

Here $\mathbf{B}(k)$ is one of the elliptic integrals [10].

2. The Airy's semiparabola with in-tangent line to inflection point is given by

$$\begin{aligned} \varkappa u^{\mathrm{mir}}(v) &= (1-b^2/c)\,\varkappa' v - b^3/3c^2, \qquad -\infty < v \leqslant v_0, \\ &= \varkappa' v + b\,\varkappa'^2 v^2 + (1/3)\,c\,\varkappa'^3 v^3, \qquad v_0 \leqslant v < \infty, \end{aligned}$$

where the inflection point $v_0 = -b/\varkappa' c$, $b > 0$, and $c > b^2$ due to the timelikeness of the trajectory.

$$\alpha^B_{\omega'\omega} = \sqrt{\frac{x'/x}{\varkappa\varkappa'}}(cx)^{-1/3}e^{i(b/c)(x-x')-i(2/3)w^{3/2}} \times \times[\pi\,\mathrm{Ai}(z) - i(\pi\,\mathrm{Gi}(z) - \frac{1}{z})], \quad (48)$$

$$\alpha^F_{\omega'\omega} = \frac{(cx)^{-1/3}}{\sqrt{\varkappa\varkappa'(\alpha+1)}}\,e^{i(b/c)(x-x')-i(2/3)w^{3/2}} \times \times[\frac{i\sqrt{\alpha}}{z} + \frac{1}{\sqrt{w}}\int_0^\infty dt\,\sqrt{t^2+\alpha w}\,e^{-izt-it^3/3}]. \quad (49)$$

Here $\mathrm{Ai}(z)$ and $\mathrm{Gi}(z)$ are well known Airy and Scorer functions defined as in [17], and

$$\begin{aligned} &z = (cx)^{-1/3}(x-x') - w, \quad w = (b/c)^2(cx)^{2/3}, \\ &x = \omega/\varkappa, \quad x' = \omega'/\varkappa', \quad \alpha = c/b^2 - 1, \end{aligned} \quad (50)$$

Parameter $\alpha = (1-\beta_{10})/2\beta_{10}$ is defined by the initial relative velocity β_{10} of the mirror and detector, $\beta_{20} = -1$.

$$\begin{aligned} \mathrm{tr}\,\alpha^B \;&=\; \frac{1}{2\pi}(\alpha+1)\{-\frac{\pi}{2} - i[\ln\,\frac{3(\alpha+1)^2}{\gamma\varepsilon} - 1 - \frac{1}{3}\,\ln 2]\}, \\ &\varepsilon = \nu/w_0, \end{aligned} \quad (51)$$

$$\mathrm{tr}\,\alpha^F = \frac{1}{2\pi}\{\sqrt{\alpha(\alpha+1)}(-\frac{\pi}{2} - i\ln\frac{3(\alpha+1)^2}{\gamma\varepsilon}) + i\sqrt{\alpha+1}\,J(\alpha)\}, \tag{52}$$

$$J(\alpha) = 1 + \sqrt{\alpha} + \frac{\alpha-2}{3\sqrt{\alpha+1}}\ln\frac{\alpha+\sqrt{\alpha(\alpha+1)}}{1+\sqrt{\alpha+1}} + \frac{\sqrt{4+\alpha}}{3}\ln\frac{\sqrt{\alpha(4+\alpha)}-\alpha}{4+2\sqrt{4+\alpha}}.$$

Note, that $\alpha^{B,F}_{\omega'\omega}$ depend on two dimensionless parameters b, c, but the traces $\mathrm{tr}\,\alpha^{B,F}$ depend only on their combination α, i.e. only on the subluminal relative velocity β_{10}.

Airy semiparabola with out-tangent line is time-reversed to the considered trajectory and can be obtained from it by the changes $v \rightleftarrows -u$, $\varkappa \rightleftarrows \varkappa'$. This leads to the change $x \rightleftarrows x'$ in the expressions for $\alpha^{B,F}_{\omega'\omega}$. The $\mathrm{tr}\,\alpha^{B,F}$ do not change at all, but it must be understood that the parameter α is now defined by the final (and negative) relative velocity β_{20} of the mirror and detector: $\alpha = -(1+\beta_{20})/2\beta_{20} > 0$, while $\beta_{10} = 1$.

The infrared logarithmic singularities of $\mathrm{tr}\,\alpha^{B,F}$ were regularized by nonzero momentum transfer $\nu \ll w_0$. Their coefficients are in accordance with general consideration of Section 5. These singularities disappear from $\mathrm{tr}\,\alpha^F$ at luminal velocities of the ends, and $\mathrm{tr}\,\alpha^F$ becomes pure imaginery positive.

We do not consider here the coefficients $\beta^{B,F*}_{\omega'\omega}$. They can be obtained from $\alpha^{B,F}_{\omega'\omega}$ by the changes $\omega \to -\omega$, $\sqrt{\omega} \to -i\sqrt{\omega}$, and division on i in Bose-case, see (2).

The symmetry between processes induced by the mirror in two-dimensional and by the charge in four-dimensional space-times predicts not only the value $e_0^2 = 1$ for the bare charge squared that corresponds to the bare fine structure constant $\alpha_0 = 1/4\pi$. It predicts also the appearance of scalar particles in ultra high-energy collisions in 3+1-space and the decreasing their interaction with scalar source with increasing of the energy.

It is very interesting that the bare fine structure constant has the purely geometrical origin, and, also, that its value is small: $\alpha_0 = 1/4\pi \ll 1$. The smallness of α_0 has the essential meaning for the quantum elecrodynamics where it explanes the smallness of α and justifies a priori the applicability of the perturbation theory.

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