Space-time revisited

Yu. F. Pirogov
Theory Division, Institute for High Energy Physics, Protvino, RU-142281 Moscow Region, Russia

Abstract

The metric space-time is revised as a priori existing. It is substituted by the world continuum endowed only with the affine connection. The metric, accompanied by the tensor Goldstone boson, is to emerge during the spontaneous breaking of the global affine symmetry. Implications for gravity and the Universe are indicated.

1 Introduction

According to the present-day paradigm, the physical sciences start with the space-time possessing metric as the primordial structure. I propose to go beyond this paradigm and substitute the metric space-time, at the underlying level, by the world continuum which possesses only the affine connection. The metric is to appear at the effective level during the world structure formation. In other words, the space time is to change its status from a priori existing to emerging. Ultimately, this approach results in the nonlinear model $GL(4,R)/SO(1,3)$ for the gravity, with the graviton as the tensor Goldstone boson corresponding to the spontaneously broken global affine symmetry (in detail, see ref. [1]). This is the further development of the Goldstone approach to gravity [2, 3].

2 Affine symmetry

Affine connection Assume that the forebear of the space-time is the world continuum equipped only with the affine connection. Let x^μ, $\mu = 0, \ldots, 3$ be

the world coordinates. There being, prior to metric, no partition of the continuum onto the space and time, the index 0 has yet no particular meaning. In ignorance of the underlying "dynamics", consider all the structures related to the continuum as the background ones. Let $\bar{\psi}^{\lambda}{}_{\mu\nu}(x)$ be the background affine connection and let $\bar{\xi}^{\alpha}$ be the coordinates where the connection have a particular, to be defined, form $\bar{\psi}^{\gamma}{}_{\alpha\beta}(\bar{\xi})$.[1] For reason not to be discussed, let the antisymmetric part of the background connection (the torsion) be absent identically. As for the symmetric part, one is free to choose the special coordinates to make the physics description as clear as possible.

So, let P be a fixed but otherwise arbitrary point with the world coordinates X^{μ}. One can nullify the symmetric part of the connection in this point by adjusting the proper coordinates $\bar{\xi}^{\alpha}(x, X)$. In the vicinity of P, the connection becomes:

$$\bar{\psi}^{\gamma}{}_{\alpha\beta}(\bar{\xi}) = \frac{1}{2}\,\bar{\rho}^{\gamma}{}_{\alpha\delta\beta}(\bar{\Xi})\,(\bar{\xi} - \bar{\Xi})^{\delta} + \mathcal{O}((\bar{\xi} - \bar{\Xi})^{2}), \tag{1}$$

with $\bar{\rho}^{\gamma}{}_{\alpha\delta\beta}(\bar{\Xi})$ being the background curvature tensor in the point P and $\bar{\Xi} \equiv \bar{\xi}^{\alpha}(X, X)$. Let us consider the whole set of the coordinates with the property $\bar{\psi}^{\alpha}{}_{\beta\gamma}|_{P} = 0$. The allowed group of transformations of such coordinates is the inhomogeneous general linear group $IGL(4, R) = T_4 \times GL(4, R)$ (the affine one):

$$(A, a)\ :\ \bar{\xi}^{\alpha} \to \bar{\xi}'^{\alpha} = A^{\alpha}{}_{\beta}\bar{\xi}^{\beta} + a^{\alpha}, \tag{2}$$

with A being an arbitrary nondegenerate matrix. Under these, and only under these transformations, the affine connection in the point P remains to be zero. The group is the global one in the sense that it transforms the local, i.e., the point P related coordinates in the global manner, i.e., for all the continuum at once. The respective coordinates will be called the local affine ones.[2] In these coordinates, the continuum in a neighbourhood of the point is approximated by the affinely flat manifold. In particular, the underlying covariant derivative in the affine coordinates in the point P coincides with the partial derivative.

Beyond the special relativity According to the special relativity, the present-day physical laws are invariant relative to the choice of the inertial

[1] The bar sign refers to the background. The indices α, β, etc, refer to the special coordinates, while λ, μ, etc, to the world ones.

[2] The term "local" will be omited for short.

coordinates, with the space-time symmetry being the Poincare one. Postulate the principle of *the extended relativity*, stating the invariance relative to the choice of the affine coordinates. The physics invariance symmetry extends now to the affine group. The latter is 20-parametric and supplements the 10-parameter Poincare group $ISO(1,3)$ with the ten special affine transformations. There being known no exact affine symmetry, the latter should be broken to the Poincare symmetry in transition from the underlying level to the effective one.

Metric and symmetry breaking Assume that the affine symmetry breaking is achieved due to the spontaneous emergence of the background metric $\bar{\varphi}_{\mu\nu}(x)$ in the world continuum. The metric is assumed to have the Minkowskian signature and to look in the affine coordinates as:

$$\bar{\varphi}_{\alpha\beta}(\bar{\xi}) = \bar{\eta}_{\alpha\beta} - \frac{1}{2}\bar{\rho}_{\gamma\alpha\delta\beta}(\bar{\Xi})\,(\bar{\xi}-\bar{\Xi})^{\gamma}(\bar{\xi}-\bar{\Xi})^{\delta} + \mathcal{O}((\bar{\xi}-\bar{\Xi})^3). \tag{3}$$

Here one puts $\bar{\eta}_{\alpha\beta} \equiv \bar{\varphi}_{\alpha\beta}(\bar{\Xi})$ and $\bar{\rho}_{\gamma\alpha\delta\beta}(\bar{\Xi}) = \bar{\eta}_{\gamma\delta}\,\bar{\rho}^{\delta}{}_{\alpha\delta\beta}(\bar{\Xi})$. The metric (3) is such that the Christoffel connection $\bar{\chi}^{\gamma}{}_{\alpha\beta}(\varphi)$, determined by the metric, matches with the affine connection $\bar{\psi}^{\gamma}{}_{\alpha\beta}$ in the sense that the connections coincide locally, up to the first derivative: $\bar{\chi}^{\gamma}{}_{\alpha\beta} = \bar{\psi}^{\gamma}{}_{\alpha\beta} + \mathcal{O}((\bar{\xi}-\bar{\Xi})^2)$. This is reminiscent of the well-known fact that the metric in the Riemannian manifold may be approximated locally, up to the first derivative, by the Euclidean metric. In the wake of the background metric, there appears the (yet preliminary) partition of the world continuum onto the space and time.

Under the affine symmetry, the background metric ceases to be invariant. But it still possesses an invariance subgroup. Viz., without any loss of generality, one can choose among the affine coordinates the particular ones with $\bar{\eta}_{\alpha\beta}$ being in the Minkowskian form $\eta = \mathrm{diag}\,(1,-1,-1,-1)$. The respective coordinates will be called the background inertial ones. They are to be distinguished from the effective inertial ones (see later on). Under the affine transformations, one has

$$(A,a)\ :\ \eta \to \eta' = A^{-1T}\eta A^{-1} \neq \eta, \tag{4}$$

whereas the Lorentz transformations $A = \Lambda$ still leave η invariant. It follows that the subgroup of invariance of η is the Poincare group $ISO(1,3) \in IGL(4,R)$, the translation subgroup being intact. Thus under the appearance of the metric, the $GL(4,R)$ group is broken spontaneously to the residual

Lorentz one

$$GL(4,R) \xrightarrow{M_A} SO(1,3). \tag{5}$$

For the symmetry breaking scale M_A, one expects a priori $M_A \sim M_{Pl}$, with M_{Pl} being the Planck mass. The relation between the scales will be discussed later on.

3 Gravity

Affine Goldstone boson Let $\bar{\xi}^\alpha$ be the background inertial coordinates adjusted to the point P. Attach to this point the auxiliary linear space T, the tangent space in the point. By definition, T is isomorphous to the Minkowski space-time. The tangent space is the structure space of the theory, whereupon the realizations/representations of the physics space-time symmetries, the affine and the Poincare ones, are defined. Introduce in T the coordinates ξ^α, the counterpart of the background inertial coordinates $\bar{\xi}^\alpha$ in the space-time. By construction, the connection in the tangent space is zero identically. For the connection in the space-time in the the point P to be zero, too, the coordinates are to be related as $\xi^\alpha = \bar{\xi}^\alpha + \mathcal{O}((\bar{\xi} - \bar{\Xi})^3)$.

Due to the spontaneous breaking, $GL(4,R)$ should be realized in the nonlinear manner [4], with the nonlinearity scale M_A, the Lorentz symmetry being still realized linearly. The spinor representations of the latter correspond to the matter fields, as usually. In this, the finite dimensional spinors appear only at the level of $SO(1,3)$. The broken part $GL(4,R)/SO(1,3)$ should be realized in the Nambu-Goldstone mode. Accompanying the spontaneous emergence of the metric, there should appear the 10-component Goldstone boson which corresponds to the ten generators of the broken affine transformations.

According to ref. [4], the nonlinear realization of the symmetry G spontaneously broken to the symmetry $H \subset G$ can be built on the quotient space $K = G/H$, the residual subgroup H serving as the classification group. One is interested in the pattern $GL(4,R)/SO(1,3)$, with the quotient space consisting of all the broken affine transformations. Let $\kappa(\xi) \in K$ be the coset-function on the tangent space. To restrict κ by the quotient space, one should impose on the representative group element some auxiliary condition, eliminating explicitly the extra degrees of freedom. Under the arbitrary affine

transformation $\xi \to \xi' = A\xi + a$, the coset is to transform as

$$(A, a) \ : \ \kappa(\xi) \to \ \kappa'(\xi') = A\kappa(\xi)\Lambda^{-1}, \tag{6}$$

where $\Lambda(\kappa, A)$ is the appropriate element of the residual group, here the Lorentz one. This makes the transformed group element compatible with the auxiliary condition. In the same time, by the very construction, the Minkowskian η stays invariant under the nonlinear realization:

$$(A, a) \ : \ \eta \to \eta' = \Lambda^{-1T}\eta\Lambda^{-1} = \eta \tag{7}$$

(in distinction with the linear representation eq. (4)).

Otherwise, one can abandon any auxiliary condition extending the affine symmetry by the hidden local symmetry $\hat{H} \simeq H$. In the tangent space, one should distinguish now two types of indices: the Lorentz ones, acted on by the local Lorentz transformations $\Lambda(\xi)$, and the affine ones, acted on by the global affine transformations A. Designate the Lorentz indices as a, b, etc, while the affine ones as before α, β, etc. The Lorentz indices are manipulated by means of the Minkowskian η_{ab} (respectively, η^{ab}). The Goldstone field is represented by the arbitrary 4×4 matrix $\hat{\kappa}^{\alpha}_{a}$ (respectively, $\hat{\kappa}^{-1}{}^{a}_{\alpha}$) which transforms similar to eq. (6) but with arbitrary $\Lambda(\xi)$.[3] The extra Goldstone degrees of freedom are unphysical due to the gauge transformations $\Lambda(\xi)$. This is the linearization of the nonlinear model, with the proper gauge boson $v_{ab\gamma}$ being expressed, due to the equation on motion, through $\hat{\kappa}^{a}_{\alpha}$ and its derivatives. With this in mind, the abrupt expressions entirely in terms of $\hat{\kappa}^{a}_{\alpha}$ and its derivatives are used in what follows. The versions differ by the higher order corrections.

Matter and radiation For the matter fields ϕ one puts

$$\phi(\xi) \to \phi'(\xi') = \hat{\rho}_{\phi}(\Lambda)\phi(\xi), \tag{8}$$

with $\hat{\rho}_{\phi}$ taken in the proper Lorentz representations. As for the gauge bosons, they constitutes one more separate kind of fields, the radiation. By definition, the gauge fields V_{α} transform under A linearly as the derivative $\partial_{\alpha} \equiv \partial/\partial\xi^{\alpha}$. The modified fields $\hat{V}_{a} \equiv \hat{\kappa}^{\alpha}_{a}V_{\alpha}$ transform as the Lorentz vectors

$$\hat{V}(\xi) \to \hat{V}'(\xi') = \Lambda^{-1T}\hat{V}(\xi) \tag{9}$$

and are to be used in the model building.

[3]The hat sign refers to the hidden local Lorentz symmetry.

Nonlinear model To explicitly account for the residual symmetry it is convenient to start with the objects transforming only under the latter symmetry. Clearly, any nontrivial combinations of $\hat{\kappa}$ and $\hat{\kappa}^{-1}$ alone transform explicitly under A. Thus the derivative terms are inevitable. To describe the latter ones, introduce the Cartan one-form chosen as follows:

$$\hat{\omega} = \eta \hat{\kappa}^{-1} d\hat{\kappa}. \tag{10}$$

The one-form transforms as the Lorentz quantity:

$$\hat{\omega}(\xi) \to \hat{\omega}'(\xi') = \Lambda^{-1T} \hat{\omega}(\xi) \Lambda^{-1} + \Lambda^{-1T} \eta d\Lambda^{-1}. \tag{11}$$

In the component notation, the one-form looks like $\hat{\omega}_{ab}$. Decompose it into the symmetric and antisymmetric parts $\hat{\omega}^{\pm}_{ab}$, respectively:

$$\hat{\omega}_{ab} \equiv \sum_{\pm} \hat{\omega}^{\pm}_{ab} = \sum_{\pm} [\eta \hat{\kappa}^{-1} d\hat{\kappa}]^{\pm}_{ab}. \tag{12}$$

One can see that $\hat{\omega}^{\pm}_{ab}$ transform independently as

$$\hat{\omega}^{\pm}(\xi) \to \hat{\omega}'^{\pm}(\xi') = \Lambda^{-1T} \hat{\omega}^{\pm}(\xi) \Lambda^{-1} + \delta^{\pm}, \tag{13}$$

where

$$\begin{aligned} \delta^{-} &= \Lambda^{-1T} \eta d\Lambda^{-1}, \\ \delta^{+} &= 0. \end{aligned} \tag{14}$$

For the derivative of the one-form one gets:

$$\hat{\omega}^{\pm}_{abc} = \hat{\kappa}^{\gamma}_{c}\, \hat{\omega}^{\pm}_{ab} / \partial \xi^{\gamma} = [\eta \hat{\kappa}^{-1} \hat{\partial}_{c}\, \hat{\kappa}]^{\pm}_{ab}, \tag{15}$$

where $\hat{\partial}_c \equiv \hat{\kappa}^{\gamma}_{c} \partial_{\gamma} = \hat{\kappa}^{\gamma}_{c} \partial / \partial \xi^{\gamma}$ is the effective partial derivative. Transforming inhomogeneously, $\hat{\omega}^{-}_{abc}$ could be used as the minimal connection for the nonlinear realization.

The transformation properties of the nonlinear covariant derivative are not changed if one adds to the above minimal connection the properly modified terms $\hat{\omega}^{+}_{abc}$, the latter ones transforming homogeneously. For consistency reason (see later on), choose for the nonminimal connection the following special combination:

$$\hat{\omega}_{abc} = \hat{\omega}^{-}_{abc} + \hat{\omega}^{+}_{cab} - \hat{\omega}^{+}_{cba}. \tag{16}$$

By means of this connection, one can define the nonlinear derivatives of the matter fields $\hat{D}_a\phi$, the gauge strength $\hat{F}_{ab}$, as well as the field strength for affine Goldstone boson $\hat{R}_{abcd}$ and its contraction $\hat{R} \equiv \eta^{ab}\eta^{cd}\hat{R}_{abcd}$.

The above objects can serve as the building blocks for the nonlinear model $GL(4,R)/SO(1,3)$ in the tangent space. Postulate *the equivalence principle* in the sense that the tangent space Lagrangian should not depend explicitly on the background parameter-functions $\bar{\rho}^d{}_{abc}$ (cf. eq. (1)). Thus, the Lagrangian may be written as the general Lorentz (and, thus, affine) invariant built of $\hat{R}$, $\hat{F}_{ab}$, $\hat{D}_a\phi$ and ϕ. As usually, one restricts himself by the terms containing two derivatives at the most.

Once such a Lagrangian is built, one can rewrite it by means of $\hat{\kappa}_a^\alpha$ and $\hat{\kappa}^{-1a}{}_\alpha$ in terms of the affine quantities. This makes explicit the geometrical structure of the theory and relates the latter with the gravity. Under the above choice for the nonlinear connection, the Lagrangian for the affine Goldstone boson, radiation and matter becomes

$$L = c_g M_A^2 R(\gamma_{\alpha\beta}) + L_r(F_{\alpha\beta}) + L_m(D_\alpha\phi, \phi). \tag{17}$$

Here

$$\gamma_{\alpha\beta} = \hat{\kappa}^{-1a}{}_\alpha \eta_{ab} \hat{\kappa}^{-1b}{}_\beta \tag{18}$$

transforms as the affine tensor

$$(A, a) \ : \ \gamma_{\alpha\beta} \to \gamma'_{\alpha\beta} = A^{-1\gamma}{}_\alpha \gamma_{\gamma\delta} A^{-1\delta}{}_\beta. \tag{19}$$

It proves that $R(\gamma_{\alpha\beta}) = \hat{R}(\hat{\omega}_{abc})$ can be expressed as the contraction $R = R^{\alpha\beta}{}_{\alpha\beta}$ of the tensor $R^\gamma{}_{\alpha\delta\beta} \equiv \eta^{\gamma\gamma'}\hat{\kappa}^{-1a}{}_\alpha \hat{\kappa}^{-1b}{}_\beta \hat{\kappa}^{-1d}{}_\delta \hat{\kappa}^{-1c}{}_{\gamma'} \hat{R}_{cadb}$, the latter in turn being related with $\gamma_{\alpha\beta}$ as the Riemann-Christoffel curvature tensor with the metric. In this, all the contractions of the affine indices are understood with $\gamma_{\alpha\beta}$ (respectively, $\gamma^{\alpha\beta}$). Similarly, $D_\alpha\phi \equiv \hat{\kappa}^{-1a}{}_\alpha \hat{D}_a\phi$ look like the covariant derivatives of the matter fields with the spin-connection $\omega_{ab\gamma} \equiv \hat{\omega}_{abc}\hat{\kappa}^{-1c}{}_\gamma$. The gauge strength $F_{\alpha\beta}$ has the usual form containing the partial derivative ∂_α.

Clearly, the Lagrangian L looks like the generally covariant one in the tangent space considered as the Riemannian manifold with the effective[4] metric $\gamma_{\alpha\beta}$, the Riemann-Christoffel curvature $R^\gamma{}_{\alpha\delta\beta}$, the Ricci curvature $R_{\alpha\beta}$, the Ricci scalar R, the spin-connection $\omega_{ab\gamma}$ and the tetrad $\hat{\kappa}^{-1a}{}_\alpha$ (the inverse

[4]The term "effective" will be omitted for short, while that "background" will, in contrast, be retained.

one $\hat{\kappa}_a^\alpha$). This is in no way accidental. Namely, as it is shown in ref. [3], under the special choice of the nonlinear connection eq. (16), the Lagrangian becomes conformally invariant, too. Further, according to ref. [5], the theory which is invariant both under the conformal symmetry and the global affine one is generally covariant. After the choice of the metric, this imposes the Riemannian structure onto the tangent space. Precisely the last property justifies the above special choice for the nonlinear connection. The affine Goldstone boson proves to be the graviton in disguise.

General Relativity and beyond The preceding construction referred to the tangent space T in the given point P. Accept the so defined Lagrangian as that for the space-time, being valid in the background inertial coordinates in the infinitesimal neighbourhood of the point. After multiplying the Lagrangian by the generally covariant volume element $(-\gamma)^{1/2}\, d^4\bar{\Xi}$, with $\gamma \equiv \det\gamma_{\alpha\beta}$, one gets the infinitesimal contribution into the action in the given coordinates.

The relation between the background inertial and world coordinates is achieved by means of the background tetrad $\bar{e}_\mu^\alpha(X)$. Now, introduce the effective tetrad related with the background one as

$$e_\mu^a(X) = \hat{\kappa}^{-1}{}^a_\alpha(X)\, \bar{e}_\mu^\alpha(X). \tag{20}$$

The effective tetrad transforms as the Lorentz vector:

$$e_\mu(X) \to e'_\mu(X) = \Lambda(X)\, e_\mu(X). \tag{21}$$

Due to the local Lorentz transformations $\Lambda(X)$, one can eliminate six components out of e_μ^a, the latter having thus ten physical components. In this terms, the effective metric in the world coordinates is

$$g_{\mu\nu} \equiv \bar{e}_\mu^\alpha \gamma_{\alpha\beta} \bar{e}_\nu^\beta = e_\mu^a \eta_{ab} e_\nu^b. \tag{22}$$

In other words, the tetrad e_μ^a defines the effective inertial coordinates. Physically, eq. (20) describes the disorientation of the effective inertial and background inertial frames depending on the distribution of the affine Goldstone boson.

By means of e_μ^a, the tangent space quantities result in the world coordinates in the usual expressions of the Riemannian geometry containing metric $g_{\mu\nu}$ and spin-connection $\omega_{ab\mu}$. One gets for the total action:

$$I = \int \left(\frac{1}{2} M_{Pl}^2 R(g_{\mu\nu}) + L_r(F_{\mu\nu}) + L_m(D_\mu\phi, \phi) \right) (-g)^{1/2}\, d^4X, \tag{23}$$

with $g \equiv \det g_{\mu\nu}$. Note that due to the weight factor $\sqrt{-g}$, the affine Goldstone boson enters the action also with the derivativeless couplings. Finally, one arrives at the General Relativity (GR) equation of motion for gravity:

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = M_{Pl}^{-2} T_{\mu\nu}. \tag{24}$$

Here $T_{\mu\nu}$ is the energy-momentum tensor of the radiation and matter.

In the above, the constants are such that $c_g M_A^2 = 1/2\, M_{Pl}^2 \equiv 1/(16\pi G_N)$, with G_N being the Newton's constant. Superficially, the (effective) Riemannian geometry is valid at all the space-time intervals. Nevertheless, its accuracy worsen at the smaller and smaller intervals, requiring more and more terms in the decomposition over the ratio of the energy to the symmetry breaking scale M_A, as it should be for the effective theory. Thus, the scale M_A (or rather, the Planck mass M_{Pl}) is the inverse minimal length in the nature.

In the GR, after fixing the Lagrangian the theory becomes unique, independent of the choice of the coordinates. Under extension of the tangent space Lagrangian beyond the general covariance, the theory in the space-time ceases to be generally covariant and thus unique. It depends not only on the Lagrangian but on the choice of the coordinates. Relative to the general coordinate transformations, the obtained GR extensions divide into the inequivalent classes, each of which is characterised by the particular set of the background parameter-functions. A priori, no one of the sets is preferable. Which one is suitable (if any), should be determined by observations. Each class consists of the equivalent extensions related by the residual covariance group. Among the inequivalent extensions, there appears the natural hierarchy according to whether the affine symmetry is explicitly violated or not. For details, see ref. [1].

The Universe and beyond Let the formation of the Universe be the result of the actual transition between the two phases of the continuum, the affinely connected and metric ones. This transition is thus *the "Grand Bang"*, the origin of the Universe and the very space-time. There is conceivable the appearance (as well as disappearance and coalescence) of the various bubbles of the metric phase inside the affinely connected one (and v.v.). These bubbles are to be associated with the multiple universes. One of the latter ones happens to be ours. Hopefully, this may shed light on the long-standing problem of the fine tuning of our Universe.

4 Conclusion

In conclusion, the new physics paradigm realizes consistently the approach to gravity as the Goldstone phenomenon. The theory constructed proceeds, in essence, from two basic symmetries: the spontaneously broken global affine symmetry and the general covariance. The theory embodies the GR as the lowest approximation. Its distinction with the GR are twofold. At the effective level, the theory predicts the natural hierarchy of the GR extensions depending on the mode of the affine symmetry. At the underlying level, it presents the new physics comprehension of the gravity and the Universe.

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