# Optical activity of intergalactic space

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**Abstract**

The birefringence of a neutrino sea is discussed in the Standard Model. We demonstrate that the optical activity of a neutrino sea in the SM is dominated by the contribution of the two-loop diagrams that are five order of magnitudes larger than one-loop diagrams.

My talk is based on an unpublished paper in collaboration with G. Karl.

# 1 Introduction. Preliminaries

In the last decade a number of authors raised a question on the possibility of the violation of Lorentz and $CPT$ invariance.[1] To get a taste of this activity one can look at the paper written by Glashow and Coleman [1]. The most interesting scenario is that the speed of light in vacuum (i.e. in intergalactic space, in practice) is slightly different for different polarizations states of light, i.e. vacuum is birefringent. It is clear that such effect would indicate a small violation of Lorentz and $CPT$ invariance at the fundamental level.

There is no many choices to get Lorenz violation at the level of Lagrangian. If we wish to preserve gauge invariance and renormalizability of QED there is more or less one way to modify QED Lagrangian. That is

$$L = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \Delta L, \;\; \Delta L = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}b_\mu A_\nu F_{\alpha\beta} \;\; , \tag{1}$$

where $A_\mu$, $F_{\alpha\beta}$ are potential and field-strength tensor for electromagnetic field, and $b_\mu$ is some new field. The condensation of vector field $b_\mu$ in vacuum, i.e. a constant vector field in eq.(1), generates violation of Lorentz and $CPT$ symmetry.

One can check that in the "rest" frame where $b_\mu = (b_0, 0, 0, 0)$ the index of vacuum refraction for left and right circular polarized light $n_{+,-}$ is different from unity and is different for different polarization states:

$$n_+ = 1 + b_0/\omega, \;\; n_- = 1 - b_0/\omega \;\; . \tag{2}$$

Here $\omega$ is the light frequency.

For a liner polarized light it means that the plane of polarization rotates at angle $\Delta\phi$ when light propagates the distance $l$

$$\Delta\phi = \omega(n_+ - n_-)l \;\; . \tag{3}$$

Thus the rotatory power $\Phi = \Delta\phi(l)/l$ is

$$\Phi = \omega(n_+ - n_-) = 2b_0. \;\; . \tag{4}$$

[1]Actually every week in electronic arhive one can find one or two papers devoted to this subject.

Non-zero $b_0$ (vacuum condensation of vector field $b_\mu$) leads to a birefringence of vacuo. That is how people work with vacuum birefringence at fundamental level.

Less radical (and more solid) physical reason for light to have different speed of propagation in intergalactic space for different polarization states is $P$-odd interaction of light with neutrino sea. In this case neutrino sea in intergalactic space plays a role of optical active media, i.e. of a left-handed sugar.

The idea that intergalactic space is a birefringent medium for light due to the presence of a neutrino sea has been contemplated for a long time. Some thirty years ago, Royer [3] estimated in $V - A$ theory an effect of order $G_F$

$$\Phi \sim \alpha G_F k_F^3, \tag{5}$$

where $k_F$ is Fermi momentum of neutrino sea. Later, Stodolsky [2] noted that due to a theorem of Gell-Mann [4] there can be no such effect with massless neutrinos and a point-like coupling, i.e.

$$\Phi \sim 0. \tag{6}$$

In the early 1980's data on propagation of radio waves through intergalactic space put a stringent upper bound on possible optical activity of the neutrino sea [5] and [6] and this led to renewed estimates for the size of such effect on the assumption of a neutrino magnetic moment, which occur for a massive neutrino [6].

More recently an evaluation was made for massless neutrino and for on-shell photons [8] in one-loop approximation withing the Standard Model (SM). The effect is <u>non-zero</u> but extremely small

$$\Phi \sim \alpha G_F k_F^3 \frac{(k_F \omega)^2}{m_W^4}. \tag{7}$$

Here $m_W$ is the mass of $W$ boson. Due to Gell-Mann theorem there is additional suppression factor of the order $(k_F\omega)^2/m_W^4$ to naive estimate of Royer [3].

I my talk I demonstrate that these one-loop estimates for on-shell photon-neutrino scattering are irrelevant. The main contribution to the optical activity comes from the <u>two-loop</u> amplitude. The latter is larger than the

[2]This observation was recorded in a review [2]

[3]See also the calculations for off-mass shell photons [7]

one-loop estimates by a factor of $10^5$ or more. There is simple physics behind this amazing result.

# 2 Photon-neutrino interaction

## 2.1 Gell-Mann theorem

According to Gell-Mann's theorem [4] point-like weak interactions and massless neutrino leads to vanishing amplitude for photon-neutrino effective interaction. The theorem is easy to understand in the cross channel, i.e. for annihilation of $\nu\bar{\nu}$ pair into $\gamma\gamma$ in the center of mass frame.

For point-like interaction the orbital moment of a pair of neutrinos is exactly zero $L$=0. Thus for massless left-handed neutrino and right-handed antineutrino the total angular momentum $J$=1. On the other hand for two on-shell photons the states with $J$=1 are forbidden by Landau's theorem [9]. As a result the transition between $\nu\bar{\nu}$ and $\gamma\gamma$ states is forbidden in a point-like limit.

To escape Gell-Mann's restriction one needs non-local interactions in order to include higher orbital momenta of the pair $\nu\bar{\nu}$ into annihilation process. In the Standard Model, where the $W$ and $Z$ bosons mediate the interaction, the probability amplitude for neutrino pair to have nonzero orbital moment is proportional to some power of small factor $\sim p/m_W$, where $p$ is neutrino momenta. The factor $1/m_W$ measures the shortest separation of two neutrino during interaction (non-locality) in one-loop approximation.

## 2.2 Effective Lagrangian Approach

The standard way to deal with low-energy scattering is to use Effective Lagrangian approach.

The simplest example of the Effective Lagrangian is the four-fermion interactions of neutrinos $\nu$ with electrons $e$. For small fermion momenta one can forget about degrees of freedom associated with $W$ and $Z$ boson and can write the Effective Lagrangian only for fermionic fields:

$$L_{eff} = \frac{G_F}{\sqrt{2}}(\bar{\nu}\gamma_\alpha\nu)(\bar{e}\Gamma_\alpha e) \quad , \tag{8}$$

where $\Gamma_\alpha = g_V \gamma_\alpha + g_A \gamma_\alpha \gamma_5$. In the $SM$ $g_V = \frac{3}{2} - 2\sin^2\theta_W, g_A = \frac{3}{2}$. For momenta smaller than $m_W$ (or $m_Z$) the effective Lagrangian is as good as the fundamental Lagrangian of the $SM$.

Consider now the process $\nu(p) + \gamma(k) \to \nu(p) + \gamma(k)$. Neutrino has no electric charge and interaction between neutrino and photons takes place only at the level of loop diagrams. They are numerous and a bit complicated. For small momenta $(pk/m_W^2) \ll 1$ one can expand $\nu(p) + \gamma(k) \to \nu(p) + \gamma(k)$ amplitudes in the power series in this small parameter. The lowest terms of this expansion can be represented as a matrix element of the appropriate operators, i.e. of the particular terms of the Effective Lagrangian.

Each term of the Effective Lagrangian has to be Lorenz-invariant combination of gauge-invariant electromagnetic field tensor $F_{\mu\nu}$ and left-handed neutrino field $\nu_L = \frac{1}{2}(1+\gamma_5)\nu$ and their derivatives. The operator may have high dimensions $D$. To preserve correct dimension of the Lagrangian $[L] \sim [m]^4$ the coefficients in front of this operator should be proportional to appropriate power of $(1/m)$, where $m$ is the scale of mass that walk inside the loops. The actual calculation of the diagrams gives numerical coefficient.

## 2.3 P-even scattering amplitude

Consider how all these work in the case of $P$-even $\nu\gamma$-scattering. In this case the amplitude should be the same for right-handed photons and left-handed photons. One has to construct appropriate Lorenz invariant operators from the fields $F_{\mu\nu}$ and $\nu_L$. The simplest combination of the fields, that satisfies all these conditions, looks as follows:

$$L_{eff} \sim \frac{e^4}{m^4}[F_{\mu\alpha}F_{\mu\beta}]\bar{\nu}\gamma_\alpha\partial_\beta(1+\gamma_5)\nu + h.c. \quad , \tag{9}$$

It has dimension $D = 8$. Matrix element of $L_{eff}$ for forward scattering gives the amplitude

$$T \sim \frac{e^4}{m^4}(pk)^2\epsilon(k)^*\epsilon(k) \quad , \tag{10}$$

If we identify parameter $m$ in eq.(9) with the largest mass in diagrams (i.e. with $m_W$) we reproduce well known result for neutrino-photon amplitude [11] up to the numerical constant

$$T \sim G_F \alpha \frac{(pk)^2}{m_W^2}\epsilon(k)^*\epsilon(k) \quad , \tag{11}$$

Of course it is important to calculate numerical coefficients. But even before any calculations from this simple exercise we have learn two lessons.

First, non-zero term for $\gamma\nu$ scattering appears only in the second order in $(pk)^2$. It indicates immediately that any calculations that give effect of the order $(G_F\alpha) \sim \alpha^2/m_W^2$ (such as in e.g. of ref. [4, 10]) are erroneous. It is impossible to violate Gell-Mann theorem in the framework of Effective Lagrangian approach. Appropriate operators do not exist.

Second, the actual calculations for Effective Lagrangian are simplified enormously. Indeed now one expands every individual Feynman diagram up to the order of $(pk)^2$. To calculate the numerical factor one can put external momenta in remaining integrals to be equal zero, i.e. $p = 0$ and $k = 0$. For zero external momenta Feynman integration becomes trivial and the whole problem reduces to a pure algebraic (though rather boring) calculations of traces and products.

## 2.4 Optical activity. P-odd scattering amplitude

Now let us come back to $P$-odd effects in $\nu\gamma$ scattering and find an appropriate operators in $L_{eff}$ responsible for optical activity. The simplest Lagrangian that depend on $P$- odd combinations of photons amplitudes looks as follows

$$L_{eff} \sim \frac{1}{m^4}[F_{\mu\alpha}\tilde{F}_{\mu\beta}][\bar{\nu}\gamma_\alpha\partial_\beta(1+\gamma_5)\nu] + h.c. \ , \tag{12}$$

where $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F_{\alpha\beta}$.

The surprise is that this operator does not work in our case. Indeed refraction index is proportional to the photon forward scattering amplitude. One can check that the matrix element of $F_{\mu\alpha}\tilde{F}_{\mu\beta}$ between photons with the same momenta and polarization ( forward scattering) is identically zero. Thus operators of $D = 8$ do not contribute into $P$-odd forward scattering and effect is zero in $(pk)^2/m^4$ order!

Now we have to look for operators of higher dimension $D = 10$. One of these operators looks like follows

$$L_{eff} \sim \frac{1}{m^6}[F_{\mu\alpha}(\partial_\gamma\tilde{F}_{\mu\beta})][\bar{\nu}\gamma_\alpha\partial_\beta\partial_\gamma(1+\gamma_5)\nu] + h.c. \tag{13}$$

With this $L_{eff}$ the forward scattering amplitude of a photon of momentum $k$ from a neutrino of momentum $p$ is equal to

$$T = C(e^4/8\pi^2)(pk/m^2)^2\epsilon_{\mu\nu\alpha\beta}\epsilon_\mu^(k)\epsilon_\nu^*(k)(p_\alpha k_\beta/m^2). \quad (14)$$

This amplitude has different contribution to left-handed and to right-handed photons scattering: $T_{LL} = -T_{RR}$.

### 2.5 One-loop calculations of P-odd effect

The actual calculation of the coefficient C has been done in one loop-approximation in [8] with the results

$$T = C(e^4/8\pi^2 s^2)(pk/m_W^2)^2\epsilon_{\mu\nu\alpha\beta}\epsilon_\mu^(k)\epsilon_\nu^*(k)(p_\alpha k_\beta/m_W^2)\ , \quad (15)$$

where

$$C = 4/3(\ln(m_W^2/m^2) - 11/3), \quad (16)$$

in the first reference in [8] and

$$C = 4/3(\ln(m_W^2/m^2) - 8/3), \quad (17)$$

in the second one[8]. The reason for that discrepancy is unknown and it would be interesting to understand whether there is a correct one-loop result.

The message of this talk is that for a $P$-odd effect there is enormous enhancement factor in two-loop approximation. Thus any one-loop results (correct or erroneous) are irrelevant.

## 3 Two-loop calculations

### 3.1 P-odd effect

The physical reason for the dominance of the two-loop diagrams under the one-loop is simple.

To escape Gell-Mann's restriction one needs non-local interactions in order to include higher orbital momenta of the pair $\nu\bar{\nu}$ into annihilation process. In the one loop approximation the only source of non-locality is the $W$ boson exchange. Thus the one-loop probability amplitude $T^{(1)}$ for neutrino pair to have nonzero orbital moment is $\sim p/m_W$. The factor $1/m_W$ measures the shortest separation of two neutrino in one-loop approximation.

In the two-loop approximation there is a set of diagrams in which two neutrinos are emitted at a separation of the order of the electron Compton wavelength. This is due to $e^-e^+\nu$ in the intermediate state. Thus in two loop approximation the amplitude $T^{(2)}$ for neutrino pair to have nonzero orbital moment is $\sim p/m_e$.

Thus moving to the next order in electro-weak interaction we loose a small factor $\alpha/2\pi$ , but win a great factor $(m_W^2/m_e^2)$. The net effect is

$$(T^{(2)}/T^{(1)}) \sim (\alpha/2\pi)(m_W^2/m_e^2) \sim 10^7. \tag{18}$$

Actual calculation is rather lengthy. The result is

$$T^{(2)} = (13/27)(G^2e^2/64\pi^4)(g_V^2+g_A^2)(pk)^2\epsilon_{\mu\nu\alpha\beta}\epsilon_\mu(k)\epsilon_\nu^*(k)(p_\alpha k_\beta/m_e^2). \tag{19}$$

The enhancement factor is

$$T^{(2)}/T^{(1)} = (\alpha/64\pi sin^2\theta_W)(m_W^2/m_e^2)(13/27)(g_V^2+g_A^2)/C \sim 10^5, \tag{20}$$

where $C$ is one-loop coefficient from eq.(15). We have lost two order of magnitude compared with naive estimate. This happened mainly due to the large logarithmic factor in one-loop coefficient $C$. Still the effect is very large $\sim 10^5$!

## 3.2 Two-loop estimates of P-even effect

It is interesting to understand wheather similar enhancement factor takes place for main P-even amplitude, e.g. for cross section of $\nu\bar{\nu}$ annihilation into two photon. The answer is negative.

The arguments are the following. Consider two-loop amplitudes with $(e^+e^-\nu)$ in intermediate state between two external neutrino vertexes. In local four-fermion approximation we expect that these diagrams are proportional to $G_F^2\alpha/m_e^2$ . Thus to preserve correct dimension in effective Lagrangian we need operator of dimension $D = 10$. Appropriate effective Lagrangian is

$$L_{eff} \sim \frac{G_F^2\alpha}{m_e^2}[F_{\mu\alpha}(\partial_\gamma F_{\mu\beta})][\bar{\nu}\gamma_\alpha\partial_\beta\partial_\gamma(1+\gamma_5)\nu] + h.c. \tag{21}$$

For this $L_{eff}$ the scattering amplitude is of the third order in $(pk)$

$$T \sim C\frac{G_F^2\alpha}{m_e^2}(pk)^3\epsilon(k)\epsilon^*(k). \tag{22}$$

Thus for P-even scattering second order loops give correction of the order $(G_F M_W^2)(pk/m_e^2)$, i.e. small correction to the one-loop result.

# 4 Numerical estimates

In spite of a huge missing factor $10^5$ in the one-loop estimates of optical activity of neutrino sea [12] we find that the physical effect is still tiny. It is rather unlikely that direct measurements of such small rotation of the plane of polarization of light when it travels in intergalactic space are ever possible. But maybe correct estimates of P-odd effect give hint for more subtle experiments. In any case it is nice to have correct estimate.

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# References

[1] S.Coleman and S.Glashow, Phys. Rev. **D59** (1999) 116008; R.Jackiw and V.A.Kostelecky, Phys. Rev. Lett. **82** (1999) 3572, and references therein.

[2] G.Karl, Canadian Journal of Physics **54** (1976) 568.

[3] J.Royer, Phys. Rev. **174** (1968) 1719.

[4] M.Gell-Mann, Phys. Rev. Lett. **6** (1961) 70.

[5] J.N.Clarke, P.P.Kronberg, M. Simard-Normandin, Mon.Not.Roy. **190** (1980) 205.

[6] J.N.Clarke, G.Karl and P.J.S.Watson, Canadian Journal of Physics, **60** (1982) 1561.

[7] S.Mohanty, J.F.Nieves and P.Pal, Phys. Rev. **D58** (1998) 093007.

[8] G.Karl and V.Novikov, hep-ph/0009012;

A.Abbasabadi and W.Repko, Phys. Rev. **D64** (2001) 113007;Phys. Rev. **D67** (2003) 073018.

[9] L.D.Landau, Dokl. Akad. Nauk USSR **60**, 207 (1948); C.N.Yang, Phys. Rev. **77**, 242 (1950)

[10] S.G.Matinyan and N.N. Tsilosuni, Zurn.Eksp.Teor.Fiz.**41** (1961) 1691; Sov.Phys. JEPT**14** (1961) 1195.

[11] M.J.Levine, Nuovo Cimento **A48** (1967) 67.

[12] A.Abbasabadi and W.Repko, Phys. Rev. **D67** (2003) 073018.