Review of measurements of the angles of the CKM matrix from the BABAR experiment

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Abstract

Experimental results on CP violation measurements in the B meson system from the BABAR experiment at the PEP-II asymmetric B Factory are reviewed.

1 Introduction

CP violation is elegantly incorporated in the Standard Model by a single non-vanishing phase in the unitary Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix $V_{\text{CKM}}$ [1]. A useful parameterization of the CKM matrix [2] stems from the observation that the couplings among quark families show a hierarchy in terms of the parameter $\lambda \equiv |V_{ub}/V_{cb}|$. Three other quantities: $A$, $\varphi$ and $\eta$, appear in this parameterization.

The unitarity of the CKM matrix can be represented in the complex plane by a triangle with angles conventionally called $\alpha$, $\beta$ and $\gamma$, and apex $(\varphi, \eta)$. Sides and angles of this unitarity triangle (UT) as measured from different processes should be the same if the underlying theory is valid. Figure 1 shows the constraints in the $\varphi-\eta$ plane obtained combining several such measurements, most of which come from B physics [3]. The measurement of the UT angles from time-dependent CP asymmetries in different $B$-meson decays is

![Figure 1: Constraints in the complex $\varphi-\eta$ plane.](image)

1In terms of the CKM matrix elements, the angles read: $\alpha = \arg(-V_{tb}^*V_{td}/V_{ub}^*V_{ud})$, $\beta = \arg(-V_{ub}^*V_{cd}/V_{ub}^*V_{ud})$, $\gamma = \arg(-V_{ub}^*V_{ud}/V_{cd}^*V_{cd})$. In the Wolfenstein parameterization, $V_{td} \simeq |V_{td}|e^{-i\beta}$, $V_{ud} \simeq |V_{ud}|e^{-i\gamma}$ (and $\alpha = \pi - \beta - \gamma$).
a central part of the experimental program of the BABAR and Belle experiments, operating at the SLAC and KEK asymmetric $B$ Factories. Such measurements are aimed at probing the validity of the Standard Model, precisely measuring the SM parameters, and possibly identifying patterns for new physics.

2 The BABAR experiment at PEP-II

The BABAR experiment [4] is a general-purpose detector, albeit asymmetric to match the asymmetry of PEP-II beams (see Section 3). From inside out, BABAR is composed by: a silicon detector for the precision measurement of decay vertices and a low-mass drift chamber to detect charged tracks, both immersed in a solenoidal magnetic field of 1.5 T; a CsI(Tl) electromagnetic calorimeter for the detection of photons and electrons; a ring-imaging Cherenkov detector for charged particle identification up to 4 GeV/c (the measurement of $dE/dx$ in the tracking system is also used at lower momenta); a muon and $K^0_L$ detection system made by resistive plate counters positioned in the segmented iron of the magnet return yoke.

The results shown in the following sections have been made possible by the high quality of the data collected by the BABAR detector, but also by the exceptionally good performance of PEP-II, whose instantaneous luminosity exceeded $9.2 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$, i.e. more than three times the design value, and has further increased the integrated luminosity with the almost continuous ‘trickle’ injection technique, yet keeping under control the background levels in the detector. At the moment of writing the present paper, BABAR has collected an integrated luminosity of $232 \text{ fb}^{-1}$ (of which $22 \text{ fb}^{-1}$ below the $\Upsilon(4S)$ peak). Many new and updated results based on the full BABAR dataset have been presented at the Summer conferences [5]; following an explicit request from the Organizing Committee of the Quarks 2004 seminar however, only results presented in my original talk will be quoted in the present paper: they refer to smaller datasets of 88 or $122 \text{ fb}^{-1}$, depending on the analysis.

3 Experimental considerations

$B$ mesons from the decay of the $\Upsilon(4S)$ resonance are produced in a $J^{PC} = 1^{--}$ state (neutral $B$ pairs evolve coherently after production). The innova-
tive concept distinguishing PEP-II from previous first-generation B Factories is the $\Upsilon(4S)$ boost in the laboratory system, obtained with asymmetric beam energies (3.1 GeV for the $e^+$ beam and 9.0 GeV for the $e^-$ beam); such boost separates the $B$-meson production vertices by $\approx 260 \, \mu m$ on average, and for the first time allowed to study the time evolution of the system at the $B$-meson production threshold.

The study of time-dependent asymmetries in $\text{BaBar}$ proceeds through techniques which are common to the measurement of all three UT angles described in the following sections. First the ‘$CP$’ final state is reconstructed, and its production vertex measured; to identify the signal events we take advantage of two nearly uncorrelated variables, the energy difference $\Delta E \equiv E^*_B - E^*_{\text{beam}}$ and the ‘energy substituted mass’ $m_{ES} \equiv \sqrt{E^2_{\text{beam}} - p^2_B}$, which in signal events accumulate at zero and at the $B^0$ mass, respectively $^2$. Once the ‘$B\text{reco}$’ meson has been reconstructed, all remaining tracks in the event are used to build the ‘tag’ meson, $B\text{tag}$. We exploit the correlation of the charge of the $B\text{tag}$ decay products, mainly for leptons and kaons, to infer the flavour ($B^0$ or $\bar{B}^0$) of $B\text{tag}$ at the moment of its decay. Typical effective tag efficiencies (the relevant quantity is $Q \equiv \varepsilon\text{tag}(1 - 2w)^2$, where $\varepsilon\text{tag}$ is the tagging efficiency, and $w$ the mistag probability) are around 30%$^3$. The measured decay vertices are used to determine the proper time separation between the two $B$ decays, $\Delta t \equiv (z_{\text{reco}} - z_{\text{tag}})/(\beta\gamma c)$, where $\beta\gamma = 0.56$ is the $\Upsilon(4S)$ boost. The time-dependent decay rate to a given final state $f$ is:

$$N^+_f(\Delta t) = \frac{e^{-|\Delta t|/\tau_{\text{B}^0}}}{4\tau_{\text{B}^0}} [1 \pm S_f \sin(\Delta m_d \Delta t) \mp C_f \cos(\Delta m_d \Delta t)], \quad (1)$$

where the + (−) sign applies for a $B^0$ ($\bar{B}^0$) tag, $\Delta m_d$ is the $B^0\bar{B}^0$ mixing frequency and $\tau_{\text{B}^0}$ the $B^0$ lifetime. The $S_f$ coefficient describes $CP$ violation in the interference between decay and mixing, while the cosine coefficient, $C_f$, is responsible for $CP$ violation in the decay, and necessitates of at least two competing amplitudes with different phases in the decay.

$^2$ $E^*_B$ and $p^*_B$ are the reconstructed $B$ energy and momentum in the Center of Mass (CM) system. The constraint of the beam energy $E^*_{\text{beam}}$, known with much better precision than the reconstructed $B$ energy, is used to improve the precision on $m_{ES}$.

$^3$ The tagging techniques have steadily improved in $\text{BaBar}$ over the years: $Q$, which can be measured directly from the data in channels where the flavour is auto-tagged, such as e. g. $B^0 \to D^{*-}\pi^+$, was 28.5% in 2002 and is 30.5% in 2004.
4 The angle $\beta$

The interference between $B^0$ decays to $CP$ eigenstates with and without $B^0\bar{B}^0$ mixing gives rise to $CP$ asymmetries in $\Delta t$ distributions from which $\sin 2\beta$ can be measured.

4.1 Charmonium modes

The measurement of $\sin 2\beta$ in charmonium modes, the gold-plated $B^0 \rightarrow J/\psi K^0_S$ in primis, conclusively established $CP$ violation in the $B$ sector in year 2002 [6], really becoming a precision measurement. Figure 2 is based on a sample of about 88 million $B\bar{B}$ pairs [6], where the $B_{\text{reco}}$ is fully reconstructed in $CP$-odd ($\eta_f = -1$) modes ($B^0 \rightarrow J/\psi K^0_S$, $\psi(2S)K^0_S$, $\chi_{c1}K^0_S$, $\eta\eta K^0_S$, $J/\psi K^0\pi$) in $B^0 \rightarrow J/\psi K^0_{L\bar{L}}$, which has $\eta_f = +1$, and in $J/\psi K^{*0}$, which has both $CP$-odd and $CP$-even components. The figure shows the distribution of $m_{ES}$ for the $CP$-odd states and $J/\psi K^{*0}$, and of $\Delta E$ for $B^0 \rightarrow J/\psi K^0_{L\bar{L}}$. The latter is the second most abundant $CP$ eigentstate after $J/\psi K^0_S$, albeit with a higher background level, due to the fact that only the direction but not the energy

![Figure 2: a) $m_{ES}$ distribution in $CP$-odd charmonium states; b) $\Delta E$ in $J/\psi K^0_S$; c) $\Delta t$ distribution for $B^0$ and $\bar{B}^0$-tagged events with $CP$-odd final states; and d) projection of the corresponding asymmetry; e) $\Delta t$ for $B^0$ and $\bar{B}^0$-tagged $J/\psi K^0_L$ events; and f) projection of the corresponding asymmetry.](image-url)
of $K_L^0$'s can be measured in the detector. On the right-hand side of the same figure the time evolution as well as the raw $CP$ asymmetries are shown for events selected in the signal regions. The average over the quoted channels is $\sin^2\beta = 0.741 \pm 0.067$ (stat) $\pm 0.034$ (syst). It should be noted that the dominant source of uncertainty is still statistical.

This measurement is perfectly consistent with the SM prediction obtained from measurements of the sides of the UT triangle (mainly $|V_{ub}|$, $|V_{cb}|$, $\varepsilon_K$, $\Delta m_d$, $\Delta m_s$): $\sin^2\beta = 0.676 \pm 0.090$ [7] (see again Figure 1), and thus represents a first important precision test of the SM in the CKM sector.

As expected, since as mentioned only one amplitude is largely dominant in these decay channels, the measured $C_f$ coefficient is consistent with 0.

### 4.2 Penguin dominated modes

Comparing $\sin^2\beta$ measured in decay channels involving $b \to s\bar{s}s$ quark transitions with the one precisely measured in the charmonium $b \to \phi K_s^0$ modes is part of the $B$-Factory program. Some of such modes, as $B^0 \to \phi K_s^0$, are particularly interesting since they are expected to proceed to a good approximation through a single dominant amplitude and therefore – analogously to the golden charmonium modes – to be relatively clean from the theoretical point of view. In practice, this ‘single amplitude dominance’ is not guaranteed to hold equally well for all modes; in [8] estimates are performed, based on different assumptions, that $|\sin^2\beta_{b \to s} - \sin^2\beta| \lesssim 0.05 \div 0.3$. Another attractive feature of these $b \to s$ decays is that they are dominated by penguin diagrams, where effects of new physics from e. g. heavy, unknown particles circulating in the virtual loops are in principle easier to discover. The price to pay is a much smaller branching ratio\(^5\), as well as a considerably less clean signature; these modes are generally plagued by large contaminations from continuum events, only partially tamed by using dedicated rejection techniques. In spite of all this, clear signal peaks are observed, even in the most challenging modes involving $K_L^0$'s (see Figure 3). The analysis [9] is based on 114 million $B\bar{B}$ pairs, and finds $S_{\phi K^0} = 0.47 \pm 0.34^{+0.08}_{-0.07}$, fully compatible with the charmonium result, and $C_{\phi K^0} = 0.01 \pm 0.33 \pm 0.10$, as expected if a single amplitude dominates the decay. Other interesting final states which

\(^4\)Throughout this paper, unless otherwise specified, the first quoted error is statistical and the second systematic.

\(^5\)Compare e. g. $B'$, the total branching fractions including the intermediate decay modes: $B'(B^0 \to J/\psi K_s^0) \simeq 36.0 \times 10^{-6}$, $B'(B^0 \to \phi K_s^0) \simeq 1.4 \times 10^{-6}$. 

5
\section*{Figure 3:} a) $m_{ES}$ distribution of selected $B^0 \rightarrow \phi K^0$ candidates; b) $\Delta E$ distribution of selected $B^0 \rightarrow \phi K^0$ candidates.

$BABAR$ has studied include $K^+K^-K^0_S$, $f_0K^0_S$, $\eta'K^0_S$ and $\pi^0K^0_S$ (in the latter a novel technique was pioneered to reconstruct the decay vertex of this all-neutral final state, extrapolating the $K^0_S$ decay vertex back to the beam spot region in the transverse plane).

A compilation of these results, also including measurements from Belle, is presented in Figure 4. Although a hint of a trend is observed that penguin modes give on average smaller $S_f$ values than charmonium modes, at present one should be very cautious in drawing conclusions, since the information conveyed by these measurement of the effective $\sin^2\beta$ depends on how important the sub-dominant diagrams are. While more theoretical work is needed to assess this point, it is clearly important to improve the measurements, some of which still have rather large experimental errors.

\begin{figure} [H]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{Summary of $S_f$ measurements from $\sin^2\beta$.}
\end{figure}
5 The angle $\alpha$

The CKM angle $\alpha = \pi - (\beta + \gamma)$ can be accessed in charmless $B^0 \rightarrow h^+ h^-$ decays, where $h = \pi$ or $\rho$, through the interference of $b \rightarrow u \pi d$ transitions (which introduce a phase $\gamma$ in the amplitude), and $B^0 \overline{B}^0$ oscillations (which contribute with a phase $\beta$). The connection between measured parameters in the time-dependent asymmetry and CKM matrix elements is however not straightforward: since in these decays the penguin diagram is expected to contribute significantly, the process is not dominated by a single amplitude, and the measured phase can be shifted with respect to the simple picture. In the general case we therefore expect direct $CP$ violation, $C_{hh} \neq 0$, and $S_{hh} = \sqrt{1 - C_{hh}^2 \sin^2 \alpha_{eff}}$. In order to relate the measured $\sin 2\alpha_{eff}$ with $\sin 2\alpha$, a bound due to Grossman and Quinn [10] on the shift induced by the penguin contribution can be used: $\sin^2(\alpha_{eff} - \alpha) \leq B(B^0 \rightarrow h^0 h^0)/B(B^\pm \rightarrow h^0 h^\pm)$. This is a somewhat simplified version of triangular relations which can be constructed assuming SU(2) symmetry [11], but which require measuring tagged branching fractions to five different final states ($h^+ h^-$, $h^0 h^0$, $h^\pm h^0$, both from $B$ and $\overline{B}$ decays), and are therefore difficult experimentally and still statistically limited.

5.1 $B \rightarrow \pi\pi$

The time-dependent $CP$ asymmetry in $B^0 \rightarrow \pi^+ \pi^-$ has been measured by $BABAR$ using 123 million $B\overline{B}$ pairs. Charmless decay are experimentally challenging given the small branching fractions and the contamination both from $B$ and continuum decays, addressed in this analysis combining several event-shape variables into a Fisher discriminant. Our preliminary measurement is [12]: $S_{\pi\pi} = -0.40 \pm 0.22 \pm 0.03$, $C_{\pi\pi} = -0.19 \pm 0.19 \pm 0.05^{6}$. $BABAR$ has made the first measurement of $B(B^0 \rightarrow \pi^0 \pi^0) = (2.1 \pm 0.6 \pm 0.3) \times 10^{-6}$ [14]. This result unfortunately gives a rather weak Grossman-Quinn bound: $|\alpha_{eff} - \alpha| < 45^\circ$ at 90% CL.

\footnote{It is worth to remark that this measurement contradicts the results from the Belle collaboration [13] both of large indirect $CP$ violation: $S_{\pi\pi} = -1.00 \pm 0.21 \pm 0.07$, and of direct $CP$ violation: $C_{\pi\pi} = -0.58 \pm 0.15 \pm 0.07$.}
5.2 \(B \to \rho \rho\)

This process has the same quark content, and larger branching fraction of \(B \to \pi \pi\), but is a vector-vector final state, and can in general proceed through S, P and D waves, being therefore a superposition of longitudinally (\(CP\)-even) and transversely polarized states (which are not \(CP\) eigenstates).

\(B^{+}\) has recently measured the fraction of longitudinal polarization in the decay \([15]\), finding \(f_{pol}^{\rho^{+}\rho^{-}} = (99 \pm 3^{+4}_{-7})\%\). This decay is therefore essentially completely \(CP\)-even, a fortunate simplification. From the measured \(B \to \rho \rho\) branching fractions\(^7\), the Grossman-Quinn bound turns out to be quite effective for \(\rho^{+}\rho^{-}\): \(|\alpha_{eff} - \alpha| < 13^\circ\) at 68\% CL. Given the measured values: \(S_{\rho\rho} = -0.19 \pm 0.33 \pm 0.11, C_{\rho\rho} = -0.23 \pm 0.24 \pm 0.14\), this allows, neglecting interference with \(I = 1\) transition amplitudes and possible contribution from non resonant or \(\rho \pi \pi\), \(4\pi\) and \(a_1 \pi\) final states, to finally extract a measurement of \(\alpha = [102^{+16}_{-12} + 5 \pm 13(\text{penguin})]^\circ\). The constraints on \(\alpha\) from combined \(B^{+}\) and Belle measurements are presented in Figure 5; we observe that the \(\rho \rho\) system provides the most stringent constraint on \(\alpha\), and that such constraint is consistent with the one obtained from the CKM fit not including the \(B^{0} \to h^{+}h^{-}\) measurements.

6 The angle \(\gamma\)

The phase \(\gamma\) can be measured in the interference between allowed \(b \to c\) and suppressed \(b \to u\) transitions. In all cases the size of the asymmetry is driven by the parameter \(r = |A(B^- \to \bar{D}^{0}K^-)/A(B^- \to D^{0}K^-)|\), the ratio of the suppressed and the allowed decay amplitudes.

\(^7\)B(\(B^{0} \to \rho^{0}\rho^{0}\)) < 2.1 \times 10^{-6}, \ B(\(B^{\pm} \to \rho^{0}\rho^{\pm}\)) = (26.4 \pm 6.4) \times 10^{-6}.
6.1 $B \to D K$ decays

The amplitudes for the $B^- \to \overline{D}^0 K^-$ and $B^- \to D^0 K^-$ can interfere if $D^0$ and $\overline{D}^0$ decay to the same final state. Two main strategies exist: $D^0$ decaying to $CP$ eigenstates such as $\pi^+\pi^-$ or $K^+K^-$, proposed by Gronau, London and Wyler (GLW) [16], and the Atwood-Dunietz-Soni (ADS) method [17], where the total decay amplitudes are ‘equalized’. The idea is to select decays with either $B^- \to D^0 K^- (b \to c)$ followed by $D^0 \to K^+\pi^-$ (doubly Cabibbo-suppressed) transitions, or $B^- \to \overline{D}^0 K^- (b \to u, \text{color-suppressed})$ followed by $\overline{D}^0 \to K^+\pi^-$ (Cabibbo-allowed) transitions, thus maximising the expected asymmetry. We then extract $\gamma$ directly from decay rates measurements: $R_{K\pi} = \mathcal{B}_s/\mathcal{B}_f = r_D^2 + r_B^2 + 2r_D r_B \cos \gamma \cos \delta$, where the suppressed branching fraction $B_s \equiv \mathcal{B}(B^- \to [K^-\pi^+]K^-) + \mathcal{B}(B^+ \to [K^+\pi^-]K^+)$, and the favoured one is $B_f \equiv [\mathcal{B}(B^- \to [K^+\pi^-]K^-) + \mathcal{B}(B^+ \to [K^+\pi^-]K^+)]$. The suppressed/favoured ratio for the $D^0$ decay amplitudes $r_D$ is measured, while the corresponding ratio $r_B \simeq 0.1 \div 0.3$ is not precisely known. Note that the result also depends on the strong phase difference $\delta$. We measure $R_{K\pi} = 0.004 \pm 0.012 (< 0.025 \text{ at } 90\% \text{ CL})$, and use this measurement to calculate the upper limit $r_B < 0.20 (< 0.22 \text{ at } 90\% \text{ CL using (not using)}$ the constraint $48^\circ < \gamma < 73^\circ$ suggested by the global fit [3]. This low value of $r_B$ is certainly not favourable for the measurement of $\gamma$ with this method.

6.2 $B \to D^{(*)}\pi$ decays

The decay $B^0 \to D^{(*)}\pi^+$ can proceed either through a favoured $(b \to c)$ transition, or through a $B^0 \to \overline{D}^0$ oscillation (with phase $2\beta$ respect to the direct case), followed by the doubly Cabibbo-suppressed $\overline{D}^0 \to D^{(*)}\pi^+$ decay (bringing in an additional $\gamma$ phase shift). This method therefore measures $\sin(2 \beta + \gamma)$. Defining the ratio of the competing amplitudes by $r_{(s)}^{\phi(\gamma)} \equiv A(\overline{D}^0 \to D^{(*)}\pi^+)/A(B^0 \to D^{(*)}\pi^+)$, the sine and cosine coefficients of eqn. 1 read therefore: $C^{(*)} \simeq 1$, $S^{(*)} \simeq 2 r_{(s)}^{\phi(\gamma)} \sin(2 \beta + \gamma \pm \delta^{(*)})$.

$BABAR$ has measured $\sin(2 \beta + \gamma)$ in $B^0 \to D^{(*)}\pi^+$ decays using both fully and partially reconstructed $B$ decays. The partial reconstruction technique uses only the information from the high-momentum pion $\pi_h$ from the $B$ decay and from the slow-momentum pion $\pi_s$ from the decay $D^{(*)} \to \overline{D}^0 \pi^-$.

\footnote{Terms of order $r_{(s)}^2$ have been neglected for simplicity. This is justified since (unfortunately for the measurement) $r_{(s)}$ is expected to be small, of the order of 2%, due to the double Cabibbo-suppression.}
without explicitly reconstructing the \( \bar{D}^0 \). The four momentum of the ‘missing’ \( \bar{D}^0 \) is calculated applying kinematic constraints, and the mass recoiling against the \( \pi_h - \pi_s \) system in the hypothesis of a \( B^0 \rightarrow D^{*-} \pi^+ \) decay can be used to identify the signal events. The advantage of the method, only in part spoiled by the augmented background level, is the high reconstruction efficiency, particularly important in this analysis where a small \( CP \) asymmetry is expected. With this technique \( \text{BABAR} \) measures [18]:

\[
a^* \equiv (S^{++} + S^{*-})/2 \simeq 2r_* \sin(2\beta + \gamma) \cos \delta^* = -0.063 \pm 0.024 \pm 0.014,\]

which deviates from zero by 2.3\( \sigma \). The measurement is also performed using full \( B^0 \) reconstruction, both in \( B^0 \rightarrow D^* \pi \) and \( B^0 \rightarrow D \pi \) decays. Using SU(3) to estimate \( r \) and \( r_* \) from the measured ratio \( B(B^0 \rightarrow D_i(\pi^*)^+)/B(B^0 \rightarrow D^{(\pi^*)+}) \) yields the constraints \(|\sin(2\beta + \gamma)| > 0.87 \) (0.58) at 68\% (95\%) CL from the combination of the full and partial reconstruction measurements.

7 Conclusions

The PEP-II asymmetric \( B \) Factory has been operating successfully since 1999, and \( \text{BABAR} \) has produced many interesting results on the CKM phases with the data collected. \( CP \) violation has been firmly established in the \( B \) sector, and the measurements confirm the validity of the SM, where the origin of \( CP \) violation is only in the CKM complex phase. The focus is now moving on making precision measurements in the sin2\( \beta \) sector. The first interesting constraints have been placed on the phase \( \alpha \), and an intense activity is being carried out to measure \( \gamma \) at the \( B \) Factories, while up to recent times this angle was thought to be measurable only at hadronic facilities.

References


