Cosmological constant problem and long-distance modifications of Einstein theory

A.O.Barvinsky

Theory Department, Lebedev Physics Institute, Leninsky Prospect 53, Moscow 117924, Russia

Abstract

We construct the covariant nonlocal action for recently suggested long-distance modifications of gravity theory motivated by the cosmological constant and cosmological acceleration problems. This construction is based on the special nonlocal form of the Einstein-Hilbert action explicitly revealing the fact that this action within the covariant curvature expansion begins with curvature-squared terms.

New approach to the solution of the cosmological constant problem consists in the assumption that, instead of adjusting the vacuum energy of quantum matter to zero (or to small value of the cosmological acceleration), one should modify the purely gravitational sector of the theory in far infrared region. Matter sources with wavelengths comparable with the horizon size of the present Universe $L \sim 1/H_0 \sim 10^{28}$ cm gravitate with the long-distance gravitational constant G_{LD} which is much smaller than the conventional Planckian value G_P. Therefore, the vacuum energy $\mathcal{E}$, $T_{\mu\nu} = \mathcal{E} g_{\mu\nu}$, of TeV or even Planckian scale (necessarily arising in all conceivable models with spontaneously broken SUSY or in quantum gravity) will not generate a catastrophically big spacetime curvature incompatible with the tiny observable Hubble constant $H_0^2 \sim G_{LD}\mathcal{E} \ll G_P\,\mathcal{E}$. This mechanism is drastically different from the old suggestions of supersymmetric cancellation of $\mathcal{E}$ [1], because it relies on the fact that the nearly homogeneous vacuum energy gravitates very little, rather than it is itself very small.

This idea implies that the gravitational coupling constant should be promoted to the level of the operator, $G_P \Rightarrow G(\Box)$, which for sake of covariance can be regarded as a function of the covariant d'Alembertian $\Box = g^{\alpha\beta}\nabla_\alpha\nabla_\beta$, interpolating between the Planck scale of the gravitational coupling constant $G_P = 1/16\pi M_P^2$ for local matter sources of size $\ll L$ and the long distance gravitational constant $G_{LD} = G(0)$ with which the sources nearly homogeneous at the horizon scale are gravitating [2]. The modified equations of motion were suggested to have the form of Einstein equations

$$M^2(\Box)\Big(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\Big) = \frac{1}{2}\,T_{\mu\nu} \tag{1}$$

with "nonlocal" inverse gravitational constant or Planck mass,

$$\frac{1}{16\pi G(\Box)} \equiv M^2(\Box) = M_P^2\,(1 + \mathcal{F}(L^2\Box\,)), \tag{2}$$

being some function of the dimensionless combination of $\Box$ and the additional scale L – the length at which infrared modification becomes important. If the function of $z = L^2\Box$ satisfies the conditions, $\mathcal{F}(z) \to 0$ at $z \gg 1$, and $\mathcal{F}(z) \to \mathcal{F}(0) \gg 1$ at $z \to 0$, then the long-distance modification is inessential for processes varying in spacetime faster than $1/L$ and is large for slower phenomena at wavelengthes $\sim L$ and longer.

One difficulty with this construction is that for any nontrivial form factor $\mathcal{F}(L^2\Box\,)$ the left hand side of (1) does not satisfy the Bianchi identity and, therefore, cannot be generated by generally covariant action. Obviously, this makes the situation unsatisfactory because of a missing off-shell extension of the theory, problems with its quantization, etc. Here we suggest to circumvent this problem by resorting to the *weak field* approximation, which is certainly valid in the infrared regime. This means that Eq. (1) should be understood only as a first term of the perturbation expansion in powers of the curvature. Its left hand side should be modified by higher than linear terms in the curvature, and the modified nonlocal action $S_{NL}[\,g\,]$ should be found from the variational equation

$$\frac{\delta S_{NL}[\,g\,]}{\delta g_{\mu\nu}(x)} = M_P^2\,g^{1/2}\Big(1 + \mathcal{F}(L^2\Box\,)\Big)\Big(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R\Big) + \mathrm{O}\,[\,R_{\mu\nu}^2\,]. \tag{3}$$

Flexibility in higher orders of the curvature allows one to guarantee the integrability of this equation and to construct the nonlocal action as a generally covariant (but nonlocal) curvature expansion. Here we explicitly present this construction along the lines of covariant curvature expansion of [3]. As a starting point we consider a special nonlocal form of the Einstein-Hilbert action revealing its basic property – the absence of a linear in metric perturbation part (on flat-space background). Then we introduce a needed long-distance modification by a simple replacement of the nonlocal form factor in the curvature-squared term of the obtained action [4]. The paper is accomplished by a discussion of the nature of nonlocalities in quantum-gravitational and brane-induced models of [5]. In particular, the fact that curvature expansion for the action begins with the quadratic order is revisited from the viewpoint of the running gravitational coupling constant and the issue of acausality of nonlocal effective equations, raised in [2], is reconsidered.

The Einstein action in the Euclidean asymptotically-flat spacetime

$$S_E[\,g\,] = -M_P^2 \int dx\, g^{1/2}\, R(\,g\,) + M_P^2 \int_{|x|\to\infty} d\sigma^\mu \,(\partial^\nu h_{\mu\nu} - \partial_\mu h\Big). \tag{4}$$

includes the bulk integral of the scalar curvature and the Gibbons-Hawking surface integral over spacetime infinity, $|x| \to \infty$. The latter is usually called the Gibbons-Hawking action which in the covariant form contains the trace of the extrinsic curvature of the boundary (with an appropriate subtraction of the flat space background contribution). This surface term guarantees the consistency of the variational problem for this action which yields as a metric variational derivative the Einstein tensor.

The action (4) is explicitly linear in the curvature, but this linearity is misleading, because its variational derivative — the Einstein tensor — is also linear in the curvature. Therefore, it is at least linear in metric perturbation on flat-space background, $R_{\mu\nu} \sim h_{\mu\nu}$, and the perturbation theory for $S_E[\,g\,]$ should start with the quadratic order, $\mathrm{O}\,[\,h_{\mu\nu}^2\,] \sim \mathrm{O}\,[\,R_{\mu\nu}^2\,]$. This is a well known fact from the theory of free massless spin-2 field. Our goal is to make this $h_{\mu\nu}$-expansion manifestly covariant by converting it to the covariant *curvature* expansion. A systematic way to do this is to use the technique of covariant perturbation theory of [3], which begins with the derivation of the expression for the metric perturbation in terms of the curvature.

Expand the Ricci curvature in $h_{\mu\nu}$ on flat-space background

$$R_{\mu\nu} = -\frac{1}{2}\,\Box\, h_{\mu\nu} + \frac{1}{2}\Big(\nabla_\mu X_\nu + \nabla_\nu X_\mu\Big) + \mathrm{O}\,[\,h_{\mu\nu}^2\,], \tag{5}$$

$X_\mu \equiv \nabla^\lambda h_{\mu\lambda} - \frac{1}{2}\,\nabla_\mu h$, and solve it by iterations as a nonlocal expansion in powers of the curvature. This expansion starts with the following terms

$$h_{\mu\nu} = -\frac{2}{\Box} R_{\mu\nu} + \nabla_\mu f_\nu + \nabla_\nu f_\mu + \mathrm{O}\,[\,R_{\mu\nu}^2\,]. \tag{6}$$

Here $1/\Box$ denotes the action of the Green's function of the *covariant metric-dependent* d'Alembertian on the space of symmetric second-rank tensors with zero boundary conditions at infinity and the term $\nabla_\mu f_\nu + \nabla_\nu f_\mu$ in (6) reflects the gauge ambiguity in this solution (it originates from the harmonic-gauge terms in the right-hand side of (5)).

Now restrict ourselves with the approximation quadratic in $R_{\mu\nu}$ (or equivalently $h_{\mu\nu}$) and integrate the variational equation for $S_E[\,g\,]$. Since the variational derivative is at least linear in $h_{\mu\nu}$, $\delta S_E/\delta g_{\mu\nu} \sim h_{\alpha\beta}$, the quadratic part of the action in view of this equation is given by the integral

$$S_E[\,g\,] = \frac{1}{2}\int dx\, h_{\mu\nu}(x) \frac{\delta S_E[\,g\,]}{\delta g_{\mu\nu}(x)} + \mathrm{O}\,[\,R_{\mu\nu}^3\,]. \tag{7}$$

Substituting the Einstein tensor for $\delta S_E/\delta g_{\mu\nu}$ and (6) for $h_{\mu\nu}$ and integrating by parts one finds that the contribution of the gauge parameters f_μ vanishes in view of the Bianchi identity for the Einstein tensor, and the final result reads

$$S_E[\,g\,] = M_P^2 \int dx\, g^{1/2}\, \Big\{ -\Big(R^{\mu\nu} - \frac{1}{2}\, g^{\mu\nu} R\Big) \frac{1}{\Box} R_{\mu\nu} + \mathrm{O}\,[\,R_{\mu\nu}^3\,] \Big\}. \tag{8}$$

This is the covariant *nonlocal* form of the *local* Einstein action [6, 7, 4]. This nonlocal incarnation of (4) explicitly features the absence of the linear in curvature term, which can be clarified by the subtraction effect of the Gibbons-Hawking term.

In asymptotically-flat (Euclidean) spacetime with the asymptotic behavior of the metric $g_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}$, $h_{\mu\nu} = \mathrm{O}\,\big(1/|x|^{d-2}\big)$, $|x| \to \infty$, the Gibbons-Hawking term in Cartesian coordinates can be transformed to the bulk integral of the integrand $\partial^\mu\big(\partial^\nu h_{\mu\nu} - \partial_\mu h\big)$ – the linear in $h_{\mu\nu}$ part of the scalar curvature. Similarly to the above procedure this integral can be covariantly expanded in powers of the curvature. Up to quadratic terms inclusive this expansion reads [4]

$$\int_\infty d\sigma^\mu \big(\partial^\nu h_{\mu\nu} - \partial_\mu h\big) = \int dx\, g^{1/2} \Big\{ R - \Big(R^{\mu\nu} - \frac{1}{2}\, g^{\mu\nu} R\Big) \frac{1}{\Box} R_{\mu\nu} + ... \Big\}. \tag{9}$$

As we see, when substituting to (4) its linear term cancels the Ricci scalar part and the quadratic terms reproduce those of (8). Obviously, this type of expansion can be extended to arbitrary order in curvature.

Long distance modification of the Einstein action that would generate (3) as the left-hand side of the gravitational equations of motion now can be simply obtained from the nonlocal form of the Einstein action (8). It is just enough to make the following replacement in the quadratic part of (8), $1/\Box \to (1+\mathcal{F}(L^2\Box))/\Box$. Indeed, the variation of the Ricci tensor here and integration by parts "cancel" $\Box$ in the denominator. All commutators of covariant derivatives with the $\Box$ in $\mathcal{F}(L^2\Box)$ give rise to the curvature-squared order which is beyond our control. This recovers the Einstein tensor term of (3) with the needed "nonlocal" Planckian mass $M_P^2\Big(1+\mathcal{F}(L^2\Box\,)\Big)$.

Thus, the long-distance modification in question takes the form

$$S_{NL}[\,g\,] = -\int dx\, g^{1/2}\left\{\Big(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R\Big)\frac{M^2(\Box)}{\Box}\,R_{\mu\nu} + \mathrm{O}[R_{\mu\nu}^3]\right\}. \qquad (10)$$

It is manifestly generally covariant, and its variational derivative (the left hand side of the modified Einstein equations) exactly satisfies the Bianchi identity and does not suffer from the concerns of [2]. This action is not unique though, because it is defined by a given form factor $\mathcal{F}(L^2\Box\,)$ only in quadratic order, while we do not have good principles to fix its higher-order terms thus far.

One of the main mechanisms for nonlocalities of the above type is the contribution of graviton and matter loops to the quantum effective action. In quantum theory the concept of a nonlocal form factor replacing a coupling constant is not new. In fact this concept underlies the notion of the running coupling constants and sheds new light on the cosmological constant problem also from the viewpoint of the renormalization theory. For simplicity, consider QED or Yang-Mills theory in the quadratic order in gauge field strength $F_{\mu\nu}^2$. The transition from classical to quantum effective action, $S \to S_{\rm eff}$, boils down to the replacement of the local invariant by

$$g^{-2}\int dx\, F_{\mu\nu}^2 \to \int dx\, F_{\mu\nu}\, g_{\rm eff}^{-2}(-\Box)\, F^{\mu\nu}. \qquad (11)$$

Here the effective coupling constant $g_{\rm eff}^{-2}(-\Box)$ is a nonlocal form factor which can be obtained from the corresponding solution of renormalization-group equation. Obviously, it plays the role of $\mathcal{F}(L^2\Box)$ above.

This concept, however, fails when applied to the gravitational theory in the sector of the cosmological and Einstein-Hilbert terms[1]. Indeed, naive replacement of ultralocal cosmological and gravitational coupling constants by nonlocal form factors,

$$\int dx\, g^{1/2} \left(\Lambda - M_P^2\, R \right) \to \int dx\, g^{1/2} \left(\Lambda(\Box) - M_P^2(\Box)\, R \right), \tag{12}$$

is meaningless because the action of the covariant d'Alembertian on the right hand side always picks up its zero mode, and both form factors reduce to their numerical values in far infrared, $\Lambda(0)$, $M_P^2(0)$. Therefore, even if one has solutions of renormalization group equations for Λ and M_P^2, like those of [8], one cannot automatically recover the corresponding pieces of effective action or the corresponding nonlocal correlation functions.

The construction above suggests that the running coupling constant "delocalization" of M_P^2 should be done in the (already nonlocal) representation of the Einstein action (8). As its curvature expansion begins with the quadratic order, one can insert the nonlocal form factor $M_P^2(\Box)$ *between two curvatures* so that no integration by parts would result in its degeneration to a trivial constant. It would be interesting to see how a similar mechanism works for the nonlocal cosmological "constant" $\Lambda(\Box)$.[2] The mechanisms of its generation due to infrared asymptotics of the effective action, or late-time asymptotics of the corresponding heat kernel, are discussed in [12].

Nonlocalities of the type (10) also arise in a certain class of braneworld models [13, 5]. They cannot appear in models of the Randall-Sundrum type with strictly localized zero modes, because in these models nontrivial form factors basically arise in the transverse-traceless sector of the action (as kernels of nonlocal quadratic forms in *Weyl* tensor [7]). In contrast to these models, the nonlocal part of (10) is not quadratic in the Weyl tensor, $\int dx\, g^{1/2} W_{\mu\nu\alpha\beta}^2 \sim \int dx\, g^{1/2} (R_{\mu\nu}^2 - \frac{1}{3} R^2)$ (with the insertion of a nonlocal form factor between the curvatures). Rather, (10) includes the structure

[1] When applied to formally renormalizable (albeit non-unitary) curvature-squared gravitational models [8].

[2] One should expect that a quadratic action for the cosmological term would read as $\Lambda \int dx\, g^{1/2} R^{\mu\nu} (1/\Box^2) R_{\mu\nu}$. This structure (also suggested in [9] and discussed within the renormalization group theory) appears in two-brane models [7] and expected as a covariant completion of the mass term in models of massive gravitons [10] and discussions of the van Damm-Veltman-Zakharov discontinuity [11].

$\int dx\, g^{1/2}(R_{\mu\nu}^2 - \frac{1}{2}R^2)$ which contains the *conformal* sector. It is this sector which is responsible for the potential resolution of the cosmological constant problem. It becomes dynamical in models with metastable graviton like the Gregory-Rubakov-Sibiryakov model [13] or brane induced gravity models of the Dvali-Gabadaze-Porrati (DGP) type [5]. In particular, for the (4+1)-dimensional DGP model with the bulk $G_{AB}(X)$ and induced on the brane $g_{\mu\nu}(x)$ metrics

$$S_{DGP}[\,G\,] = M^3 \int d^5X\, G^{1/2}\; {}^5\!R(G_{AB}) + M_P^2 \int d^4x\, g^{1/2}\; {}^4\!R(g_{\mu\nu}) \tag{13}$$

(we disregard here relevant Gibbons-Hawking terms) the effective braneworld action takes the form (10) with the form factor $\mathcal{F}(L^2\Box)$ which is singular at $\Box \to 0$ [5, 14]

$$\mathcal{F}(L^2\Box) = \frac{1}{L\sqrt{-\Box}}, \quad L = \frac{M_P^2}{M^3}, \tag{14}$$

where $M \sim 10^{-21} M_P \sim 100$ MeV is a mass scale of the bulk gravity as opposed to the Planckian scale of the Einstein term on the brane $M_P \sim 10^{19}$ GeV. This model is interesting, because its approximate cosmological FRW equation of motion,

$$H^2 - \frac{H}{L} = \frac{\rho}{6M_P^2}\,, \tag{15}$$

admits the self-accelerating regime with $H = 1/L$ at late stages of evolution with matter density $\rho \to 0$ [14]. Unfortunately, this model can hardly serve as a consistent candidate for the description of our Universe because of the presence of the sufficiently low strong-coupling scale $(M_P/L^2)^{1/3} \sim 10^{-8}\mathrm{cm}^{-1}$, which invalidates its applications to phenomena already at distances smaller than 1000 km [15] (see, however, Ref. [16] advocating that this difficulty can be circumvented by special type of regularization).

Form factors like (14) are unambiguously defined only in the Euclidean space with the negative semi-definite d'Alembertian $\Box$. This raises the problem of their continuation to the Lorentzian spacetime where the issues of causality and unitarity become important. The principles of this continuation depend on the physical origin of nonlocality in $\mathcal{F}(L^2\Box)$. In particular, scattering problems for in-out matrix elements, $\langle\, out\,|\,\hat{\varphi}\,|\,in\,\rangle$, in spacetime with asymptotically-flat past and future imply a usual Wick rotation. The problem for in-in mean value of the quantum field, $\phi = \langle\, in \,|\, \hat{\varphi} \,|\, in \,\rangle$, is more complicated and incorporates the Schwinger-Keldysh diagrammatic technique [17]. In this technique the effective equations for ϕ cannot be obtained as variational derivatives of some one-field action functional. However, there exists a special case of the initial quantum state — the *Poincare-invariant in-vacuum* in asymptotic past, $|\, in \,\rangle = |\, in, vac \,\rangle$. Effective equations for ϕ in this vacuum can be obtained by the following procedure [3]. Calculate the *Euclidean* effective action in asymptotically-flat spacetime, take its variational derivative containing the nonlocal form factors which are uniquely specified by zero boundary conditions at Euclidean infinity. Then formally go over to the Lorentzian spacetime signature with the *retardation* prescription for all nonlocal form factors. These retarded boundary conditions uniquely specify the nonlocal effective equations and guarantee their causality. This procedure was proven in [3] and also put forward in a recent paper [18] as the basis of the covariant nonlocal model of MOND theory.

This procedure justifies the Euclidean setup used above and suggests that in this setting no contradiction arises between the nonlocal nature of the *Euclidean* action and *causal* nature of nonlocal equations of motion in Lorentzian spacetime. In this respect the situation is essentially different from the assumptions of [2] where acausality of equations of motion is necessarily attributed to the nonlocal action. No such assumptions are needed in effective equations for expectation values which are fundamentally causal despite their nonlocality. These equations have interesting applications in quantum gravitational context and, in particular, show the phenomenon of the cosmological acceleration due to infrared back-reaction mechanisms [19].

The situation with brane induced nonlocalities and their causality status is more questionable and conceptually open. For example, the branch point of the square root in the nonlocal form factor (14) is apparently related to different branches of cosmological solutions of (15) including the scenario of cosmological acceleration [14]. Therefore, in contrast to tentative models of [2] with finite $\mathcal{F}(0) \gg 1$, which only interpolate between two Einstein theories with different gravitational constants $G_{LD} \sim G_P/\mathcal{F}(0) \ll G_P$, the DGP model is anticipated to suggest the mechanism of the cosmological acceleration. This implies the replacement of the asymptotically-flat spacetime by the asymptotically-deSitter one. For small values of asymptotic curvature (as is the case of the observable horizon scale $H_0^2/M_P^2 \sim 10^{-120}$) the curvature expansion used above seems plausible, although the effect of the asymptotic curvature might be in essence nonperturbative. Therefore, the above construction might have to be modified accordingly. In particular, the expansion in powers of the curvature should be replaced by the expansion in powers of its deviation from the asymptotic value $R_{\mu\nu} - \frac{1}{d} g_{\mu\nu} R_\infty$ (d is a spacetime dimensionality). This would introduce in the formalism as a free parameter the value of the curvature in far future, $R_\infty \sim H_0^2$, reflecting the measure of acausality in the model. The resulting modifications in the above construction are currently under study and will be presented elsewhere.

The author would like to thank G.Gabadadze, V.Mukhanov, S.Solodukhin and R.Woodard for helpful stimulating discussions. This work was supported by the Russian Foundation for Basic Research under the grant No 02-02-17054 and the LSS grant No 1578.2003.2.

References

[1] S.Weinberg, Rev. Mod. Phys. **61** (1989) 1.

[2] N.Arkani-Hamed, S.Dimopoulos, G.Dvali and G.Gabadadze, Nonlocal modification of gravity and the cosmological constant problem, hep-th/0209227.

[3] A.O.Barvinsky and G.A.Vilkovisky, Nucl. Phys. **B 282** (1987) 163, Nucl. Phys. **B333** (1990) 471; A.O.Barvinsky, Yu.V.Gusev, G.A.Vilkovisky and V.V.Zhytnikov, J. Math. Phys. **35** (1994) 3525-3542; J. Math. Phys. **35** (1994) 3543-3559.

[4] A.O.Barvinsky, Phys. Lett. **B 572** (2003) 109.

[5] G.Dvali, G.Gabadadze and M.Porrati, Phys. Rev. Lett. **B485** (2000) 208, hep-th/0005016.

[6] A.O.Barvinsky, Phys. Rev. **D 65** (2002) 062003, hep-th/0107244.

[7] A.O.Barvinsky, A.Yu.Kamenshchik, A.Rathke and C.Kiefer, Phys. Rev. **D67** (2003) 023513, hep-th/0206188.

[8] E.S. Fradkin and A.A. Tseytlin, Phys.Lett. **B104** (1981) 377; A.O.Barvinsky and I.G.Avramidi, Phys.Lett. **B159** (1985) 269.

[9] E.V.Gorbar and I.L.Shapiro, JHEP **0302** (2003) 021.

[10] M.Porrati, Phys. Lett. **B534** (2002) 209, hep-th/0203014.

[11] H. van Damm and M.J.Veltman, Nucl. Phys. **D22** (1970) 397; V.I.Zakharov, JETP Lett. **12** (1970) 312; M.Porrati, Phys.Lett. **B498** (2001) 92, hep-th/0011152.

[12] A.O.Barvinsky, Yu.V.Gusev, V.F.Mukhanov and D.V.Nesterov, Phys. Rev. **D 68** (2003) 105003, hep-th/0306052.

[13] R.Gregory, V.A.Rubakov and S.M.Sibiryakov, Phys. Rev. Lett. **84** (2000) 5928, hep-th/0002072.

[14] C.Deffayet, Phys.Lett. **B502** (2001) 199, hep-th/0010186; C.Deffayet, G.Dvali and G.Gabadadze, Phys. Rev. **D65** (2002) 044023, astro-ph/0105068.

[15] M.A.Luty, M.Porrati and R.Rattazzi, JHEP **0309** (2003) 029, hep-th/0303116; V.A.Rubakov, hep-th/0303125; S.L.Dubovsky and M.V. Libanov, JHEP **0311** (2003) 038, hep-th/0309131.

[16] M.Kolanovich, M.Porrati and J.W.Rombouts, Phys. Rev. **D68** (2003) 064018, hep-th/0304148; M.Porrati and J.W.Rombouts, Phys. Rev. **D69** (2004) 122003, hep-th/0401211.

[17] J.Schwinger, J. Math. Phys. **2** (1961) 407; J.L.Buchbinder, E.S.Fradkin and D.M.Gitman, Fortschr. der Phys. **29** (1981) 187; R.D.Jordan, Phys. Rev. **D33** (1986) 444.

[18] M.E.Soussa and R.P.Woodard, Phys. Lett. **B578** (2004) 253-258, astro-ph/0302030.

[19] L.R.Abramo and R.P.Woodard, Phys. Rev. **D65** (2002) 063516; V.K.Onemli and R.P.Woodard, Class.Quant.Grav. **19** (2002) 4607, gr-qc/0204065.