Lepton MHD and mean magnetic field generation by $\alpha-$ effect driven by neutrinos in early universe plasma

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1 Introduction

The main problem of primordial magnetic field generation is an inconsistency of their values B and correlation lengths L_0 obtained in different scenarios. There are many ways how to generate small-scale random magnetic fields with large values of $B_{rms} = \sqrt{< B^2 >}$, e.g. using some causal mechanisms like bubble collisions at phase transitions, while the correlation length of such magnetic fields evolved (via inverse cascade) during the expansion of universe into large-scale magnetic fields turns out to be too small at present time, $L_0 \sim tens\ parsecs$, to reach the size $L_0 \sim 100\ kps$ for galactic magnetic field, or even more ($\gtrsim$ Mps) for extragalactic magnetic fields. The other way

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using the inflation scenario allows, vice versa, to get large-scale (a few Mps) magnetic fields while their strength occurs too small for observable magnetic fields.

We suggest a new mechanism [1] for amplification of large-scale magnetic fields in the hot plasma of early universe which is based on the *parity violation* in weak interactions. Such large-scale (mean) magnetic field is originated from the small-scale ones generated before in phase transitions and we explore how this field evolves during the cooling of universe in the presence of powerful neutrino fluxes. The mechanism suggested in [1] can result in a self-excitation of an (almost) *uniform* cosmological magnetic field that solves the problem of a large-scale field seeding galactic magnetic fields, or surely such mean field survives the recombination time. On the other hand, there appears the remarkable property of the cosmological magnetic field $\mathbf{B}$ amplified through weak interactions: in addition to the usual axial vector part, obeying the transformation $\mathbf{B}_1 = \mathrm{P}\mathbf{B}_1\mathrm{P}^{-1}$ under spatial inversion, it gets a pure vector component violating parity $\mathbf{B}_2 = -\mathrm{P}\mathbf{B}_2\mathrm{P}^{-1}$ in the macroscopic magnetohydrodynamics (MHD).

There remains a problem how to observe these *"wrong"* magnetic fields which are pure vectors like electric fields in QED. They are produced by the collective neutrino interaction with charged leptons populating the main Landau level in the field $\mathbf{B}_1$ and their observation would be sensitive to the relic neutrino density asymmetry $\delta n_\nu = n_\nu - n_{\bar{\nu}}$ (see below Eq. 6).

It is well-known that many astrophysical objects are magnetized, in particular, planets (a few Gauss), stars ($\sim 10^{12}$ Gauss for neutron stars), spiral galaxies, clasters of galaxies ($\sim \mu G = 10^{-6}\ Gs$). Large-scale fields have scales exceeding $L \geq 1$ AU=1.49×10^{13} cm, even $L \geq 1\ pc = 3\times10^{18}$ cm.

The observation of galactic (intergalactic) and stellar magnetic fields started many years ago being initiated by the well-known theoretical predictions. First, in 1943 Alfvén showed that magnetic fields survive in a highly conducting plasma. Then in 1949 assuming that galactic magnetic fields are the primordial ones, Fermi estimated the strength of galactic magnetic fields, $B_{gal} \sim 10^{-6}\ Gs$, which keep (tangling) observable cosmic rays. This estimate comes immediately from the energy equilibrium of cosmic rays with magnetic fields, $w_{cr} \sim B^2_{gal}/8\pi \sim 1\ eV/cm^3$. Simultaneously (1949) Hiltner and independently Hall observed polarization of starlight- an effect of galactic magnetic field aligning the dust grains.

Let us recall the experimental ways for a measurement of the usual (axial vector) magnetic fields $\mathbf{B}_1$. There are three ways. Unfortunately, the Doppler broadening embarrasses the simplest way to measure the longitudinal component causing the Zeeman splitting of spectral lines (neutral hydrogen, $f = 1420\ \ MHz$, $\lambda = 21\ cm$):

$$\Delta\nu_{Zeeman} = \frac{\mu_B B_\parallel}{\pi} \sim 3\ \ Hz \ll \Delta\nu_{Doppler} = \frac{v_T}{c}\nu \sim 30\ \ kHz.$$

The second way, the observation of synchrotron emission gives ability to estimate the radio emission intensity for the typical size of the source L : $(B_{tot,\perp})^{(\alpha+1)/2}W_0L$, then the transversal magnetic field can be found from the electron distribution: $n_edE = n_0E^{-\alpha}dE$ for which the index α should be defined somehow.

First, this way was used for synchrotron emission of Crab nebula (in 50-60-th) (measurement of $B_\perp$ in region $\nu \sim GHz$) where the intensity maximum is given by the frequency,

$$\frac{\nu_{\max}}{MHz} \simeq 15\left(\frac{B_{tot,\perp}}{\mu G}\right)\left(\frac{E_e}{GeV}\right)^2,$$

which again depends on the parent electron energy E_e.

The most popular way to observe astrophysical magnetic fields is the measurement of *Faradey rotation of polarization plane.*

Lyne, Smith suggested in 1968: the knowledge of the column of electron density along light ray (from pulsars), called dispersion measure $DM \propto \int n_e dl$, allows to find the longitudinal component $B_\parallel$ from the Faradey rotation of polarization plane $\phi = \phi_0 + RM \times \lambda^2$, in dependence on the wave length λ, where $RM = \Delta\phi/\Delta\lambda^2 \propto DM \times B_\parallel$:

$$RM = 0.8119\int\left(\frac{B_\parallel}{\mu G}\right)\left(\frac{n_e}{cm^{-3}}\right)d\left(\frac{l}{pc}\right) \qquad \frac{rad}{m^2}$$

In the next section we recall how the usual dynamo mechanisms look in standard MHD without neutrinos. Then in section 3 we generalize Faradey equation in SM with neutrinos and in section 4 we discuss the evolution of mean magnetic fields in hot plasma of early universe. In section 5 we compare the new collective mechanism of the mean magnetic field generation with the collision one suggested in [2]. In section 6 we resume our results.

2 Two dynamo mechanisms of magnetic field amplification

¿From Maxwell equation $\partial \mathbf{H}/\partial t = -\nabla \times \mathbf{E}$ and Ohm's law $\mathbf{E} = -\mathbf{v} \times \mathbf{H} + \mathbf{j}/\sigma = -\mathbf{v} \times \mathbf{H} + (\nabla \times \mathbf{H})/4\pi\sigma$ one obtains Faradey equation ($\nu_m = (4\pi\sigma_{cond})^{-1}$)

$$\frac{\partial \mathbf{H}}{\partial t} = \nabla \times \mathbf{v} \times \mathbf{H} + \nu_m \Delta \mathbf{H}$$

for the total magnetic field $\mathbf{H} = \mathbf{B} + \mathbf{b}$ and velocity field $\mathbf{v} = \mathbf{V} + \mathbf{u}$, which include small-scale (random) components $< \mathbf{b} > = < \mathbf{u} > = 0$.

Krause and Rädler [3] define the ponderomotive force as the mean one $\vec{\varepsilon} = < \mathbf{u} \times \mathbf{b} > = \nu_{turb} \nabla \times \mathbf{B} - \alpha \mathbf{B}$ where the second term is stipulated by the fluid vortices in a turbulent medium, that leads to the evolution equation for large-scale magnetic field ($\alpha\Omega-$ dynamo):

$$\begin{array}{l} \frac{\partial \Omega}{\partial r}, \frac{\partial \Omega}{\partial \theta} \text{ (Parker)} \quad +\eta \nabla^2 \mathbf{B} \\ \uparrow\uparrow\uparrow\uparrow\uparrow\uparrow \qquad\qquad\qquad \nearrow\nearrow\nearrow \end{array}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{V} \times \mathbf{B} - \nabla \times \nu_{turb} \nabla \times \mathbf{B} + \nabla \times \alpha \mathbf{B} \tag{1}$$

Notice that hydrodynamic diffusion is much bigger than the microscopic one caused by the plasma conductivity, $\nu_{turb} \gg \nu_m$. The first term in the r.h.s. of Eq. (1) is called dynamo term which depends on the differential rotation of a medium and vanishes, e.g. for rigid rotation of a magnetized body. Namely, the differential rotation plus α-effect given by last term in (1) mean the standard $\alpha\Omega$-dynamo mechanism. Nevertheless, even in the absence of such rotation the existence of α-term can lead to so-called α^2-dynamo, or to the α-effect (compare below Eq. (8). **Important**: the hydrodynamic helicity $\alpha = -\tau < \mathbf{u} \cdot (\nabla \times \mathbf{u}) > /3-$ is the *pseudoscalar* in standard MHD (without neutrino!), or obviously, Eq. (1) preserves parity as it should be in QED. The detailed analysis of the dynamo mechanisms in macroscopic electrodynamics is done in the book [4].

3 MHD in polarized medium and parity non-conservation

We refer here to the work [5] where MHD derived from the kinetic equations [6] is generalized in SM of particle physics both for an isotropic plasma and a magnetized medium. The Euler equation for $\sigma-$component of plasma (accounting for collisions in $\tau-$approximation) takes the form:

$$\begin{aligned}\frac{\mathrm{d}\mathbf{P}_\sigma}{\mathrm{dt}} = & -\nu_\sigma^{em}\delta\mathbf{P}_\sigma - (\nu_{\sigma\nu} + \nu_{\sigma\bar{\nu}})\mathbf{P}_\sigma - \frac{\nabla p_\sigma}{n_\sigma} + \\ + & \ q_\sigma(\mathbf{E} + [\mathbf{V}_\sigma \times \mathbf{B}]) + \mathbf{f}_\sigma^{weak},\end{aligned} \tag{2}$$

where weak force includes two terms $\mathbf{f}_\sigma^{weak} = \mathbf{f}_\sigma^{(V)} + \mathbf{f}_\sigma^{(A)}$:
the weak vector force $\mathbf{f}_\sigma^{(V)}$ derived by another way (Lagrangian method) in [7]

$$\begin{aligned}\mathbf{f}_\sigma^{(V)} = -(sign\ \sigma)G_F\sqrt{2}\sum_{\nu_a} c_V^a\Big[-\nabla\delta n_{\nu_a}(\mathbf{x},t) - \\ -\frac{\partial\delta\mathbf{j}_{\nu_a}(\mathbf{x},t)}{\partial t} + \mathbf{V}_\sigma \times \nabla \times \delta\mathbf{j}_{\nu_a}(\mathbf{x},t)\Big],\end{aligned} \tag{3}$$

and the weak axial vector force $\mathbf{f}_\sigma^{(A)}$ [5] appearing in magnetized plasma:

$$\begin{aligned}\mathbf{f}_\sigma^{(A)} = \frac{G_F\sqrt{2}\delta_{\sigma e}(sign\ \sigma)}{n_\sigma}\sum_{a=e,\mu,\tau} c_{\sigma\nu_a}^{(A)}\Big[n_{0\sigma}\widehat{\mathbf{b}}\frac{\partial\delta n_{\nu_a}}{\partial t} + \\ + N_{0\sigma}\nabla(\widehat{\mathbf{b}}\cdot\delta\mathbf{j}_{\nu_a})\Big];\quad \mathrm{ort}\ \widehat{\mathbf{b}} = \mathbf{B}/\mathrm{B}.\end{aligned} \tag{4}$$

Here $\delta j^\mu_{\nu_a} = (\delta n_{\nu_a}, \delta\mathbf{j}_{\nu_a}) = j^\mu_{\nu_a}(\mathbf{x},t) - j^\mu_{\bar{\nu}_a}(\mathbf{x},t)$ is the neutrino four-current density asymmetry, $\mu = 0,1,2,3$; $\mathbf{B} = \mathbf{B}_1$ is the usual (axial-vector) large-scale magnetic field; $n_{0\sigma} =\mid e \mid BT\ln 2/2\pi^2$ is the charged lepton density at the main Landau level; $N_{0\sigma}$ is the relativistic correction [5] to this density, $N_{0\sigma} \to n_{0\sigma}$ in non-relativistic plasma; $c_V^a = 2\xi \pm 0.5$, $c^{(A)}_{\sigma\nu_a} = \mp 0.5$ are vector and axial vector couplings in SM (upper sign for electron neutrinos), $\xi = \sin^2\theta_W = 0.23$ is the Weinberg parameter.

Multiplying Euler equation by the electric charge q_σ, summing over σ and dividing by $Q^2 = \sum_\sigma q_\sigma^2$, we find the electric field $\mathbf{E} = -\sum_\sigma(q_\sigma^2/Q^2)\mathbf{V}_\sigma \times \mathbf{B} +$

..., from where using Maxwell equation $\partial_t \mathbf{B} = -\nabla \times \mathbf{E}$ one obtains Faradey equation for evolution of MEAN (large-scale) magnetic field in SM:

$$\partial_t \mathbf{B} = \nabla \times \alpha \mathbf{B} + \eta \nabla^2 \mathbf{B} \tag{5}$$

Here diffusion coefficient $\eta = (4\pi 137\ T)^{-1}$ - is given by the conductivity of relativistic plasma.

Let us neglect the neutrino vorticity, $\nabla \times \delta \mathbf{j}_\nu = 0$, (neutrino gas rotation is absent), or weak vector force (3) does not contribute to (5). From the axial-vector weak force given by Eq. (4) that implies **PARITY VIOLATION!!!** using $\alpha\delta_{ij}$-term and without any action of Coriolis force that leads in standard MHD to mirror asymmetry of left-handed and right-handed fluid vortices, in other words, **NO FLUID ROTATION!!!** we derive the *scalar* α-parameter entering Faradey equation (5):

$$\alpha = \frac{G_F}{2\sqrt{2} \mid e \mid B} \sum_a c^{(A)}_{e\nu_a} \left[\left(\frac{n_{0-} + n_{0+}}{n_e} \right) \frac{\partial \delta n_{\nu_a}}{\partial t} \right] \simeq$$

$$\simeq \frac{\ln 2}{4\sqrt{2}\pi^2} \left(\frac{10^{-5} T}{m_p^2 \lambda^{(\nu)}_{\text{fluid}}} \right) \left(\frac{\delta n_\nu}{n_\nu} \right) \tag{6}$$

Here $\lambda^{(\nu)}_{\text{fluid}}$ is the scale of neutrino gas inhomogeneity.

For small neutrino chemical potentials $\xi_{\nu_a}(T) = \mu_{\nu_a}(T)/T \ll 1$, the neutrino density asymmetry entering (6) is of the form

$$\frac{\delta n_\nu}{n_\nu} = \sum_a c^{(A)}_{e\nu_a} \frac{\delta n_{\nu_a}}{n_{\nu_a}} = \frac{2\pi^2}{9\zeta(3)} [\xi_{\nu_\mu}(T) + \xi_{\nu_\tau}(T) - \xi_{\nu_e}(T)],$$

where one can use the bound on the electron neutrino chemical potential coming from the BBN constraints, $\mid \xi_{\nu_e} \mid \lesssim 0.07$ [8].

4 Amplification of large-scale magnetic fields in early universe

The spatial scale of the mean magnetic field obeying the evolution equation (5) $\Lambda = \eta/\alpha$ is given by [1]:

$$\frac{\Lambda}{l_H} = 1.6 \times 10^9 \left(\frac{T}{\text{MeV}} \right)^{-5} \left(\frac{\lambda^{(\nu)}_{\text{fluid}}}{l_\nu(T)} \right) \mid \xi_{\nu_e}(T) \mid^{-1} \tag{7}$$

The growth rate $\alpha^2/4\eta$ defines the mean magnetic field amplitude

$$B(t) = B_{\max}\exp\left(\int_{t_{\max}}^{t}\frac{\alpha^2(t')}{4\eta(t')}dt'\right). \tag{8}$$

Accounting for (6), the expansion time $t = 2.4\ sec\ (T/MeV)^{-2}/\sqrt{g^*}$ and $(T/2\times 10^4\ \mathrm{MeV}) \to \mathrm{x} < \mathrm{x}_{\max} = 1$, such amplitude takes the form

$$B(x) = B_{\max}\exp\left[25\int_{x}^{1}\left(\frac{\xi_{\nu_e}(x')}{0.07}\right)^2 x'^{10}dx'\right] \tag{9}$$

Thus, during the cooling of universe the scale of mean magnetic field (7) overcomes horizon, $\Lambda \gtrsim l_H$, somewhere at the temperature $T \lesssim 100$ MeV, while its amplitude (9) increases by $\approx$ ten orders of magnitude from a small initial value B_{max} at the high temperature $T \sim 20$ GeV $\ll \mathrm{T_{EW}} \sim 100$ GeV.

5 Comparison with weak collision mechanism

Accounting for the weak collision terms in Euler equation (2) one can find that the difference of weak cross-sections $\sigma_{\nu e^-} - \sigma_{\nu e^+} = 7G_F^2T^2$ leads to the friction force separating electrons and positrons, or to the electric current caused by the neutrino density asymmetry [2]:

$$\begin{aligned} J_{ext}^{collision} &\simeq \frac{en_e\delta n_\nu}{3}(\sigma_{\nu e^-} - \sigma_{\nu e^+})\tau_e = \\ &= 4\times 10^{-20}eT^3\left(\frac{T}{\mathrm{MeV}}\right)^3\left(\frac{\delta n_\nu}{n_\nu}\right) \end{aligned} \tag{10}$$

Such current generates field: $\partial_t\mathbf{B} = ... + \sigma_{cond}^{-1}\nabla\times\mathbf{J}_{ext}^{collision}$

Let us compare the collision current (10) with its analogue caused by collective mechanism, e.g. with the weak vector current originated by the force (3)

$$\mathbf{J}_{ext}^{collective} = \frac{eG_F\sqrt{2}c_V}{\alpha}\sigma_{cond}\left[\mathbf{V}\times\nabla\times\mathbf{V}_\nu\delta n_\nu\right]$$

One obtains the ratio

$$\frac{J_{ext}^{collision}}{J_{ext}^{collective}} = 2\times 10^8\left(\frac{T}{MeV}\right)^{-3}\left(\frac{\lambda_{fluid}^{(\nu)}}{l_\nu(T)}\right), \tag{11}$$

or at high temperatures, $T \gtrsim 1\, GeV$, and for the neutrino fluid inhomogeneity scale $\lambda^{(\nu)}_{fluid} \leq l_\nu(T)$, collective mechanism is more efficient. And vice versa, near the decoupling time (low temperatures $T \sim$ MeV) collision mechanism is more important.

6 Discussions and conclusions

We rely here on the following scenario of the mean magnetic field generation. First, a small-scale magnetic field is generated in phase transitions in early Universe. These are 1a) GUT phase transition violating the symmetry of strong and electroweak interactions SU(5) $\longrightarrow SU(3)_c \otimes (SU(2) \otimes U(1)_Y$ at $T \sim 10^{15}$ GeV$\sim 10^{28}$ K; 1b) Electroweak (EW) phase transition violating symmetry $SU(2) \otimes U(1)_Y \longrightarrow U(1)_{em}$ at $T \sim M_{W,Z} \sim 10^2$ GeV$\sim 10^{15}$ K; $B_{rms} \sim M^2_W/e \sim 10^{24}$ Gauss; 1c) Adronization of quark-gluon plasma, QCD-phase transition, $B_{rms} \sim T^2_{QCD}/e \sim 10^{18}$ Gauss.

Then at the step 2) domains of these strong small-scale (random) fields are merged through inversed cascade (in Fourier space via decays $\omega = \omega_1 + \omega_2$, $\mathbf{k} = \mathbf{k_1} + \mathbf{k_2}$) leading to a large-scale field of a small amplitude $\longrightarrow B_{\max} \ll T^2_{\max}/e$- seed field in our model of mean fields.

The scale of mean field , $L \ll l_H$, , increases very efficiently due to collective interactions with inhomogeneities of neutrino gas resulting in B becomes **global uniform**, $L \geq l_H$. For instance, for the neutrino inhomogeneity scale $\lambda^{(\nu)}_{\rm fluid} \sim l_\nu(T)$ magnetic field reaches superhorizon size at $T \sim 100$ MeV. Such field does not dissipate (ohmic losses are negligible), survives the recombination time and can be a seed for galactic magnetic field.

Accordingly (9) the amplitude B increases exponentially from an initial $B_{\max}$ only at the relativistic stage $T \gg m_e$ ($\alpha^2 - mechanism$), then magnetic field (frozen-in) cools down as $B \sim T^2$.

Let us emphasize that the mean magnetic field $\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2$ includes an unusual part $\mathbf{B}_2$ which is a pure vector (violating parity) and the search for its issues in astrophysical observations remains the challenge for future exploration.

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