Photon conversion into sterile neutrino $\gamma \to \nu_s \tilde{\nu}_s$ via Z' bozon in a magnetized plasma

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Abstract

Neutrino–photon interaction in a magnetized plasma is investigated in the framework of an extended standard model when neutrino–electron coupling is caused by exchange of Z' boson. The process of the photon conversion into right neutrino pair $\gamma \to \nu_R + \bar{\nu}_R$ is analyzed as an additional channel of star energy loss. The comparison of luminosity due to right neutrino emission in the process $\gamma \to \nu_R + \bar{\nu}_R$ from supernova core with the total neutrino luminosity allows to get a new restriction on the mass of Z' boson.

1 Introduction.

The neutrino processes in an active medium (plasma and external magnetic field) arouse steady interest over last decades. In particular, neutrino play a paramount role in astrophysical phenomena like a supernova explosion when a large number of neutrino are produced in a collapsing stellar core [1]. Super dense ($\rho \sim 10^{14}$ g/cm^3) and hot ($T \sim 35$ MeV) substance

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of supernova core is opaque for the standard left neutrino. At the same time the nonstandard right neutrino are the "sterile" neutrino state with respect to the standard weak interaction and can escape from the hot and dense stellar interior. So, the processes of the right neutrino emission could give an additional contribution to the energy losses by stellar objects. By this means, the investigation of the right neutrino involving processes under extreme conditions of hight density and/or temperature of matter and strong magnetic field are of great interest [1].

In this paper we study a photon conversion into pair right neutrino, $\gamma \to \nu_R + \bar{\nu}_R$, in a plasma with the presence of an external magnetic field. Such process becomes possible in the framework of the extended standard model, for example, in the minimal quark–lepton symmetry model [2], when neutrino–electron interaction is due to exchange by Z' boson.

We consider the process $\gamma \to \nu_R + \bar{\nu}_R$ in degenerate ultrarelativistic plasma under conditions when the chemical potential of plasma electrons, μ, is the dominate physical parameter, whereas the magnetic field is not so strong[1]

$$\mu^2 \gg T^2 \gg eB \gg m_e^2, \tag{1}$$

where m_e is the electron mass.

Notice, that the magnetic field being relatively weak (1) could be strong enough in comparison with the critical value $B_e = m_e^2/e = 4.41 \times 10^{14}$ G. Really, under conditions of a supernova core when the typical chemical potential is assumed to be $\mu \sim 250$ MeV, plasma temperature is $T \sim 35$ MeV [1] from (1) we have

$$\frac{\mu^2}{m_e^2} \sim 10^5 \gg \frac{B}{B_e} \gg 1 \tag{2}$$

Relation (2) demonstrates that even very strong magnetic fields up to the 10^{16} G satisfy the conditions (1) and can be considered as a relatively weak field.

In the presence of both components of active medium the process $\gamma \to \nu_R + \bar{\nu}_R$ is depicted by diagrams in Fig.1 and Fig.2. The loop diagram in Fig.1 corresponds to the field–induced neutrino–photon interaction, where double lines indicate that the influence of the external field is taken into account in the propagators of virtual fermions. The processes of forward photon "scattering" into neutrino pair on plasma electrons (see two diagrams in Fig 2.) give the plasma contribution to the conversion $\gamma \to \nu_R + \bar{\nu}_R$.

[1]We use natural units in which $c = \hbar = 1$, $e > 0$ is the elementary charge.

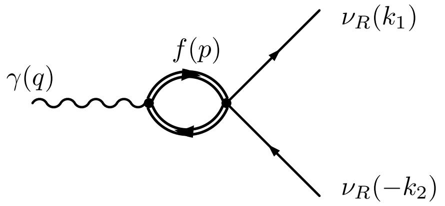

Figure 1: Neutrino photon interaction in an external magnetic field.

The interaction of Z' boson with electrons and neutrino can be written in the general form

$$L = -\frac{e}{\cos\theta_W} Z'_\mu \, [\bar{\nu}\gamma^\mu (v + a\gamma_5)\nu + \bar{e}\gamma^\mu (g_v + g_a\gamma_5)e], \tag{3}$$

where θ_W is the Weinberg angle, v, g_v and a, g_a are the model depended vector and axial coupling constant of Z' boson to fermions [2, 3]. We will perform calculations under the assumption of a relatively small momentum transferred $q^2 \ll m_{Z'}^2$, where $m_{Z'}$ is the mass of Z' boson. Therefore we can investigate the process $\gamma \to \nu_R + \bar{\nu}_R$ in the local limit when the line of Z' boson is contracted to a point. In view of this fact from (3) one can obtained the effective lagrangian of neutrino-electron interaction

$$L^{eff} = -\frac{rG_F}{\sqrt{2}} \, (\bar{e}\gamma^\mu (g_v - g_a\gamma_5)e) \, j_\mu. \tag{4}$$

Here $r = 4\sin^2\theta_W (v - a) m_Z^2 / m_{Z'}^2$ and $j_\mu = \bar{\nu}\gamma_\mu (1 + \gamma_5)\nu$ is the right neutrino current, m_Z is the mass of the standard Z boson.

Because of the vector (V) character of a photon–electron interaction, there are exist two types of transition $V \to V$ and $V \to A$, corresponding to the

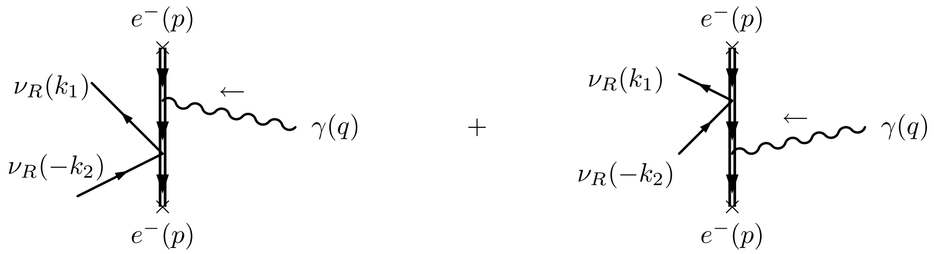

Figure 2: The conversion $\gamma \to \nu_R + \bar{\nu}_R$ in the presence of magnetized plasma.

vector and axial-vector coupling of electrons with the right neutrino. Below we will separate these processes by the indexes V and A correspondingly.

The differential probability of the process $\gamma \longrightarrow \nu_R + \bar{\nu}_R$ is defined by expression [4]

$$dW = \frac{1}{8(2\pi)^2} \, \frac{d^3k_1 d^3k_2}{\omega\omega_1\omega_2} \, \delta^4(q - k_1 - k_2) \; | \, M \, |^2 \, . \tag{5}$$

Here $q^\mu = (\omega, \vec{k})$, $k_1^\mu = (\omega_1, \vec{k}_1)$, $k_2^\mu = (\omega_2, \vec{k}_2)$ are the four - momenta of the photon, neutrino and antineutrino correspondingly, M is the invariant amplitude which is connected with a S matrix element by following way:

$$S = \frac{i(2\pi)^4 \, \delta^4(q - k_1 - k_2)}{\sqrt{2\omega V}\sqrt{2\omega_1 V}\sqrt{2\omega_2 V}} \; M.$$

From the point of view of a practical application of the result obtained the star energy-loss due to a neutrino emission is of more interest than the probabilities of the process considered. The volume density of plasma energy loss in unite time can be presented in the terms of invariant amplitude

$$\dot{\varepsilon} \equiv \frac{1}{8(2\pi)^5} \int \frac{d^3k}{e^{\omega/T} - 1} \, \frac{dk_1 dk_2}{\omega_1\omega_2} \, \delta^4(q - k_1 - k_2) \; | \, M \, |^2 \, . \tag{6}$$

2 Average amplitude squared of the process $\gamma \longrightarrow \nu_R + \bar{\nu}_R$.

As one can see from (6) there is a need to calculate the averaged amplitude squared which is defined as

$$\mathcal{M}^2 \equiv \int \frac{d^3k_1 d^3k_2}{\omega_1\omega_2} \, \delta^4(q - k_1 - k_2) \; | \, M \, |^2 \, . \tag{7}$$

Taking into account that the amplitude of the process considered can be presented in the form

$$M = (J_\alpha \; j^\alpha),$$

where J_α is a 4 – vector which does not depends on neutrino momentum k_1^μ, k_2^μ, the calculation of the amplitude squared (7) reduces to the computation of the integral

$$\int \frac{d^3k_1 d^3k_2}{\omega_1\omega_2}\,\delta^4(q-k_1-k_2)\,j_\alpha j^*_\beta = \frac{16\pi}{3}(q_\alpha q_\beta - q^2 g_{\alpha\beta}), \tag{8}$$

We do no present here the details of calculation the amplitude of the process considered, which will be published in an extended paper. Performing the calculations with (8) we have obtained the following result for the axial contribution into the plasma averaged amplitude squared

$$\left(\mathcal{M}_A^{pl}\right)^2 = \alpha\,\frac{16G_F^2\,r^2\,g_a^2}{3\pi^2}\,\mu^2\,(q^2)^2\,\frac{(1-x^2)\,(1-l(x))^2}{x^2}, \tag{9}$$

where $\alpha = e^2/4\pi$ is the fine structure constant, $x = \mid \vec{k} \mid /\omega$ and function $l(x)$ is determined as

$$l(x) = \frac{1}{2x}\,\ln\left(\frac{1+x}{1-x}\right).$$

To obtain the expression for vector contribution into the amplitude of the process $\gamma \to \nu_R + \bar{\nu}_R$ we use amplitude of the transition $\gamma \to \gamma$, which can be represented in the form

$$M_{\gamma\to\gamma} = \varepsilon^*_\alpha \Pi^{\alpha\beta}\varepsilon_\beta, \tag{10}$$

where $\Pi_{\alpha\beta}$ is the photon polarization operator, ε_α is the photon 4 - polarization vector.

Replacing in the (10) one polarization vector to the right neutrino current with corresponding coefficient, $\varepsilon_\alpha \to -G_F r g_v\, j_\alpha/e\sqrt{2}$, we have

$$M_V^{pl} = -\frac{G_F r g_v}{e\sqrt{2}}(j_\alpha \Pi^{\alpha\beta}\varepsilon_\beta). \tag{11}$$

In view of the relative smallness of the magnetic field on the scale of the plasma parameters (1) a property of the polarization operator are basically determined by plasma. In this case the tensor structure of the photon polarization operator is usually decomposed into transversal and longitudinal parts

$$\begin{aligned}
\Pi_{\alpha\beta} &= P^t\Pi^t_{\alpha\beta} + P^l\Pi^l_{\alpha\beta}, \qquad (12)\\
\Pi^t_{\alpha\beta} &= -\left(g_{\alpha\beta} - \frac{q_\alpha q_\beta}{q^2} - \frac{l_\alpha l_\beta}{l^2}\right),\ \Pi^l_{\alpha\beta} = \frac{l_\alpha l_\beta}{l^2},
\end{aligned}$$

where P^t and P^L are the eigenvalues of the transversal and longitudinal modes of the $\Pi_{\alpha\beta}$, l_α is the longitudinal eigenvector corresponding to the longitudinal plasmon

$$l_\alpha = \sqrt{\frac{q^2}{(uq)^2 - q^2}} \left(u_\alpha - \frac{(uq)}{q^2} q_\alpha \right) .$$

Here u^α is the medium-velocity 4-vector, which in the plasma rest frame has the form $u^\alpha = (1, \vec{0})$. Eigenvalues P^t and P^L in the degenerate ultrarelativistic plasma can be presented in the form [5]

$$P^t = \frac{2\alpha}{\pi} \mu^2 \frac{(1 - (1 - x^2) l(x))}{x^2}, \tag{13}$$

$$P^l = \frac{4\alpha}{\pi} \mu^2 \frac{(1 - x^2)(l(x) - 1)}{x^2}. \tag{14}$$

The result of our calculations for the averaged amplitude squared caused by the conversion transversal (γ_t) and longitudinal (γ_L) plasmon into pair right neutrino is

$$\left(\mathcal{M}^{pl}_{V,\gamma_t}\right)^2 = \frac{8 G_F^2 r^2 g_v^2}{3\pi} \mu^2 (q^2)^2 \frac{(1 - (1 - x^2) l(x))}{x^2}, \tag{15}$$

$$\left(\mathcal{M}^{pl}_{V,\gamma_L}\right)^2 = \frac{8 G_F^2 r^2 g_v^2}{3\pi} \mu^2 (q^2)^2 \frac{(1 - x^2)(l(x) - 1)}{x^2}, \tag{16}$$

where the dispersion laws for the transversal and longitudinal photon ($P^{t,l} = q^2$) where taken into account.

As one can see from (9), (15) and (16) the contribution into the plasma averaged amplitude squared due to the axial-vector neutrino - electron interaction contains the suppression associated with the fine structure constant α, and can be neglected in comparison with the vector one.

The field-induced contribution into the amplitude of the process caused by the loop diagram (Fig.1). The result of our calculations for the field-induced part of the averaged amplitude squared can be reduced to the form

$$\left(\mathcal{M}^F_V\right)^2 \simeq \alpha G_F^2 r^2 g_v^2 q^2 q_\parallel^2 \left(\frac{eB}{q_\parallel^2}\right)^{4/3} \{a_1 (\varepsilon\varphi q)^2 + b_1 (\varepsilon\tilde{\varphi} q)^2\}, \tag{17}$$

$$\left(\mathcal{M}^F_A\right)^2 \simeq \alpha G_F^2 r^2 g_a^2 q^2 q_\parallel^2 \left(\frac{eB}{q_\parallel^2}\right)^2 \{a_2 (\varepsilon\tilde{\varphi} q)^2 + b_2 (\varepsilon\varphi\varphi q)^2\}, \tag{18}$$

where a_1, a_2, b_1, b_2 are the dimensionless coefficients of the order of unit, $\varphi_{\alpha\beta} = F_{\alpha\beta}/B$ and $\tilde{\varphi}_{\alpha\beta} = \epsilon_{\alpha\beta\mu\nu}\varphi_{\mu\nu}/2$ are the dimensionless magnetic field tensor and the dual tensor, $q_{\parallel}^2 = q\tilde{\varphi}\tilde{\varphi}q = \omega^2 - k_3^2$ (the magnetic field directed along the third axis, $\vec{B} = (0, 0, B)$).

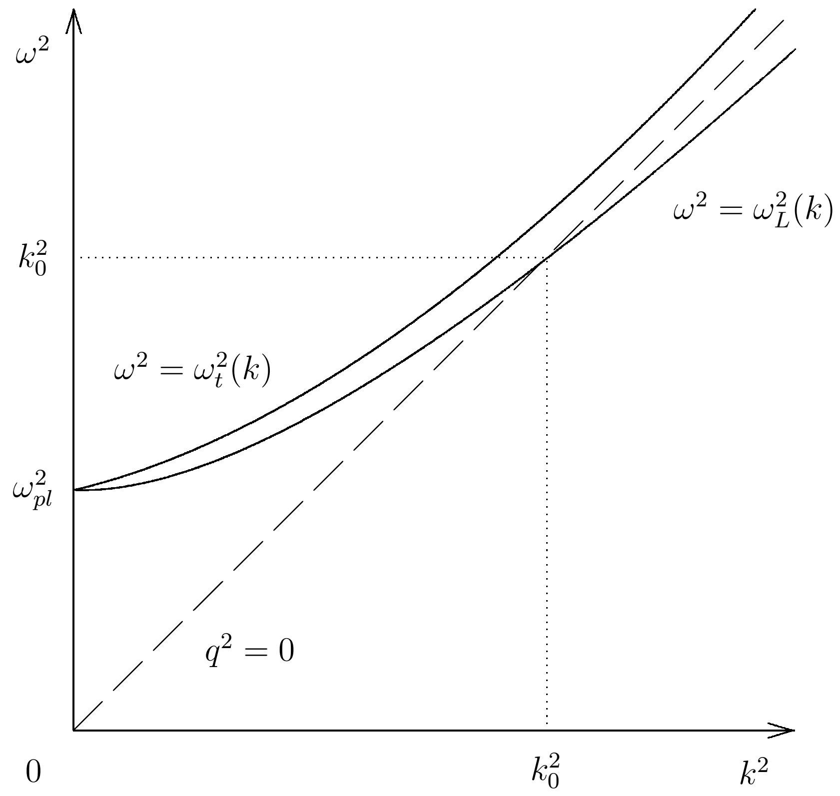

Figure 3: Dispertion curves for transversal plasmon $\omega^2 = \omega_t^2(k)$ (upper solid line), longitudinal plasmon $\omega^2 = \omega_L^2(k)$ (lower solid line) and vacuum photon $q^2 = 0$ (broken line).

Notice, that the averaged amplitude squared (9), (15)-(18) and, consequently the probabilities of the processes considered not being zero only because of photon dispersion law in a plasma ($q^2 = P^t \neq 0$ or $q^2 = P^l \neq 0$) in contrast to vacuum one ($q^2 = 0$). The dispersion curves for longitudinal and transversal plasmon are shown in Fig.3, where ω_{pl} and k_o are the plasmon frequency and point of intersection of the longitudinal plasmon and vacuum lines correspondingly. In the degenerate ultrarelativistic plasma these parameters are [5]:

$$\omega_{pl}^2 = \frac{4\alpha}{3\pi}\,\mu^2,\;\; k_0^2 = \frac{4\alpha}{\pi}\,\mu^2\,\ln\left(\frac{2\mu}{m_e} - 1\right).$$

The necessary condition of the photon conversion into pair neutrino $q^2 > 0$ is realized for the transversal photon in the region $\omega_{pl}^2 < \omega_t^2 < \infty$ and for the longitudinal plasmon in the region $\omega_{pl}^2 < \omega_L^2 < k_0^2$. Since under conditions of a supernova core where the typical photon energy is of the order of plasma temperature, $q_{\|}^2 \sim T^2$, we can estimate the ratio of field averaged amplitude square to the plasma one:

$$\frac{\left(\mathcal{M}_V^F\right)^2}{\left(\mathcal{M}_V^{pl}\right)^2} \sim \left(\frac{eBT}{\mu^3}\right)^{4/3} \ll 1,\;\; \frac{\left(\mathcal{M}_A^F\right)^2}{\left(\mathcal{M}_V^{pl}\right)^2} \sim \frac{g_a^2}{g_v^2}\left(\frac{eB}{\mu^2}\right)^2 \ll 1. \qquad (19)$$

Thus under conditions considered (1) the main contribution into the process $\gamma \to \nu_R + \bar{\nu}_R$ arises from Compton - like process of neutrino scattering on plasma electrons caused in main by vector part of neutrino-electron interaction in the lagrangian (4).

3 Neutrino emissivity.

To illustrate a possible astrophysical application of the result obtained, we have calculate the stellar energy loss due to neutrino emission in the process $\gamma \to \nu_R + \bar{\nu}_R$ from a supernova core. In our case the volume density of energy loss per unit time is defined in (6). Substituting expressions (16), (15) in (6) and carrying out the integration over angles, we obtain the plasma energy losses due to conversion of longitudinal and transversal photons into neutrino pair in the form

$$\begin{aligned}
\dot{\varepsilon}_{\gamma_L \to \nu\tilde{\nu}} &= \frac{G_F^2 g_v^2}{96\alpha\pi^4}\,\omega_{pl}^6\,T^3 \int\limits_v^{v^*} \frac{dz\,z\sqrt{z^2 - v^2\beta_l(z)}}{e^z - 1}\,\beta_l^3(z),\\
\dot{\varepsilon}_{\gamma_t \to \nu\tilde{\nu}} &= \frac{G_F^2 g_v^2}{48\alpha\pi^4}\,\omega_{pl}^6\,T^3 \int\limits_v^{\infty} \frac{dz\,z\sqrt{z^2 - v^2\beta_t(z)}}{e^z - 1}\,\beta_t^3(z),
\end{aligned} \qquad (20)$$

where the variable z defines the photon energy, $z = \omega/T$, and the following designations are used

$$v = \frac{\omega_{pl}}{T}, \quad v^* = \frac{k_0}{T}, \quad \beta_l(z) = \frac{P^l(z)}{\omega_{pl}^2}, \quad \beta_t(z) = \frac{P^t(z)}{\omega_{pl}^2}.$$

Below we estimate the star energy losses due to emission of right neutrino in the process $\gamma_t \to \nu_R + \bar{\nu}_R$ from a supernova core during the first few second after collapse. For estimation we take $\mu \sim 250$ MeV, $T \sim 35$ MeV:

$$\dot{\varepsilon}_{\gamma_t \to \nu\bar{\nu}} \sim r^2 \; g_v^2 \; 10^{55} \text{erg/s}. \tag{21}$$

The vector and axial-vector coupling could be identified, for example, in a minimal quark–lepton symmetry model, where $g_V^2 \simeq 1$ and parameter r with a good precision is a ratio between mass of Z and Z' bosons, $r \simeq m_Z^2/m_{Z'}^2$. On the assumption that the result (21) does not exceed of 10% from neutrino luminosity under the same conditions $L_\nu \sim 10^{52}$ erg/s [1], we obtained the following bound on the mass of Z' boson

$$m_{Z'} > 1000 \text{ GeV},$$

which is a greater then existing one [2, 6]

4 Conclusion.

We have studied the photon conversion into pair right neutrino in a plasma with the presence of a relatively weak magnetic field. It is shown that under typical conditions of a supernova core the plasma process dominates over field one and mainly caused by the vector coupling of right neutrino and plasma electrons. We have calculated the plasma energy losses due to right neutrino emission in the process $\gamma \to \nu_R + \bar{\nu}_R$ from the supernova core and obtained the new bound to the mass of Z' boson.

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