BCS-Bose Crossover in Color Superconductivity

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Abstract

It is shown that the onset of the color superconducting phase occurs in the BCS-BE crossover region.

During the last 2-3 years color superconductivity became one of the QCD focal points –see review papers [1]-[3]. To a large extent (but not completely) the basic ideas of the subject are traced back to the BCS theory of superconductivity and its later development. It is known that the discovery of high temperature superconductors (HTSC) gave rise to new ideas and approaches and revealed interest to the problem of the transition from the BCS regime to the Bose-Einstein (BE) condensation. The BCS-BE crossover is important for the physics of HTSC since the underlying distinction of HTSC from ordinary superconductors is that they are characterized by much smaller value of the dimensionless parameter $\xi n^{1/3}$, where ξ is the coherence length and n is the carrier density. In the BCS, or weak coupling regime, $\xi^3 n \sim (10^8 - 10^{10})$ while in the opposite strong coupling case $\xi^3 n \lesssim 1$ and we are dealing with the compact pairs of composite bosons, which may undergo BE condensation. It has been suggested (see e.g. [4]) that the description of the HTSC might require an intermediate approach between the BCS and BE limits. The evolution from weak to strong coupling was theoretically investigated [5] before the discovery of HTSC. It was shown that the transition proceeds via a smooth crossover though the two limits are physically quite different (see also [6]).

Having reminded these well known facts we may formulate the question which forms the core of the present note. As model calculations show [7] the onset of the color superconducting phase in two flavor QCD(the so called 2SC phase) occurs at rather low quark densities n, namely at n only three times larger than the quark density in normal nuclear matter, or even at lower ones [8].

Thus it is natural to ask in which region regarding the BCS-BE crossover does it happen?

According to [7] in QCD with two massless flavors transition to the superconducting $2SC$ phase occurs at $n^{1/3} \simeq 0.2$GeV (the dimension of n is $1/fm^3$ or GeV^3). As for the corresponding value of ξ we may only rely on some estimates since accurate calculations are lacking. One should also keep in mind possible distinction between the correlation length and the pair size. The two quantities coincide in the BCS regime [9] while in the BE region the pair size is smaller than the coherence length [10, 6]. With these reservations being made we quote the value $\xi \simeq 0.8$ fm from [11]. Rather close result, namely $\xi \simeq 0.6$ fm, follows from the BCS estimate $\xi \simeq 1/\pi\Delta$ [12], where $\Delta \simeq 0.1$ GeV [1, 7]. Thus the diquark pair in the "newly born" color superconducting phase is rather compact. This is easy to understand from simple physical considerations. In color antitriplet $\overline{3}$ state the one-gluon exchange leads to the quark-quark potential which is only a factor of two weaker than the quark-antiquark one. Instanton or NJL models also result in a rather strong $q-q$ attraction. Consider the NJL "weak coupling" solution [1, 3] $\Delta = 2\omega_D \exp(-1/\rho_{NJL})$, where ω_D is the Debye frequency, $\rho_{NJL} = 8g^2\mu^2/\pi^2$, $g^2 \simeq 2GeV^{-2}$ is the NJL coupling constant, $\mu \simeq 0.4GeV$ is the chemical potential corresponding to the onset of the 2SC phase. We immediately see that $\rho_{NJL} \simeq 0.3 > \rho_{BCS}$, i.e. the quark-quark interaction is stronger than phonon mediated electron-electron interaction and in this sense the "newly born" color superconducting phase does not correspond to the standard BCS weak coupling limit.

We conclude that the onset of the color superconducting phase corresponds to $\xi n^{1/3} \sim (1fm)(0.2GeV) \sim 1, \ \ \xi\mu \sim (fm)(0.4GeV) \sim 2$.

These values are at least two orders of magnitude smaller than those corresponding to the BCS regime. In order to understand to which region of the BCS-BE "phase diagram" they correspond one has to resort to model calculations performed for the system of electrons. Most results have been obtained for the system in two dimensions [6]. Crossover in three dimensions has been studied in [13]. Transition between the two regimes occurs in a narrow range of the parameter $\xi n^{1/3}$ and for the electron systems with simple model potentials (finite range, separable, Gaussian) the value $\xi n^{1/3} \simeq 1$ corresponds to the lower limit of the BCS-like region. Needless to say that the extrapolation of this result to the system of massless quarks may be considered only as an educated guess and the problem deserves a dedicated study.

The importance of the BCS-BE crossover for the color superconductivity problem was first outlined on [3] and [14] -[15]. In [14] the low density regime was investigated by extrapolating the single-gluon exchange model which is an adequate tool at asymptotically high densities. The smooth transition from $\xi n^{1/3} \gg 1$ to $\xi n^{1/3} = 10$ at $\mu = 0.8$ GeV was observed. See also Ref.[18] for the detailed description of the mean-field dynamics of the low density region.

Let us now explicitly show the evolution of the pair wave function (PWF) from one regime to another within the mean field approximation. We follow the arguments presented in Ref. [5]. The general expression for the PWF in coordinate representation reads

$$\langle \psi_+(\mathbf{r}_1)\psi_-(\mathbf{r}_2) = \frac{1}{4V}\sum_p \frac{\Delta_p}{\varepsilon_p} th \frac{\varepsilon_p}{2T} e^{i\mathbf{p}(\mathbf{r}_1-\mathbf{r}_2)}, \tag{1}$$

where the indices $\pm$ correspond to spins up and down, $\varepsilon_p = (\xi_p^2 + \Delta^2)^{1/2}$, $\xi_p = p^2/2m - \mu$. At $T = 0$ the PWF in momentum space is

$$\varphi_p = \frac{\Delta_p}{\varepsilon_p} = \frac{\Delta_p}{\xi_p}(1 - 2n_p), \tag{2}$$

$$n_p = \frac{1}{2}(1 - \frac{\xi_p}{\varepsilon_p}). \tag{3}$$

In the BCS regime binding is effectively located within a thin layer around the Fermi surface and the distribution n_p approaches a unit step function $n_p = \theta(p_f - p)$, while in Bose (delute, or strong coupling) regime the distribution n_p is broadened over a wide interval. Next we consider the gap equation

$$\Delta_p = \frac{g^2}{2V}\sum_{p'} \frac{\Delta_{p'}}{\varepsilon_{p'}} th \frac{\varepsilon_{p'}}{2T}. \tag{4}$$

At $T = 0$ it can be rewritten as

$$(\frac{p^2}{m} - 2\mu)\varphi_p = \frac{g^2}{V}(1 - 2n_p)\varphi_p. \tag{5}$$

In the delute regime $n_p \ll 1$ and (5) reduces to the ordinary Schrodinger equation for a single bound pair. In the opposite BCS case when n_p approaches a step function the gap equation leads to the famous exponential BCS solution

$$\Delta = 2\omega_D \exp\left\{-\frac{2\pi^2}{g^2 m p_F}\right\}, \tag{6}$$

where ω_D is the Debye frequency which cuts the summation of high frequencies in the gap equation (4). The generalization of the above equations for the case of massless quarks with color and flavor is straightforward.

Another question is what are the physical consequences of the fact that the formation of the color superconducting gap is at least partly due to the existence of the preformed Bose pairs of quarks. At present we can again rely only on the corresponding studies of the electron systems [5, 17]. The key point here is that the physical origin of the critical temperature T_c is absolutely different in the limits of weak and strong coupling [5]. In the BCS region T_c corresponds to the breaking of Cooper pairs while in the BE limit T_c corresponds to the pairs center-of mass motion and to the population of zero-momenta state. Transition from the weak to strong coupling regimes results in the decrease of T_c. comparing to mean-field value. Formally this should also follow from equations (1)-(2).

Finally we note that calculations of the parameters μ and T at which the transition into color superconducting phase occurs have been performed neglecting the gluon condensate. General arguments presented in [18] show that color-magnetic field which is "frozen" into the quark system in the form of the gluon condensate shifts the transition towards higher densities. (see also [19]). Therefore equations (1) for the BCS-BE crossover should be embedded into the background gluon field.

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