# Study of the Decay $K^+ \to \pi^+ \nu \bar{\nu}$

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#### Abstract

The predictions of the Standard Model for rare flavor-changing neutral current processes  $K \to \pi \nu \bar{\nu}$  are briefly reviewed. An additional evidence for the rare kaon decay  $K^+ \to \pi^+ \nu \bar{\nu}$  observed in the E949 experiment at BNL is presented. The new value of branching ratio is  $BR(K^+ \to \pi^+ \nu \bar{\nu}) = (1.47 + 1.30 - 0.89) \times 10^{-10}$ .

### 1 Introduction

The standard mechanism to incorporate CP violation in the Standard Model (SM) is the CKM complex matrix  $V_{CKM}$  for 3 families of quarks [1]:

$$\begin{pmatrix} d'\\s'\\b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub}\\V_{cd} & V_{cs} & V_{cb}\\V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d\\s\\b \end{pmatrix} = V_{CKM} \begin{pmatrix} d\\s\\b \end{pmatrix}$$
(1)

The CKM matrix can be expressed in terms of 4 parameters (Wolfenstein parameterization)  $\lambda, A, \rho, \eta$ , where  $\eta$  represents the only CP violating parameter:

$$V_{CKM} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$
(2)

In the SM, a single parameter  $\eta$  is the only source of CP violation which makes the predictions for CP violation phenomena quite constrained. The study of rare flavour-changing neutral current processes  $K \to \pi \nu \bar{\nu}$  can shed further light on CP violation in the SM. The most interesting of these processes are the "golden" decays  $K^+ \to \pi^+ \nu \bar{\nu}$  and  $K_L^0 \to \pi^0 \nu \bar{\nu}$ . These decays are uniquely sensitive to  $|V_{td}|$  and to the CKM CP-violation parameter  $\eta$  respectively. They are strongly GIM-suppressed and their leading contributions arise from loops involving weak bosons and heavy quarks. The connection between the rates of these processes and the fundamental parameters of the SM is extremely well-determined because the matrix element connecting the short-distance interaction to the initial and final state hadrons is measured by the rate of  $K_{e3}$  decay [2].

The leading electroweak diagrams of the  $K_L^0 \to \pi^0 \nu \bar{\nu}$  decay are shown in Fig. 1.



Figure 1: The leading electroweak diagrams inducing  $K_L \to \pi^0 \nu \bar{\nu}$  decays. For  $K^+ \to \pi^+ \nu \bar{\nu}$  decay *d* quark is changed to *u* quark.

The  $K_L^0 \to \pi^0 \nu \bar{\nu}$  is driven by direct CP violation due to the CP properties of  $K_L^0$  and  $\pi^0$  and the relevant short-distance hadronic transition current. Since  $K_L^0$  is predominantly a coherent, CP odd superposition of  $K^0$  and  $\bar{K}^0$ , only the imaginary part of  $V_{ts}^* V_{td}$  survives in the amplitude. The branching ratio of the  $K_L^0 \to \pi^0 \nu \bar{\nu}$  decay can be written as

$$BR(K_L^0 \to \pi^0 \nu \bar{\nu}) = r_{IB}B(K^+ \to \pi^0 e^+ \bar{\nu}) \\ \times \frac{\tau(K_L)}{\tau(K^+)} \frac{3\alpha^2}{2\pi^2 \sin^4 \Theta_W} \eta^2 A^4 \lambda^8 X^2(x_t), \qquad (3)$$

where

$$X(x) = \eta_X \cdot \frac{x}{8} \Big[ \frac{x+2}{x-1} + \frac{3x-6}{(x-1)^2} lnx \Big].$$
(4)

Here  $\eta_X = 0.985$ ,  $\mathbf{x}_t = m_t^2/M_W^2$ , and  $\lambda$ , A,  $\rho$ ,  $\eta$  are the usual Wolfenstein parameters. The coefficient  $\mathbf{r}_{IB} = 0.944$  summarizes the leading isospin breaking corrections in relating  $K_L^0 \to \pi^0 \nu \bar{\nu}$  to  $K^+ \to \pi^0 e^+ \bar{\nu}$ . As a consequence  $BR(K_L^0 \to \pi^0 \nu \bar{\nu}) \sim \eta^2$ . Using current values of SM parameters, the branching ratio for  $K_L^0 \to \pi^0 \nu \bar{\nu}$  is expected to be in the range of about  $(3.0 \pm 0.6) \times 10^{-11}$  [4].

The decay  $K^+ \to \pi^+ \nu \bar{\nu}$  is sensitive primarily to the matrix element  $V_{td}$ with a small charm correction. The Standard Model predictions [3] for the  $K^+ \to \pi^+ \nu \bar{\nu}$  branching ratio looks as follows

$$BR(K^{+} \to \pi^{+} \nu \bar{\nu}) = \frac{r_{K^{+}} \alpha^{2} B(K^{+} \to \pi^{0} e^{+} \nu)}{V_{us}^{2} 2 \pi^{2} \sin^{4} \theta_{W}} \times \sum_{l=e,\mu,\tau} |V_{cs}^{*} V_{cd} X_{NL}^{l} + V_{ts}^{*} V_{td} X(x_{t})|^{2}.$$
(5)

Here,  $r_{K^+} = 0.901$  is an isospin-breaking correction,  $X_{NL}^l$  is a charm function calculated in [3]. The recently updated SM prediction based on the current data for the CKM matrix elements gives  $BR(K^+ \to \pi^+ \nu \bar{\nu}) = (7.8 \pm 1.2) \times 10^{-11}$  [4].

The unitarity nature of the CKM matrix leads to the relationships between the elements

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$
(6)

As a result of the unitarity of the CKM matrix, the quantities  $V_{ub}^*/A\lambda^3 = \rho + i\eta$ ,  $V_{td}/A\lambda^3 = 1 - \rho - i\eta$  and 1 form a triangle in the  $(\rho, \eta)$  plane, The unitarity relations

$$1 + \frac{V_{td}V_{tb}^{*}}{V_{cd}V_{cb}^{*}} = -\frac{V_{ud}V_{ub}^{*}}{V_{cd}V_{cb}^{*}} \equiv \bar{\rho} + i\bar{\eta}$$
(7)

determines a triangle in the  $(\bar{\rho}, \bar{\eta})$  plane as shown in Fig. 2, where the potential of  $K \to \pi \nu \bar{\nu}$  is illustrated and their relation to quantities measured in *B* decay is shown. Here,  $\bar{\rho} \simeq \rho (1 - \lambda^2/2)$  and  $\bar{\eta} \simeq \eta (1 - \lambda^2/2)$ . A clean measure of its height is provided by the  $K_L^0 \to \pi^0 \nu \bar{\nu}$  branching ratio itself and  $B(K_L^0 \to \pi^0 \nu \bar{\nu})$  plus  $B(K^+ \to \pi^+ \nu \bar{\nu})$  determine the unitarity triangle completely. An accuracy of  $\pm 10\%$  in the branching ratio measurements provides a 5% accuracy in  $\eta$  determination. Such a precise determination of the CKM parameters in  $K \to \pi \nu \bar{\nu}$  decays is comparable to what can be achieved by CP violation studies at the B factories. Because kaon decays are suppressed down to the few  $\times 10^{-11}$  level, they are also quite sensitive to physics beyond the SM. Moreover, any additional to B decays and independent measurement of CKM parameters would test the Standard Model



Figure 2: The unitarity triangle determined from the kaon and B decays.

and any significant inconsistency between unitarity relation in kaon decays  $(s \to d \text{ transitions})$  with B sector  $(b \to d \text{ transition})$  would point to new physics beyond the Standard Model [5]. Since the theoretical uncertanties in the BR's are estimated to be about 7% for  $K^+ \to \pi^+ \nu \bar{\nu}$  and ~2% for  $K_L \to \pi^0 \nu \bar{\nu}$ , several quantaties look promising for search of the new physics. For example, the comparison of  $BR(K^+ \to \pi^+ \nu \bar{\nu})$  with the ratio  $\Delta m_d / \Delta m_s$  from  $B_d - B_s$  mixing and  $\sin 2\beta$  from  $K \to \pi \nu \bar{\nu}$  with  $\sin 2\beta$  obtained from  $B_d$  decays.

With two unobservable particles in the final state, these decays present very difficult experimental challenges, but their potential is so great that they are being quite actively pursued. It is still a long and rough road ahead to rich the SM level and then to make a precise measuremet of the  $K_L \to \pi^0 \nu \bar{\nu}$  decay. But situation is very different for  $K^+ \to \pi^+ \nu \bar{\nu}$  with new result obtained in E949 at BNL.

### 2 Experiment

The E787 experiment has presented the evidence for the  $K^+ \to \pi^+ \nu \bar{\nu}$  decay and found two clean events that gave  $BR(K^+ \to \pi^+ \nu \bar{\nu}) = (1.57 + 1.75 - 1.75)$   $0.82) \times 10^{-10}$  [7]. The E949 is based on various upgrades to the techniques and technology of E787 which included an addition to the barrel photon veto to increase the number of radiation lengths that provided better photon rejection by a factor of 2; more efficient photon detection along the beam direction; higher segmentation of beam tracking elements; improved pion energy and tracking resolution; improvement in trigger and DAQ to accept higher data rates. The E949 detector is shown in Fig. 3.

It consists of a solenoidal spectrometer situated at the end of a very intense low energy separated beam, an active target, a cylindrical drift chamber, an array of scintillators and photon detectors [6]. Charged decay products of stopped  $K^+$  are tracked in a 1T magnetic field through the target, a cylindrical drift chamber and into a cylindrical range stack of scintillators (RS) where they range out. Pions are identified by kinematic correlation  $(\frac{dE}{dx}/\text{total energy/range/momentum})$  and by observing their  $\pi^+ \to \mu^+ \to e^+$ decay chain in the stopping scintillators. Photons are detected in  $4\pi$  electromagnetic calorimeter consisting of a lead/scintillator sandwich barrel detector surrounding the RS, and endcups: pure CsI detectors, Pb/scintillator sanwich collar detectors and downstream photon veto detector (see Fig. 3). The entire apparatus serves as an hermetic veto for extra tracks. The E949 data were accumulated in 2002 with number of kaons stopped in scintillating active target of  $1.8 \times 10^{12}$ .

Definitive recognition of the  $K^+ \to \pi^+ \nu \bar{\nu}$  signal requires observation of  $\pi^+$  in the momentum region 211  $MeV/c (region I above the <math>K_{\pi 2}$  monoenergetic peak in Fig.4) . and no other observable concidence activity is present in the detector. Pions are idenfitied by measuring momentum (P), range (R) and energy (E) and by observation of the  $\pi^+ \to \mu^+ \to e^+$  decay chain. The main background sources from the  $K^+$  decay modes: pions from the decay  $K^+ \to \pi^+ \pi^0$  ( $K_{\pi 2}$ ) due to missed photons or mismeasured kinematics, muons from the decay  $K^+ \to \mu^+ \nu$  ( $K_{\mu 2}$ ) and other modes (see Fig. 4). Another backgrounds come from scattered beam pions due to misidentification of  $\pi^+$  as  $K^+$ , and  $K^+$  charge excange reaction followed by  $K_L^0 \to \pi^+ l^- \bar{\nu}_l$ , where l can be electron or muon. The effective muon rejection by a factor of 10<sup>5</sup> is provided using wafeform digitizer analysis of the  $\pi^+ \to \mu^+ \to e^+$  decay sequence in the RS. The suppression factor of about 10<sup>6</sup> is obtained for events with neutral pions.

The analysis was done using so-called blind approach in which the signal region was not examined while all backgrounds are estimated and selection criteria (cuts) are developed. Each background was suppressed by two groups



Figure 3: Side (a) and end (b) view of upper half of the E949 detector which has cylindrical symmetry.

of complementary but independent cuts. For  $K_{\pi 2}$  background, the cut pair included measurement of  $\pi^+$  kinematics and photon detection in  $\pi^0 \to \gamma \gamma$ . To have the unbiased cuts, the full data sent was randomly separated into 1/3



Figure 4: Momentum of charged particles from the major  $K^+$  decay modes and  $K^+ \to \pi^+ \nu \bar{\nu}$  with their branching ratios shown in parenthesis. I–the kinematic  $K \to \pi \nu \bar{\nu}$  region above  $K_{\pi 2}$  peak used in present analysis, II–the kinematic region of  $K \to \pi \nu \bar{\nu}$  below  $K_{\pi 2}$  peak. This region can be exploited in further analysis.

and 2/3 parts. The level of signal acceptance as a function of the cut values was determined using both experimental and Monte-Carlo data. This procedure provides correct estimation of the expected background and signal rates inside and outside of the signal region at different levels of signal acceptance and background rejection. Tests of backgrounds near but outside the signal region showed good consistency between simulated and observed numbers for  $K_{\pi 2}, K_{\mu 3}$  and  $K_{\mu 2\gamma}$  decays. The background estimates from these decays are  $0.216 \pm 0.023$  ( $K_{\pi 2}$ ) and  $0.068 \pm 0.011$  ( $K_{\mu 3} + K_{\mu 2\gamma}$ ). To obtain the branching ratio the parameter space of observables was divided into a set of discrete bins. Each bin *i* was characterized by the value of  $S_i/b_i$ , where the relative probability of an event in the bin to be a  $K_{\pi\nu\bar{\nu}}$  signal is  $S_i$  or background  $b_i$ . The likelihood technique [8] was used and details are presented in [9].

The final acceptance for  $K^+ \to \pi^+ \nu \bar{\nu}$  was estimated to be 0.0022±0.0002. This number is consistent with E787 acceptance as shown in Table 1, where

	E787		E949
Number of stopped $K^+$ 's	$5.9 \times 1$	$0^{12}$	$1.8 \times 10^{12}$
Total acceptance	$0.0020 \pm 0$	0.0002	$0.0022 \pm 0.0002$
Total background	$0.14\pm0.05$		$0.30\pm0.03$
Events	1	1	1
S/b ratio	50	7	0.9
Background probability	0.006	0.02	0.07

Table 1: Sensitivity, acceptance, background for the E787 and E949 data.

the sensitivity of E787 and E949 and the total background level are presented. As seen from this table, E949 achieved a sensitivity to a single  $K_{\pi\nu\nu}$  event comparable to E787, although the number of kaons is about 3 times less. Estimated background level is dominated by  $K_{\pi 2}$  and  $K_{\mu 2}$  decay modes. The signal box was opened after completion of acceptance and background study and one candidate was observed in the signal region. Three golden events (two from E787 and one from E949) are shown in Fig. 5. The E949 event shows all the characteristics of a  $K_{\pi\nu\nu}$  event although its high momentum and 6.3 ns of the  $\pi \to \mu$  decay time indicate a higher probability (0.07 as shown in Table 1) that the two E787 events that it was due to background, particularly  $K_{\mu 2}$  decay. The obtained value of branchning ratio  $BR(K^+ \to \pi^+\nu\bar{\nu}) = (1.47 + 1.30 - 0.89) \times 10^{-10}$  included the three observed events. This result is consitent with the SM expectation.

### 3 Conclusion

E949 observed one new candidate of  $K^+ \to \pi^+ \nu \bar{\nu}$ . The measured brachning ratio  $BR(K^+ \to \pi^+ \nu \bar{\nu}) = (1.47 + 1.30 - 0.89) \times 10^{-10}$  is consistent with the SM prediction, although the central value remains twice as high. To measure this *BR* with better accuracy it would be important to complete the E949 experimental program and achieve its design goal. The E949 sensitivity can



be also increased by using the kinematic region below the  $K_{\pi 2}$  peak (region II in Fig. 4) due to improved photon veto system.

There are new experimental initiatives to measure "golden" rare kaon decays. Several experiments have beed proposed to study  $K^+ \to \pi^+ \nu \bar{\nu}$  with the capability to detect from 50 to 100 events. Two in-flight experiments, the CKM at Fermilab [10] designed to reach the  $10^{-12}$ /event level, and the NA48/3 at CERN [11] are under consideration. Another stopped kaon experiment is proposed at J-PARC [12]. One can also expect new results in  $K_L \to \pi^0 \nu \bar{\nu}$  from the running E391a experiment at KEK [13] and the KOPIO experiment at BNL [14].

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