Accretion of phantom energy onto black hole

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Abstract

Solution for a stationary spherically symmetric accretion of the relativistic perfect fluid with an equation of state $p(\rho)$ onto the Schwarzschild black hole is presented. This solution is a generalization of Michel solution and applicable to the problem of dark energy accretion. It is shown that accretion of phantom energy is accompanied with the gradual decrease of the black hole mass. Masses of all black holes tend to zero in the phantom energy universe approaching to the Big Rip.

1 Introduction

Our Universe is seems to undergo a period of accelerated expansion and it is assumed that a considerable part of the total density consists of dark energy component with negative pressure [1]. There are several candidates for the dark energy: cosmological constant (Λ) or dynamical component such as quintessence [2] and k-essence [3]. In connection with the solving of the problem of fine-tuning the models of dynamical dark energy component are seem to be more realistic as they admit to construct "tracker" [4] or "attractor" [3] solutions.

One of the peculiar feature of the cosmological dark energy is a possibility of the Big Rip [5]: the infinite expansion of the universe during a finite time. The Big Rip scenario is realized if a dark energy is in the form of the

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phantom energy with $\rho + p < 0$. In this case the scenario of Big Rip is possible when cosmological phantom energy density grows at large times and disrupts finally all bounded objects up to subnuclear scale. Note, however that the only condition $\rho + p < 0$ is not enough for the realization of Big Rip [6]. In [7] the authors analyzed the supernova data in the model independent manner and showed that the presence of the phantom energy with $-1.2 < w < -1$ is preferable in the present moment of time. The analogy between phantom and QFT in curved space-time has been developed in [8]. The entropy of the universe with phantom energy is discussed in [9].

Usually the evolution of quintessence or k-essence are considered in a view of cosmological problems. However in the presence of compact objects such as black holes the evolution of dark energy should be sufficiently different from that in the cosmological consideration. Indeed, what would be the fate of black holes in the universe filled with the phantom energy and coming to Big Rip? Recently we showed that all black holes gradually decrease their masses and very near the Big Rip they finally disappear [10]. In the present work we study in details the stationary accretion of dynamical dark energy into the black hole. As a model of DE we take the perfect fluid with negative pressure. The studying of accretion of perfect fluid on the compact objects originated from Bondi [11]. The relativistic generalization of the perfect fluid accretion were made by Michel [12]. Below we find the solution for a stationary accretion of the relativistic perfect fluid with an arbitrary equation of state $p(\rho)$ onto the Schwarzschild black hole. Using this solution we show that the black hole mass diminishes by accretion of the phantom energy. Masses of all black holes gradually tend to zero in the phantom energy universe approaching to the Big Rip. The diminishing of a black hole mass is caused by the violation of the energy domination condition $\rho + p \geq 0$ which is a principal assumption of the classical black hole 'non-diminishing' theorems [13]. The another consequence of the existence of a phantom energy is a possibility of traversable wormholes [14]. In [15, 16, 17, 18] authors studied the accretion of scalar quintessence field into the black hole, using the specific quintessence potentials $V(\phi)$ for the obtaining of the analytical solution for the black hole mass evolution. We use essentially different approach for the description of DE accretion into black hole, namely, we model the DE by the perfect fluid with the negative pressure.

2 General equations

Let us consider the spherical accretion of dark energy onto black hole. We assume that the density of the dark energy is sufficiently low so that the metric can be described by Schwarzschild metric. We model the dark energy by a perfect fluid with energy-momentum tensor: $T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p g_{\mu\nu}$, where ρ is the density and p is the pressure of the dark energy and u_μ is the four-velocity $u^\mu = dx^\mu/ds$. The integration of the time component of the energy-momentum conservation law $T^{\mu\nu}_{\ ;\nu} = 0$ gives the first integral of motion

$$(\rho + p)\left(1 - \frac{2}{x} + u^2\right)^{1/2} x^2 u = C_1, \qquad (1)$$

where $x = r/M$, $u = dr/ds$ and C_1 is a constant determined below.

Given the equation of state $p = p(\rho)$, one can introduce the function n by the relation:

$$\frac{d\rho}{\rho + p} = \frac{dn}{n}. \qquad (2)$$

The function n plays the role of concentration of the particles, though one can use n for the media without introducing any particles. In this case n is the auxiliary function. For general equation of state $p = p(\rho)$, from (2) we obtain the following solution for n:

$$\frac{n(\rho)}{n_\infty} = \exp\left(\int_{\rho_\infty}^{\rho} \frac{d\rho'}{\rho' + p(\rho')}\right), \qquad (3)$$

From the conservation of energy-momentum along the velocity $u_\mu T^{\mu\nu}{}_{;\nu} = 0$ using (3) we obtain the another first integral:

$$\frac{n(\rho)}{n_\infty} u x^2 = -A, \qquad (4)$$

where n_∞ (the concentration of the dark energy at the infinity) was introduced for convenience. In the case of inflow $u = (dr/ds) < 0$ and the constant $A > 0$. From (1) and (4) one can easily obtain:

$$\frac{\rho + p}{n}\left(1 - \frac{2}{x} + u^2\right)^{1/2} = C_2, \qquad (5)$$

where

$$C_2 = \frac{\rho_\infty + p(\rho_\infty)}{n(\rho_\infty)}. \qquad (6)$$

We will see below that the constant A which determines the flux is fixed for fluids with $\partial p/\partial\rho > 0$. This can be done through finding of the critical point. Following Michel [12] we obtain the parameters of critical point:

$$u_*^2 = \frac{1}{2x_*}, \quad V_*^2 = \frac{u_*^2}{1-3u_*^2}, \tag{7}$$

where

$$V^2 = \frac{n}{\rho+p}\frac{d(\rho+p)}{dn} - 1. \tag{8}$$

From this by using (2) it follows that $V^2 = c_s^2(\rho)$, where $c_s^2 = \partial p/\partial\rho$ is the squared effective speed of sound in the media. Combining the Eqs. (5), (6), (7) and (8) we find the following relation:

$$\frac{\rho_* + p(\rho_*)}{n(\rho_*)} = \left[1 + 3c_s^2(\rho_*)\right]^{1/2} \frac{\rho_\infty + p(\rho_\infty)}{n(\rho_\infty)}, \tag{9}$$

which gives the ρ_* for arbitrary equation of state $p = p(\rho)$. Given ρ_* one can find $n(\rho_*)$ using (3) and values x_*, u_*, using (7) and (8). Then substituting the calculated values in (4) one can find the constant A. Note that there is no critical point outside the black hole horizon ($x_* > 1$) for $c_s^2 < 0$ or $c_s^2 > 1$. This means that for unstable perfect fluid with $c_s^2 < 0$ or $c_s^2 > 1$ a dark energy flux onto the black hole depends on the initial conditions. This result has a simple physical interpretation: the accreting fluid has the critical point if its velocity increases from subsonic to trans-sonic values. In a fluid with a negative c_s^2 or with $c_s^2 > 1$ the fluid velocity never crosses such a point. It should be stressed, however, that fluids with $c_s^2 < 0$ are hydrodynamically unstable (see discussion in [20, 21]). The Eq. (5) together with (3) and (4) describe the requested accretion flow onto the black hole. These equations are valid for perfect fluid with an arbitrary equation of state $p = p(\rho)$, in particular, for a gas with zero-rest-mass particles (thermal radiation) and for a gas with nonzero-rest-mass particles. For a nonzero-rest-mass gas the couple of equations (4) and (5) is reduced to similar ones found by Michel [12]. One would note that the set of equations (3), (4) and (5) are also correct in the case of dark energy and phantom energy $\rho + p < 0$. In this case concentration $n(\rho)$ is positive for any ρ and constant C_2 in (5) is negative.

The black hole mass changes at a rate $\dot{M} = -4\pi r^2 T_0^{\ r}$ due to the fluid accretion. With the help of (4) and (5) this can be expressed as

$$\dot{M} = 4\pi A M^2 [\rho_\infty + p(\rho_\infty)]. \tag{10}$$

For the phantom energy the relation (10) leads to the diminishing of the black hole mass. That means that in the universe filled with phantom energy the black holes should melt away. This result is general, it does not depend on the equation of state $p = p(\rho)$, the only condition $p + \rho < 0$ is important.

3 The analytical models

Let us consider the model of dark energy with linear dependence of pressure from the density:

$$p = \alpha(\rho - \rho_0), \tag{11}$$

which include, among others, the ultra-relativistic gas ($p = \rho/3$) and simplest models of dark energy ($\rho_0 = 0$ and $\alpha < 0$). Introduced value α is connected with usual equation of state $w = p/\rho$ by the relation $w = \alpha(\rho - \rho_0)/\rho$. For $\alpha < 0$ there is no critical point for the flux of the fluid into the black hole. In the case of $\alpha > 0$, using (7) and (8) we find the parameters for critical point in model (11):

$$x_* = \frac{1 + 3\alpha}{2\alpha}, \quad u_*^2 = \frac{\alpha}{1 + 3\alpha}. \tag{12}$$

It should be noted that in the linear model (11) the parameters of critical point (12) determined only by $\partial p/\partial\rho = \alpha$ and do not depend on the parameter ρ_0, which determines what physical fluid is considered: relativistic gas, dark energy or phantom energy. Note also that for $\alpha > 1$ (that corresponds to the non-physical situation of superluminal speed of sound) there is no critical point outside the black hole. Let us calculate the constant A which determines the flux of the fluid into the black hole. From Eq. (3) we find:

$$\frac{n}{n_\infty} = \left|\frac{\rho_{\rm eff}}{\rho_{{\rm eff},\infty}}\right|^{1/(1+\alpha)}, \tag{13}$$

where we defined the effective density $\rho_{\rm eff} \equiv \rho + p = -\rho_0\alpha + (1 + \alpha)\rho$. Using (9) we obtain:

$$\left(\frac{\rho_{{\rm eff}_*}}{\rho_{{\rm eff},\infty}}\right)^{\alpha/(1+\alpha)} = (1 + 3\alpha)^{1/2}, \tag{14}$$

where $\rho_{{\rm eff}_*}$ is the value of effective density at the critical point and $\rho_{{\rm eff},\infty}$ is the effective density at the infinity. Substituting (14) in (13) and then using

(4) we find for linear model:

$$A = \frac{(1+3\alpha)^{(1+3\alpha)/2\alpha}}{4\alpha^{3/2}}. \tag{15}$$

It is easily seen that $A \geq 4$ for $0 < \alpha < 1$. For $\alpha = 1$ that corresponds to $c_s = 1$ the we have $A = 4$. From this we may conclude that for typical sound speeds the constant A has value around unity. For some particular choices of parameter α the values $\rho(x)$ and $u(x)$ can be calculated analytically. For example, for $\alpha = 1/3$ the fluid density is given by:

$$\rho = \frac{\rho_0}{4} + \left(\rho_\infty - \frac{\rho_0}{4}\right)\left[z + \frac{1}{3(1-2x^{-1})}\right]^2, \tag{16}$$

where

$$z = \begin{cases} -2\sqrt{\frac{a}{3}}\cos\left(\frac{2\pi}{3} - \frac{\beta}{3}\right), & 2 < x < 3, \\ 2\sqrt{\frac{a}{3}}\cos\left(\frac{\beta}{3}\right), & x > 3, \end{cases}$$

$$\beta = \cos\left[\frac{b}{2\,(a/3)^{3/2}}\right]$$

and

$$a = \frac{1}{3\left(1-\frac{2}{x}\right)^2}, \quad b = \frac{2}{27\left(1-\frac{2}{x}\right)^3} - \frac{108}{\left(1-\frac{2}{x}\right)x^4}.$$

The density distribution for another physically interesting case $\alpha = 1$ is given by:

$$\rho = \frac{\rho_0}{4} + \left(\rho_\infty - \frac{\rho_0}{4}\right)\left(1 + \frac{2}{x}\right)\left(1 + \frac{4}{x^2}\right). \tag{17}$$

The corresponding radial fluid velocity $u = u(x)$ can be calculated by substituting of (16) or (17) into (1). For $\rho_0 = 0$ the solutions (16) and (17) describe correspondingly a thermal radiation and a fluid with ultra-hard equation of state. In the case of $\rho_\infty < \alpha\rho_0/(1+\alpha)$ the solutions (16) and (17) describe the phantom energy falling onto the black hole. For example, a phantom energy flow with parameters $\alpha = 1$ and $\rho_0 = 4\rho_\infty$ results in a black hole mass diminishing with the rate $\dot{M} = -8\pi(2M)^2\rho_\infty$.

4 Black holes in the universe with Big Rip

Now we turn to the problem of the black hole evolution in the universe with the Big Rip when a scale factor $a(t)$ diverges at finite time [5]. For simplicity we will take into account only dark energy and will disregard all others forms of energy. The Big Rip solution is realized for in the linear model (11) for $\rho+p<0$ and $\alpha<-1$. From the Friedman equations for the linear equation of state model one can obtain: $|\rho+p| \propto a^{-3(1+\alpha)}$. Taking for simplicity $\rho_0=0$ we find the evolution of the density of a phantom energy in the universe:

$$\rho_\infty = \rho_{\infty,i}\left(1-\frac{t}{\tau}\right)^{-2}, \tag{18}$$

where

$$\tau^{-1} = -\frac{3(1+\alpha)}{2}\left(\frac{8\pi}{3}\rho_{\infty,i}\right)^{1/2} \tag{19}$$

and $\rho_{\infty,i}$ is the initial density of the cosmological phantom energy and the initial moment of time is chosen so that the 'doomsday' comes at time τ. From (18) and (19) it is easy to see that the Big Rip solutionis realized for $\alpha \equiv \partial p/\partial\rho < -1$. In general, the satisfying the condition $\rho+p<0$ is not enough for the possibility for Universe to come to Big Rip. From (10) using (18) we find the black hole mass evolution in the universe coming to the Big Rip:

$$M = M_i\left(1+\frac{M_i}{\dot{M}_0\,\tau}\,\frac{t}{\tau-t}\right)^{-1}, \tag{20}$$

where

$$\dot{M}_0 = (3/2)\,A^{-1}|1+\alpha|, \tag{21}$$

and M_i is the initial mass of the black hole. For $\alpha=-2$ and typical value of $A=4$ (corresponding to $u_{\rm H}=-1$) we have $\dot{M}_0=3/8$. In the limit $t\to\tau$ (i.e. near the Big Rip) the dependence of black hole mass on t becomes linear, $M \simeq \dot{M}_0\,(\tau-t)$. While t approaches to τ the rate of black hole mass decrease does not depend on both an initial black hole mass and the density of the phantom energy: $\dot{M} \simeq -\dot{M}_0$. In other words masses of all black holes in the universe tend to be equal near the Big Rip. This means that the phantom energy accretion prevails over the Hawking radiation until the mass of black hole is the Planck mass. However, formally all black holes in the universe evaporate completely at Planck time before the Big Rip due to Hawking radiation.

5 Scalar field accretion

In remaining let us confront our results with the calculations of (not phantom) scalar field accretion onto the black hole [15, 16, 17, 18]. The dark energy is usually modelled by a scalar field ϕ with potential $V(\phi)$. The perfect fluid approach is more rough because for given 'perfect fluid variables' ρ and p one can not restore the 'scalar field variables' ϕ and $\nabla\phi$. In spite of the pointed difference between a scalar field and a perfect fluid we show below that our results are in a very good agreement with the corresponding calculations of a scalar field accretion onto the black hole.

The Lagrangian of a scalar field is $L = K - V$, where K is a kinetic term of a scalar field ϕ and V is a potential. For the standard choice of a kinetic term $K = \phi_{;\mu}\phi^{;\mu}/2$ the energy flux is $T_{0r} = \phi_{,t}\phi_{,r}$. Jacobson [15] found the scalar field solution in Schwarzschild metric for the case of zero potential $V = 0$: $\phi = \dot{\phi}_\infty[t + 2M\ln(1 - 2M/r)]$, where ϕ_∞ is the value of the scalar field at the infinity. In [17] it was shown that this solution remains valid also for a rather general form of runaway potential $V(\phi)$. For this solution we have $T_0{}^r = -(2M)^2\dot{\phi}^2_\infty/r^2$ and correspondingly $\dot{M} = 4\pi(2M)^2\dot{\phi}^2_\infty$.

The energy-momentum tensor constructed from Jacobson solution completely coincides with one for perfect fluid in the case of ultra-hard equation of state $p = \rho$ under the replacement $p_\infty \to \dot{\phi}^2_\infty/2$, $\rho_\infty \to \dot{\phi}^2_\infty/2$. It is not surprising because the theory of a scalar field with zero potential $V(\phi)$ is identical to perfect fluid consideration [22]. In a view of this coincidence it is easily to see the agreement of our result (10) for $\dot{M}$ in the case of $p = \rho$ and the corresponding result of [15, 17].

To describe the phantom energy the Lagrangian of a scalar field must have a negative kinetic term [5], for example, $K = -\phi_{;\mu}\phi^{;\mu}/2$ (for the more general case of the negative kinetic term see [19]). In this case the phantom energy flux onto black hole has the opposite sign, $T_{0r} = -\phi_{,t}\phi_{,r}$, where ϕ is the solution of the same Klein-Gordon equation as in the case of standard scalar field, however with the replacement $V \to -V$. For zero potential this solution coincides with that obtained by Jacobson [15] for a scalar field with the positive kinetic term. Lagrangian with negative kinetic term and $V(\phi) = 0$ does not describe, however, the phantom energy. At the same time, the solution for scalar field with potential $V(\phi) = 0$ is the same as with a positive constant potential $V_0 = const$, which can be chosen so that $\rho = -\dot{\phi}^2/2 + V_0 > 0$. In this case the scalar field represents the required accreting phantom energy $\rho > 0$ and $p < -\rho$ and provides the decrease of black hole mass with the rate $\dot{M} = -4\pi(2M)^2\dot{\phi}_\infty^2$.

The simple example of phantom cosmology (without a Big Rip) is realized for a scalar field with the potential $V = m^2\phi^2/2$, where $m \sim 10^{-33}$ eV [23]. After short transition phase this cosmological model tends to the asymptotic state with $H \simeq m\phi/3^{1/2}$ and $\dot{\phi} \simeq 2m/3^{1/2}$. In the Klein-Gordon equation the m^2 term (with the mentioned replacement $V \to -V$) is comparable to other terms only at the cosmological horizon distance. This means that the Jacobson solution is valid for this case also. Calculating the corresponding energy flux one can easily obtain $\dot{M} = -4\pi(2M)^2\dot{\phi}_\infty^2 = -64M^2m^2/3$. For $M_0 = M_\odot$ and $m = 10^{-33}$ eV the effective time of black hole mass decrease is $\tau = (3/64)M^{-1}m^{-2} \sim 10^{32}$ yr.

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