High-energy neutrino from creating massive black hole

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Abstract

We describe the scenario of massive black hole (MBH) formation inside a supermassive star (SMS) in the galactic nucleus. A SMS is naturally formed in the dynamically evolving galactic nucleus due to destruction of normal stars in the direct collisions. The survived neutron stars (NSs) and stellar mass black holes form a compact self-gravitating subsystem deep inside the SMS. This subsystem is short-lived in comparison with a host SMS and collapses into the MBH. The frequent NS collisions in this nearly collapsing subsystem are accompanied by the generation of multiple ultra-relativistic fireballs which are the suitable place for particle acceleration. Only the secondary high-energy neutrino escape from the SMS interior. The resulting high-energy neutrino signal can be detected by underground neutrino telescope with an effective area $S \sim 1$ km^2 and can give the evidence for MBH formation in the distant galactic nucleus.

1 Introduction

Recently we proposed [1] (Paper 1) the model of very powerful but short lived hidden HE neutrino source which originates in the compact galactic nucleus just prior to its collapse into the MBH. In this model it is supposed that MBH is formed by the natural dynamical evolution of the central stellar

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cluster in the normal galactic nucleus. Dynamical evolution of dense central stellar clusters in the galactic nuclei is accompanied by the growth of the velocity dispersion of constituent stars v or, equivalently, by the growth of the central gravitational potential $\phi \propto v^2$. This process is accompanied by the contraction of the stellar cluster and terminated by the formation of the MBH, when the velocity dispersion of stars grows up to about speed of light, $v \sim c$, or equivalently, when the stellar cluster radius shrinks near to its gravitational radius (see for a review e. g. [2] and references therein). In the following we shall elaborate the hidden source model of Paper 1 for the possible case of a SMS formation in the galactic nucleus. It will be shown below that as well as in the scenario of NS cluster formation simultaneously with a massive envelope (Paper 1) the hidden source of HE neutrino also arises in the case of SMS formation, but would be shorter-lived and more intense.

2 Supermassive star formation

The supermassive star (SMS) in the galactic nucleus may be formed from the gas liberated in normal star self-destructions in the rather evolved central stellar cluster with a velocity dispersion $v \geq v_{\rm esc} = (2Gm_*/r_*)^{1/2}$, where $v_{\rm esc}$ is an escape velocity from the surface of star with mass m_* and radius r_*. For a solar type star the escape velocity is $v_{\rm esc} \simeq 620$ km s^{-1}. In the cluster with $v > v_{\rm esc}$ all normal stars are eventually disrupted in mutual collisions or in collisions with the extremely compact stellar remnants: neutron stars (NSs) or stellar mass black holes (BHs). Only these compact stellar remnants survive the stellar-destruction stage of galactic nucleus evolution ($v \simeq v_{\rm esc}$) and may form the self-gravitating subsystem. The characteristic time-scale for stellar cluster dynamical evolution is the (two-body) relaxation time:

$$t_{\rm r} = \left(\frac{2}{3}\right)^{1/2} \frac{v^3}{4\pi G^2 m^2 n \Lambda} \simeq 4.65 \cdot 10^8 N_8^2 \left(\frac{v}{v_{\rm esc}}\right)^{-3} \ {\rm yr}, \tag{1}$$

where $N = 10^8 N_8$ is the number of stars in the stellar cluster, $\Lambda = \ln(0.4N)$ is the Coulomb logarithm, v is star velocity dispersion, n is star space density, $m \simeq {\rm M}_\odot$ is typical mass of constituent stars. The first equality in Eq. (1) is valid for the local values of parameters. Respectively the second one is valid only for the mean (virial) parameters of a self-gravitating cluster. At $v > v_{\rm esc}$, where $v_{\rm esc}$ is the escape velocity from the surface of constituent star, the time-scale for self-destruction of normal stars by collisions is $t_{\rm coll} = (v_{\rm esc}/v)^4 \Lambda t_{\rm r}$ [3]. If $v \gg v_{\rm esc}$ the normal stars are eventually disrupted in mutual collisions and central stellar cluster in the galactic nucleus is converted to the SMS.

3 Neutron star cluster inside SMS

Only the relativistic compact stellar remnants such as NSs and stellar mass BHs can survive during the stellar-destruction phase of evolution of galactic nucleus at $v \geq v_{\rm esc}$ and populate the interior of the newly formed SMS. For brevity we shall refer to these compact stellar remnants as to NSs.

We approximate the gas density distribution in the SMS by the standard polytropic model with an adiabatic index $\gamma = 4/3$. For this adiabatic index the central gas density in the SMS is $\rho_c = k_{\rm c} \bar{n}_{\rm SMS} m_{\rm p}$ and the central sound velocity $c_{\rm s,c} = k_{\rm s} v_{\rm SMS}$, where $\bar{n}_{\rm SMS}$ is a mean gas number density in the SMS, $v_{\rm SMS} = (GM_{\rm SMS}/2R_{\rm SMS})^{1/2}$ is a virial velocity and numerical constants $k_{\rm c} \simeq 54.2$ and $k_{\rm s} \simeq 1.51$ respectively.

An individual NS with a mass m and local velocity V spirals down to the center of the formed SMS under the influence of dissipative dynamical friction force [4, 5] $F_{\rm df} = I \times 4\pi (Gm)^2 \rho_{\rm SMS}/V^2$ (where $\rho_{\rm SMS} = \bar{n}_{\rm SMS} m_{\rm p}$ is SMS gas density), which is directed opposite to the local NS velocity V. The dimensionless function $I \simeq 1$ for the case of $V \simeq c_{\rm s}$, where $c_{\rm s}$ is the SMS sound velocity. The corresponding effective time of a NS dynamical friction drag toward the center of the SMS is $t_{\rm df} = V/\dot{V} = mV/F_{\rm df}$. It can be easily seen that at the moment of SMS formation (when $V \sim c_{\rm s} \sim v_{\rm esc}$) the effective time of a NS friction drag is of the same order as the time-scale of SMS formation, $t_{\rm df}(v_{\rm esc}) \sim t_{\rm coll}(v_{\rm esc}) \sim t_{\rm r}(v_{\rm esc})$. We assume for simplicity that SMS contains the identical NSs with a typical NS mass $m_{\rm NS} = 1.4 {\rm M}_{\odot}$ and with a total NS cluster mass $M_{\rm NS} = f_{\rm NS} M_{\rm SMS}$. Here the NS mass fraction relative to the SMS mass is $f_{\rm NS} = 0.01 f_{-2} \ll 1$ and a total NS number in the cluster is $N_{\rm NS} = f_{\rm NS} M_{\rm SMS}/m_{\rm NS} \simeq 7.1 \cdot 10^5 M_8 f_{-2}$. The value of $f_{\rm NS}$ is a free parameter of the model and we will put $f_{-2} = 1$ in numerical estimations. The dynamical friction time-scale of the NS with a velocity $V = (GM_{\rm SMS}/2R)^{1/2}$ inside the host SMS diminishes as $t_{\rm df} \propto R^{3/2}$ during the evolution contraction of SMS radius R. On the contrary the evolution time-scale of SMS grows with SMS contraction, $t_{\rm SMS} \propto R^{-1}$. The subsystem of NSs evolves faster than the host SMS after reaching the stage when $t_{\rm df} \sim t_{\rm SMS}$.

At this time all NSs sink deep to the central part of SMS and forms there the self-gravitating cluster. From approximate equality $t_{\rm df}(R) \sim t_{\rm SMS}(R)$ we find the SMS radius at the moment of NS cluster formation inside it:

$$R_{\rm SMS} \simeq \left(\frac{9I^2}{8\pi^2}\frac{Gm_{\rm NS}^2 M_{\rm SMS}\sigma_{\rm T}^2}{c^2 m_{\rm p}^2}\right)^{1/5} \simeq 4.6\cdot 10^{15} M_8^{1/5}\ {\rm cm}, \tag{2}$$

where $M_8 = M_{\rm SMS}/(10^8{\rm M}_\odot)$, and we will use ~ 1 in subsequent numerical estimates. A mean gas number density of the SMS at this moment is

$$\bar{n}_{\rm SMS} = \frac{3}{4\pi}\frac{M_{\rm SMS}}{R_{\rm SMS}^3 m_{\rm p}} \simeq 2.9\cdot 10^{17} M_8^{2/5}\ {\rm cm}^{-3} \tag{3}$$

and the corresponding mean SMS column density is

$$X_{\rm SMS} \simeq \bar{n}_{\rm SMS} m_{\rm p} R_{\rm SMS} \simeq 2.2\cdot 10^9 M_8^{3/5}\ {\rm g\ cm}^{-2}. \tag{4}$$

Under the dynamical friction influence the NSs are sinking deep to the SMS until they are concentrated in the central region with some radius $r = R_{\rm NS} \ll R_{\rm SMS}$. At this time the total mass of NSs becomes of the same order as the mass of the ambient gas, $M_{\rm NS} = (4\pi/3)r^3\rho_{\rm c}$, and a self-gravitating NS cluster is formed. Using the above relation we find the initial parameters of the newly formed NS cluster: the radius $R_{\rm NS} = f_{\rm NS}^{1/3} k_{\rm c}^{-1/3} R_{\rm SMS}$ and virial velocity $v_{\rm NS} = f_{\rm NS}^{1/3} k_{\rm c}^{1/3} v_{\rm SMS}$. At the moment of NS cluster formation the corresponding SMS evolution time $t_{\rm SMS} = t_{\rm SMS}(R_{\rm SMS}) \simeq 7.3\cdot 10^5 M_8^{4/5}$ yrs, virial velocity $v_{\rm SMS} \simeq 0.04 M_8^{2/5} c$. Finally the dynamical evolution of NS cluster is terminated by its dynamical collapse to the MBH.

We employ the simple analytical "evaporation" model [6] for the dynamical evolution of star cluster to fix the time dependence of parameters of the NS cluster. During the process of evaporation of the fast NSs the total virial energy of the cluster remains constant $E = -Nmv^2/2 = const$. Accordingly, the NS velocity dispersion in remaining cluster grows as $v \propto N^{-1/2}$, and cluster contracts as $R \propto N^2$ with a diminishing of star number N. After reaching $v \simeq 0.3c$, which is the onset of the NS cluster global dynamical instability, the remaining NS cluster collapses to the MBH [7, 8]). The rate of NS evaporation from the cluster $\dot{N}_{\rm ev} \simeq \alpha N t_{\rm r}^{-1}$, where a relaxation time $t_{\rm r}$ is given by Eq. (1) and constant $\alpha \sim 10^{-3} - 10^{-2}$ according to the numerical Fokker-Plank calculations [6]. Integration of this equation together with a relation $E = const$ gives the non-dissipative evolution law due to the evaporation of fast stars:

$$N(t) = N_{\rm NS}\left(1 - \frac{t}{t_{\rm ev}}\right)^{2/7}. \tag{5}$$

Here the cluster evolution time $t_{\rm ev} = a_{\rm ev}t_{\rm r}$, with $a_{\rm ev} = (2/7)\alpha^{-1}$, is defined by the relaxation time $t_{\rm r}$ at the moment of NS cluster formation $t = 0$ with the initial parameters $v = v_{)^{2/7}.}(5)$Herehe following numerical estimates we will use $a_{\rm ev} = 10^2 a_2$ with $a_2 \sim 1$.

The evaporation of fast NSs is accompanied by the cluster contraction and by the increasing of the rate of direct NS collisions in the cluster. These collisions proceed most probably through the capture of two NSs into the short-lived binary due to gravitational radiative losses during flyby with a final NS coalescence [8]. The corresponding rate of NS collisions in the cluster (with the gravitational radiation losses taken into account) is [8, 9]:

$$t_{\rm cap}^{-1} = \dot{N}_{\rm cap} \approx 36\sqrt{2}\left(\frac{v}{c}\right)^{31/7}\frac{1}{N}\frac{c}{r_{\rm g}}. \tag{6}$$

Here $r_{\rm g} = 2Gm_{\rm NS}/c^2$ is the gravitational radius of NS and $t_{\rm cap}$ is the time between two successive collisions. We assume that each NS collision is accompanied by the production of energy $E = 10^{52}E_{52}$ erg with $E_{52} \sim 1$ in the form of relativistic fireball. These fireballs would be the central energy source inside the SMS. The star collision is a minor dissipative dynamical process in comparison with the nondissipative star evaporation until $\dot{N}_{\rm cap} < \dot{N}_{\rm ev}$. This inequality is equivalent to $v < v_{\rm cap}$, where

$$v_{\rm cap} = \left(\frac{4}{7\sqrt{3}}\frac{\Lambda}{a_{\rm ev}}\right)^{7/10} c \simeq 8.7 \cdot 10^{-2} a_2^{-7/10} c. \tag{7}$$

At $v > v_{\rm cap}$ the dissipative processes and mass segregation of NS collisions prevail over the NS evaporation in the dense cluster evolution [8, 10].

4 Stationary cavity inside the SMS

The numerous NS collisions produce the coexisting fireballs with a large energy release. The total energy of a single fireball is $E_0 = E_{52}10^{52}$ ergs with $E_{52} \sim 1$. The physics of fireballs was extensively elaborated in the recent years mainly with an aim of modelling the cosmological gamma-ray bursts [11]. In our case we suppose for simplicity that a final fireball after the merging of NSs is (nearly) spherically symmetric. The separation between successive fireballs is $R_{\rm cap} = c\dot{N}_{\rm cap}^{-1}$. The repeating fireballs due to frequent collisions of NSs in the dense enough NS cluster can dig out the rarefied quasi-stationary cavity inside the SMS with radius $R_{\rm cav} \ll R_{\rm SMS}$, which we first describe qualitatively below.

We model the nonstationary stage of cumulative shock expansion by the self-similar spherical shock solution for a central energy source varying in time, $E = At^k$, with $A = const$ and $k = const$ [12, 13]. The particular case of $k = 0$ will correspond to the Sedov *instant shock* solution [14]. The radius of the stationary cavity is determined from the energy flux balance on its boundary at $r = R_{\rm cav}$. The central source power or luminosity is $L = \dot{N}_{\rm cap}E_0$, where $\dot{N}_{\rm cap}$ from Eq. (6) and $E_0 = 10^{52}E_{52}$ ergs is the energy of a single fireball. Correspondingly just outside $R_{\rm cav}$ this energy flux is carried by the hydrodynamic flow $L = 4\pi R_{\rm cav}^2 \rho v(w + v^2/2)$ with the gas velocity $v \sim c_{\rm s,c}$, where $c_{\rm s,c}$ and $\rho_{\rm c}$ are the sound velocity and gas density in the central part of the SMS, respectively. The energy flux contains the enthalpy density $w = \varepsilon + p/\rho = c_{\rm s}^2/(\gamma - 1)$ because the gas produces some work under expansion. Here $\varepsilon = c_{\rm s}^2/[\gamma(\gamma - 1)]$, p, ρ and γ are correspondingly the gas internal energy density, pressure, density and adiabatic index. This energy flux balance relation determines at $\gamma = 4/3$ the radius of the stationary cavity [15]:

$$R_{\rm cav} = \left(\frac{\dot{N}_{\rm cap}E_0}{14\pi\rho_{\rm c}c_{\rm s,c}^3} \right)^{1/2}. \qquad (8)$$

The cavity is supported in the stationary state only if there are simultaneously several fireballs inside it. In other words for the stationary cavity existence the time between successive fireballs is $t_{\rm cap} = \dot{N}_{\rm cap}^{-1}$ must be less than the cavity shrinking (or spreading) time $t_{\rm cav} = R_{\rm cav}/c_{\rm s,c}$. The necessary condition for the stationary cavity existence $t_{\rm cap} < t_{\rm cav}$ with the help of Eqs. (6) and (8) can be written as $v > v_{\rm cav}$, where

$$v_{\rm cav} \simeq 7.3 \cdot 10^{-2} f_{-2}^{7/27} E_{52}^{-7/135} M_8^{91/225} c, \qquad (9)$$

with numerical values $v_{\rm cav} \simeq 4.3 v_{\rm NS} \simeq 1.8 v_{\rm SMS}$. The corresponding minimal radius $R_{\rm min} = R_{\rm cav}(v_{\rm cav})$ of a stationary cavity is

$$R_{\rm min} \simeq \left(\frac{E_0}{4\pi\rho_{\rm c}c_{\rm s,c}^2}\right)^{1/3} \simeq 2.1 \cdot 10^{12} E_{52}^{1/3} M_8^{-2/5} \text{ cm}. \quad (10)$$

The minimal radius of a stationary cavity $R_{\rm min}$ is independent of the fraction $f_{\rm NS}$ of NSs in the SMS. At the moment of a stationary cavity formation inside a SMS, $v = v_{\rm cav}$, we have respectively a NS cluster radius $R(v_{\rm cav}) \simeq 7.5 \cdot 10^{11} f_{-2}^{17/27} E_{52}^{28/135} M_8^{41/225}$ cm, a number of NSs in the cluster $N(v_{\rm cav}) \simeq 3.8 \cdot 10^4 f_{-2}^{31/27} E_{52}^{14/135} M_8^{223/225}$, a NS cluster evolution time $t_{\rm ev}(v_{\rm cav}) \simeq 0.29 a_2 f_{-2}^{41/27} E_{52}^{49/135} M_8^{173/225}$ yr and a central source l a vector item, ¡em¿item¡/em¿, from the simeq $8.6 \cdot 10^{48} E_{52}^{2/3} M_8^{4/5}$ erg/s. After the formation of a stationary cavity inside the SMS we have the hierarchy of radial scales, $R(v_{\rm cav}) \ll R_{\rm min} \ll R_{\rm SMS}$, which justifies the using of SMS central values for the gas density $\rho_{\rm c}$ and sound velocity $v_{\rm s,c}$ for the corresponding values at the cavern radius, $r = R(v_{\rm cav})$.

The power of the central source $L = \dot{N}_{\rm cap} E_0 \propto v^{45/7}$ gradually grows due to the evolutionary growing NS collision rate. So the central source evolution causes the slow expansion of the cavity with velocity

$$\dot{R}_{\rm cav} = \frac{dR_{\rm cav}}{dv}\frac{dv}{dt} = \left(\frac{3}{7}\right)^2 \frac{5}{2} \left(\frac{v}{v_{\rm NS}}\right)^7 \frac{R_{\rm cav}}{t_{\rm ev}}, \quad (11)$$

which is determined by the evolution law Eq. (5). At $v = v_{\rm cav}$ this velocity is small with respect of the ambient gas sound velocity, $\dot{R}_{\rm cav}(v_{\rm cav}) = 5.7 \cdot 10^{-5} c_{\rm s,c}$.

A stationary cavity reaches its maximum radius $R_{\rm max}$ at maximum luminosity of the central source and so the rate of NS collisions Eq. (6) in the evolving NS cluster. It is reasonable to assume that $\dot{N}_{\rm cap}$ reaches its maximum at $v \simeq v_{\rm cap}$ from Eq. (7) when NS cluster evolution time becomes equal to collision time $t_{\rm cap} = \dot{N}_{\rm cap}^{-1}$. During this time NSs pass mostly through mutual collisions and their number in the cluster reduces drastically because of the numerous stellar mass BH formation. The corresponding maximum central source power $L_{\rm max} = \dot{N}_{\rm cap}(v_{\rm cap}) E_0$ at $v = v_{\rm cap}$ due to NS collisions is

$$L_{\rm max} \simeq 2.8 \cdot 10^{49} a_2^{-9/2} f_{-2}^{-5/3} E_{52} M_8^{-9/5} \text{ erg s}^{-1}. \quad (12)$$

By substituting this luminosity in Eq. (8) we find the maximal radius of the cavity $R_{\rm max} = R_{\rm cav}(v_{\rm cap})$ at the maximum central source power

$$R_{\rm max} \simeq 3.8 \cdot 10^{12} a_2^{-9/4} f_{-2}^{-5/6} E_{52}^{1/2} M_8^{-17/10} \text{ cm}. \quad (13)$$

At $v = v_{\rm cap}$, the stationary cavity reaches its maximal radius $R_{\rm max}$. The corresponding NS cluster radius $R(v_{\rm cap}) \simeq 3.6 \cdot 10^{11} a_2^{14/5} f_{-2}^{5/3} M_8^{9/5}$ cm, number of NS in the cluster $N(v_{\rm cap}) \simeq 2.7 \cdot 10^4 a_2^{7/5} f_{-2}^{5/3} M_8^{9/5}$ and NS cluster evolution time $t_{\rm ev}(v_{\rm cap}) \simeq 31.3 a_2^{59/10} f_{-2}^{10/3} M_8^{18/5}$ days respectively. Collisions of NSs during the rather short lifetime $t_{\rm ev}$ at $v = v_{\rm cap}$ supply the total energy

$$E_{\rm max} \simeq L_{\rm max} t_{\rm ev} \simeq 7.6 \cdot 10^{55} a_2^{7/5} f_{-2}^{5/3} E_0 M_8^{9/5} \ \text{erg}$$

into the cavity. Nevertheless this energy is far less than the SMS binding energy $E_{\rm SMS} \simeq G M_{\rm SMS}^2 / 2 R_{\rm SMS} \simeq 2.9 \cdot 10^{59} M_8^{9/5}$ erg, and so the final shock does not influence the SMS state.

The gas inside the cavity is turbulized by the numerous inward and outward colliding shocks from fireballs. The particles are accelerated in such media by Fermi II mechanism. We assume an existence of the equipartition magnetic field induced by the turbulence and dynamo mechanism, $H_{eq}^2/8\pi \simeq \rho_{\rm cav} u_t^2/2$, where $\rho_{\rm cav}$ and u_t are correspondingly the gas density and velocity of turbulent motions in the cavity. As a result we have

$$H_{eq} \simeq (4\pi \rho_{\rm cav})^{1/2} c \simeq 8.1 \cdot 10^4 M_8^{8/10} \Gamma_2^{-1} \ \text{G}. \tag{14}$$

The acceleration time to the energy $E_{\rm max}$ is determined by the scattering off at the largest scales l_0 with a statistically averaged fraction of energy gain $(u_t/c)^2$. Assuming a mildly relativistic turbulence $u_t \leq c$ one obtains

$$t_{acc} \sim \frac{R_{\rm max}}{c} \sim 1.25 \cdot 10^2 a_2^{-9/4} f_{-2}^{-5/6} M_8^{-17/10} E_0^{1/2} \ \text{s}. \tag{15}$$

A typical time of energy losses, determined by *pp*-collisions, is much longer than t_{acc}, and does not prevent acceleration:

$$t_{pp} = \left(\frac{1}{E} \frac{dE}{dt} \right)^{-1} = \frac{1}{f_{\rm esc} \sigma_{pp} n_{\rm cav} c} \simeq 5.7 \cdot 10^3 M_8^{-8/5} \Gamma_2^2 \ \text{s}, \tag{16}$$

where $f_{\rm esc} \approx 0.5$ is a fraction of energy lost by the HE proton in one collision and a cross-section of *pp*-interaction is $\sigma_{pp} \simeq 3 \cdot 10^{-26}$ cm^{-2}. Since the energy losses of accelerated particles are negligible, the maximum energy of acceleration is determined by the particle confinement condition:

$$E_{\rm max} \simeq Z e H_{\rm eq} R_{\rm max} \simeq 9.2 \cdot 10^{19} Z a_2^{-9/4} f_{-2}^{-5/6} M_8^{-9/10} \Gamma_2^{-1} \ \text{eV} \tag{17}$$

with $R_{\rm max}$ given by Eq. (13).

5 Neutrino production and detection

We calculate the production of HE neutrino inside the SMS following to the results of Paper 1. Particles accelerated in the cavity interact with the gas in the SMS interior producing high-energy neutrino flux. We assume that about half of the total luminosity of the central source $L_{\rm max}$ from Eq. (12) is converted into energy of accelerated particles $L_{\rm esc} \sim 0.5 L_{\rm max}$. As estimated in Eq. (4) the column density of the SMS, $X_{\rm SMS} \simeq 2.3 \times 10^9 M_8^{3/5}$ g cm^{-2}, is a large enough to absorb all produced particles except the secondary neutrinos. The charged pions, produced in pp-collisions, with the Lorentz factors up to $\Gamma_{\rm c} \sim (\sigma_{\pi N} n_{\rm SMS} c\tau_\pi)^{-1} \sim 1.5 \cdot 10^5 M_8^{-2/5}$ freely decay in the envelope (here $\sigma_{\pi N} \sim 3 \cdot 10^{-26}$ cm^2 is πN-cross-section, $\tau_\pi \simeq 2.6 \cdot 10^{-8}$ s is the lifetime of charged pion). We assume E^{-2} spectrum of accelerated protons $Q_{\rm esc}(E) = L_{\rm esc}/(\zeta E^2)$, where $\zeta = \ln(E_{\rm max}/E_{\rm min}) \sim 20 - 30$. About half of its energy the protons transfer to high-energy neutrinos through decays of pions, $L_\nu \sim (2/3)(3/4) L_{\rm esc}$, and thus the production rate of $\nu_\mu + \bar{\nu}_\mu$ neutrinos is $Q_{\nu_\mu + \bar{\nu}_\mu}(>E) = L_{\rm esc}/(4\zeta E)$. Crossing the Earth, these neutrinos create deep underground the equilibrium flux of muons, which can be calculated as [16]:

$$F_\mu(>E) = \frac{\sigma_0 N_A}{b_\mu} Y_\mu(E_\mu) \frac{L_{\rm esc}}{4\zeta E_\mu} \frac{1}{4\pi r^2}, \tag{18}$$

where the normalization cross-section $\sigma_0 = 1 \cdot 10^{-34}$ cm^2, $N_A = 6 \cdot 10^{23}$ is the Avogadro number, $b_\mu = 4 \cdot 10^{-6}$ cm^2/g is the rate of muon energy losses, $Y_\mu(E)$ is the integral muon moment of $\nu_\mu N$ interaction [16, 17]. The most effective energy of muon detection is $E_\mu \geq 1$ TeV [16]. The rate of muon events $\dot{N}(\nu_\mu) = F_\mu S$ in the detector with an effective area S at distance r from the source is given by

$$\dot{N}(\nu_\mu) \simeq 2 \left(\frac{L_{\rm esc}}{10^{49}\ \text{erg s}^{-1}} \right) \left(\frac{S}{1\ \text{km}^2} \right) \left(\frac{r}{10^3\ \text{Mpc}} \right)^{-2} \text{day}^{-1}. \tag{19}$$

Thus, we expect a few muons per day in the neutrino telescope of the Ice Cube scale with $S \sim 1$ km from a single source at the distance $\sim 10^3$ Mpc and with a total luminosity of the cavity $L_{\rm max} = 2L_{\rm esc} \simeq 2.8 \cdot 10^{49}$ erg/s from Eq. (12). The numerical value in Eq. (19) is calculated for the suitable choice of free parameters: $a_2 = f_{-2} = E_{52} = M_8 = 1$.

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