Distribution of valence quarks in hadrons in QCD: theoretical method of calculations.

B.L.Ioffe
Institute of Theoretical and Experimental Physics.
B.Cheremushkinskaya 25. 117218 Moscow. Russia

The general method for calculation of valence quark distributions in hadrons at intermediate x is presented. The imaginary part of virtual photon forward scattering amplitude on quark current with hadron quantum number is considered in the case, when initial and final virtualities of the current p_1^2 and p_2^2 are different, negative and large: $\mid p_1^2 \mid, \mid p_2^2 \mid \gg R_c^{-2}$, where R_c is confinement radius. The operator product expansion (OPE) in p_1^2, p_2^2 up to dimension 6 operators is performed. Double dispersion representations in p_1^2, p_2^2 of the amplitude in terms of physical states contributions are used. Equalling them to those calculated in QCD by OPE the desired sum rules for quark distributions in mesons are found. The double Borel transformations are applied to the sum rules, killing non-diagonal transition terms, which deteriorated the accuracy in the previous calculations of quark distributions in nucleon. Leading order perturbative corrections are accounted. Valence quark distributions in pion, longitudinally and transversally polarized ρ-mesons and proton are calculated at intermediate x, $0.2 \lesssim x \lesssim 0.7$ and normalization points $Q^2 = 2 - 5\ GeV^2$ with no fitting parameters. In cases of pion and proton the results are in agreement with found correspondingly from the data on the Drell-Yan process and deep inelastic scattering. Valence quark distributions in transversally and longitudinally polarized ρ-mesons are essentially different one from another and also differ from those in pion.

1. Introduction

Quark and gluon distributions in hadrons are not fully understood in QCD. QCD predicts the evolution of these distributions with Q^2 in accord with the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) [1]-[3] equations, but not the initial values from which this evolution starts. The standard way of determination of quark and gluon distributions in nucleon is the following [4-8] (for theoretical reviews see [9]). At some $Q^2 = Q_0^2$ (usually, at low or intermediate $Q^2 \sim 2 - 5 GeV^2$) the form of quark (valence and sea) and gluon distributions is assumed and characterized by the number of free parameters. Then, by using DGLAP equations, quark and gluon distributions are calculated at all Q^2 and x and compared with the whole set of the data on deep inelastic lepton-nucleon scattering (sometimes also with prompt photon production, jets at high $p_\perp$ etc; for pion – with the data on Drell-Yan process). The best fit for the parameters is found and, therefore, quark and gluon distributions are

d t rmin d at all Q^2, including th ir initial valu s $q(Q_0^2, x)$, $g(Q_0^2, x)$. Evid ntly, such an approach is not compl t ly satisfactory from th or tical point of vi w - it would b d sirabl to d t rmin th initial distribution dir ctly from QCD.

I pr s nt you bas d on QCD m thod of th calculation of u and d val nc quark distributions in hadrons at low $Q^2 \sim 2-5$ G V^2 and int rm diat x. Th id a of th m thod was sugg st d in [10] and d v lop d in [11-13]. R c ntly, th m thod had b n improv d and val nc quark distributions in pion [14] and transv rsally and longitudinally polariz d ρ-m son [15] had b n calculat d, what was impossibl in th initial v rsion of th m thod. For val nc quark distributions in proton th improv d m thod [16] gav a much b tt r agr m nt with xp rim nt, than th arli r v rsion [11]. Th id a of th approach (in th improv d v rsion) is to consid r th imaginary part (in s-chann l) of a four-point corr lator $\Pi(p_1, p_2, q, q')$ corr sponding to th forward scatt ring of two quark curr nts, on of which has th quantum numb rs of hadron of int r st and th oth r is l ctromagn tic. It is supp s d that virtualiti s of th photon q^2, q'^2 and hadron curr nts p_1^2, p_2^2 ar larg and n gativ $|q^2| = |q'^2| \gg |p_1^2|$, $|p_2^2| \gg$ $_c^{-2}$, wh r $_c$ is th confin m nt radius. It was shown in [11] that in this cas th imaginary part in s-chann l [$s = (p_1 + q)^2$] of $\Pi(p_1, p_2; q_1, q')$ is dominat d by a small distanc contribution at int rm diat x. (Th standard notation is us d: x is th jork n scaling variabl , $x = -q^2/2\nu$, $\nu = p_1 q$). Th proof of this stat m nt, giv n in [11], is bas d on th fact that for th imaginary part of th forward scatt ring amplitud th position of th clos st to z ro singularity in mom ntum transf r is d t rmin d by th boundary of th Mand lstam sp ctral function and is giv n by th quation

$$t_0 = -4\frac{x}{1-x}p^2 \tag{1}$$

Th r for , if $|p^2|$ is larg and x is not small, th n v n at $t = 0$ (th forward amplitud) th virtualiti s of int rm diat stat s in t-chann l ar larg nough for OPE to b applicabl . So, in th m ntion d abov domain of q^2, q'^2, p_1^2, p_2^2 and int rm diat x $Im\Pi(p, p_2; q, q')$ can b calculat d using th p rturbation th ory and th op rator product xpansion in both s ts of variabl s $q^2 = q'^2$ and p_1^2, p_2^2. Th approach is inapplicabl at small x and x clos to 1. This can b und rstood for physical r asons. In d p in lastic scatt ring at larg $|q^2|$ th main int raction r gion in spac -tim is th light-con domain and longitudinal distanc s along th light-con ar proportional to $1/x$ and b com larg at small x [17,18]. For OPE validity it is n c ssary for th s longitudinal distanc s along light-con to b also small, that is not th cas at small x. At $1 - x \ll 1$ anoth r condition of applicability of th m thod is violat d. Th total n rgy squar $s = Q^2(1/x - 1) + p_1^2$ $Q^2 = -q^2$ is not larg at $1 - x \ll 1$. Num rically, th typical valu s to b us d b low ar $Q^2 \sim 5Ge\ ^{\prime 2}$, $p_1^2 \sim -1Ge\ ^{\prime 2}$. Th n, v n at $x \approx 0.7$, $s \approx 1Ge\ ^{\prime 2}$, i. ., at such x w ar in th r sonanc r gion. So, on may xp ct b for hand, that our m thod could work only up to $x \approx 0.7$. Th inapplicability of th m thod at small and larg x manif sts its lf in th blow-up of high r ord r t rms of OPE. Mor pr cis limits on th applicability domain in x will b found from th magnitud of th s t rms.

Th furth r proc dur is common for QCD sum rul s. On on hand th four-point corr lator $\Pi(p_1, p_2; q, q')$ is calculat d by p rturbation th ory and OPE.On th oth r hand, th doubl disp rsion r pr s ntation in p_1^2, p_2^2 in t rms of physical stat s contributions is writt n for th sam corr lator and th contribution of th low st stat is xtract d using th or l transformaion. By qualing th s two xpr ssion th d sir d quark distribution is found.

. The method

Consid r th forward 4-point corr lator:

$$\Pi(p_1,p_2;q,q') = -i \int d^4x d^4y d^4z e^{ip_1x+iqy-ip_2z} \times \langle 0|T\left\{ j^h(x),\ j^{el}(y),\ j^{el}(0),\ j^h(z)\right\}|0\rangle \quad (2)$$

H r p_1 and p_2 ar th initial and final mom nta carri d by hadronic curr nt j^h, q and $q' = q+p_1-p_2$ ar th initial and final mom nta carri d by virtual photons. (Lor nz ind c s ar omitt d). It will b v ry ss ntial for us to consid r non- qual p_1, p_2 and tr at p_1^2, p_2^2 as two ind p nd nt variabl s. How v r, w may put $q^2 = q'^2 = q^2$ and $t = (p_1-p_2)^2 = 0$. W ar int r st d in imaginary part of $\Pi(p_1^2, p_2^2, q^2, s)$ in s chann l:

$$Im\Pi(p_1^2,p_2^2,q^2,s) = \frac{1}{2i}\left[\Pi(p_1^2,p_2^2,q^2,s+i\varepsilon) - \Pi(p_1^2,p_2^2,q^2,s-i\varepsilon)\right] \quad (3)$$

In ord r to construct r pr s ntation of $Im\Pi(p_1^2, p_2^2, q^2, s)$ in t rms of contributions of physical stat s, l t us writ for $Im\Pi(p_1^2, p_2^2, q^2, s)$ th doubl disp rsion r lation in p_1^2, p_2^2:

$$Im\Pi(p_1^2,p_2^2,q^2,s) = a(q^2,s) + \int_0^\infty \frac{\varphi(q^2,s,u)}{u-p_1^2}du + \int_0^\infty \frac{\varphi(q^2,s,u)}{u-p_2^2}du$$

$$+ \int_0^\infty du_1 \int_0^\infty du_2\ \frac{\rho(q^2,s,u_1,u_2)}{(u_1-p_1^2)(u_2-p_2^2)} \quad (4)$$

Th s cond and th third t rms in th right-hand sid (rhs) of (4) may b consid r d as subtraction t rms to th last on – th prop rly doubl sp ctral r pr s ntation. Th first t rm in th rhs of (4) is th subtraction t rm to th s cond and third on s. Th r for , (4) has th g n ral form of th doubl sp ctral r pr s ntation with on subtraction in both variabl s – p_1^2 and p_2^2. Apply th doubl Bor l transformation in p_1^2, p_2^2 to (4). This transformation kills thr first t rms in rhs of (4) and w hav

$$\mathcal{B}_{M_1^2}\mathcal{B}_{M_2^2}\ Im\Pi(p_1^2,p_2^2,q^2,s) = \int_0^\infty du_1 \int_0^\infty du_2 \rho(q^2,s,u_1,u_2) exp\left[-\frac{u_1}{M_1^2} - \frac{u_2}{M_2^2}\right] \quad (5)$$

Th int gration r gion ov r u_1, u_2 may b divid d into 4 ar as:

I. $u_1 < s_0$; $u_2 < s_0$

II. $u_1 < s_0$; $u_2 > s_0$

III. $u_1 > s_0$; $u_2 < s_0$

IV. $u_1, u_2 > s_0$

Using th standard QCD sum rul mod l of hadronic sp ctrum and th hypoth sis of quark-hadron duality, i. . th mod l with on low st r sonanc plus continuum, on may s , that ar a I corr sponds to r sonanc contribution. Sp ctral d nsity in this ar a can b writt n as

$$\rho(u_1,u_2,x,Q^2) = g_h^2 \cdot 2\pi F_2(x,Q^2)\delta(u_1-m_h^2)\delta(u_2-m_h^2), \quad (6)$$

wh r g_h is d fin d as

$$\langle 0|j_h|\ \rangle = g_h \quad (7)$$

(For simplicity we consider the case of the Lorentz scalar hadronic current.) If in $Im\Pi(p_1,p_2,q,q')$ the structure, proportional to $P\,P$ $[P=(p_1+p_2)/2]$ is considered, then in the lowest twist approximation $F_2(x,Q^2)$ is the structure function, depending on the Bjorken scaling variable x and weakly on $Q^2=-q^2$.

In area (IV), where both variables $u_{1;2}$ are far from resonance region, the non-perturbative effects may be neglected, and as usual in sum rules, the spectral function of hadron state is described by the bare loop spectral function ρ^0 in the same region

$$\rho(u_1,u_2,x)=\rho^0(u_1,u_2,x) \tag{8}$$

In areas (II),(III) one of the variables is far from the resonance region, but other is in the resonance region, and the spectral function in this region is some unknown function $\rho=\psi(u_1,u_2,x)$, which corresponds to transitions like $\to$ *continuum*. The areas II-III contributions are exponentially suppressed, and, using the standard hypothesis of quark-hadron duality, we may estimate them as a bare loop contribution in the same integration region. Equating physical and QCD representation of Π and taking into account cancellation of appropriate parts in the left and right sides, one can write the following sum rules:

$$Im\ \Pi^0_{\mathsf{QCD}}+\text{Power correction}=2\pi F_2(x,Q^2)g_{\mathsf{h}}^2 e^{-m_h^2(\frac{1}{M_1^2}+\frac{1}{M_2^2})}$$

$$Im\Pi^0_{\mathsf{QCD}}=\int_0^{s_0}\int_0^{s_0}\rho^0(u_1,u_2,x)e^{-\frac{u_1+u_2}{2M^2}} \tag{9}$$

It can be shown (see below), that for bare loop diagram $\psi(u_1,u_2,x))\sim\delta(u_1-u_2)$, and, as a consequence, the areas II and III contributions are zero in our model of hadronic spectrum.

It is worthmentioning that if we would consider the forward scattering amplitude from the beginning, put $p_1=p_2=p$ and perform Borel transformation in p^2, then unlike (5), the contributions of the second and third terms in (4) would not vanish. They just correspond to the non-diagonal transition matrix elements and are proportional to

$$\langle 0|j^{\mathsf{h}}|\ ^*\rangle\ \frac{1}{p^2-m_{\mathsf{h}}^{*2}}\ \langle\ ^*|j^{\mathsf{el}}(x)j^{\mathsf{el}}(0)|\ \rangle\ \frac{1}{p^2-m_{\mathsf{h}}^2}\langle\ |j^{\mathsf{h}}|0\rangle \tag{10}$$

From decomposition

$$\frac{1}{p^2-m_{\mathsf{h}}^{*2}}\ \frac{1}{p^2-m_{\mathsf{h}}^2}=\frac{1}{m_{\mathsf{h}}^{*2}-m_{\mathsf{h}}^2}\left(\frac{1}{p^2-m_{\mathsf{h}}^{*2}}-\frac{1}{p^2-m_{\mathsf{h}}^2}\right) \tag{11}$$

it is clear that in this case (10) may contribute to the second (or third) term in (4) and after Borel transformation the contribution of the second term in (11) has the same Borel exponent $e^{-m_h^2/M^2}$ as the lowest resonance contribution. The only difference is in pre-exponent factors: they are $1/M^2$ in front of the resonance term and Const. in front of non-diagonal terms. This difference was used in order to get rid of non-diagonal terms: application of the differential operator $(\partial/\partial(1/M^2)e^{m_h^2/M^2}$ to the sum rule kills the Borel non-suppressed nondiagonal terms, but detriorates the accuracy and shrinks the applicability domain of the sum rule (particularly, the domain in x, where the sum rule is valid).

3. uark distri ution s i pio

It is nough to find th distribution of val nc u-quark in π^+, sinc $\bar{d}(x) = u(x)$. Th most suitabl hadronic curr nt in this cas is th axial curr nt

$$j_5 = \bar{u}\gamma\ \gamma_5 d \tag{12}$$

In ord r to find th u-quark distribution, th l ctromagn tic curr nt is chos n as u-quark curr nt with th unit charg

$$j^{\mathrm{el}} = \bar{u}\gamma\ u \tag{13}$$

Th bar loop Fig.1 contribution is giv n by

$$\begin{aligned} Im\ \Pi \quad &= -\Big\{\frac{3}{(2\pi)^2}\ \frac{1}{2} \quad \int d^4k\ \frac{1}{k^2}\ \frac{1}{(k+p_2-p_1)^2}\delta[(q+k)^2]\delta[p_1-k)^2] \\ &\times Tr\ \gamma\ \hat{k}\gamma\ (\hat{k}+\hat{q})\gamma\ (\hat{k}+\hat{p}_2-\hat{p}_1)\gamma\ (\hat{k}-\hat{p}_1)\Big\} \end{aligned} \tag{14}$$

It is conv ni nt to introduc

$$P = (p_1+p_2)/2, \quad q = p_1 - p_2,\ q^2 = 0 \tag{15}$$

Th t nsor structur , chos n to construct th sum rul is a structur proportional to $P\ P\ P\ P\ /\nu$. Th r asons ar th following. Th r sults of th QCD sum rul s calculations ar mor r liabl , if invariant amplitud at kin matical structur with maximal dim nsion is us d. Diff r nt $p_1 \neq p_2$ ar important only in d nominators, wh r th y allow on to s parat th t rms in disp rsion r lations. In num rators on may r strict on s lf to consid ration of t rms proportional to 4-v ctor P and ignor th t rms $\sim q$. Th n th factor $P\ P$ provid s th contribution of $F_2(x)$ structur function and th factor $P\ P$ corr sponds to contribution of spin z ro stat s. (Th factor $1/\nu$ is scaling factor : $w_2 = F_2/\nu$.)

L t us us th notation

$$\Pi \quad = (P\ P\ P\ P\ /\nu)\Pi(p_1^2, p_2^2, x) + ... \tag{16}$$

Th n $Im\Pi(p_1^2, p_2^2, x)$ can b calculat d from (14) and th r sult is [14]:

$$Im\Pi(p_1^2, p_2^2, x) = \frac{3}{\pi}x^2(1-x)\int_0^\infty du_1 \int_0^\infty du_2\ \frac{\delta(u_1-u_2)}{(u_1-p_1^2)(u_2-p_2^2)} \tag{17}$$

Th matrix l m nt of th axial curr nt b tw n vacuum and pion stat is w ll known

$$\langle 0|j_5|\pi\rangle = ip\ f \tag{18}$$

wh r $f = 131 Me$. Th us of (9), (17), (18) giv s th sum rul for val nc u-quark distribution in pion in th bar loop approximation [14]:

$$u\ (x) = \frac{3}{2\pi^2}\ \frac{M^2}{f^2}x(1-x)(1-e^{-s_0/M^2})e^{m_\pi^2/M^2}, \tag{19}$$

wh r s_0 is th continuum thr shold. In r f.[14] th following corr ctions to (19) w r account d:

1. Leading order (LO) perturbative corrections, proportional to $ln(Q^2/\mu^2)$, where μ^2 is the normalization point. In what follows the normalization point will be chosen to be equal to the Borel parameter $\mu^2 = M^2$.

2. Higher order terms of OPE. Among the latter, the dimension-4 correction, proportional to gluon condensate $\langle 0|\frac{\alpha_s}{\pi}G^n\ G^n\ |0\rangle$ was first accounted, but it was found that its contribution to the sum rule vanishes after double borelization. There are two types of vacuum expectation values (v. .v) of dimension 6: one, where only gluonic fields enter:

$$\frac{g_s}{\pi}\alpha_s f^{abc}\langle 0|G^a\ \ G^b\ \ G^c\ |0\rangle \tag{20}$$

and the other, proportional to four-quark operators

$$\langle 0|\bar{\psi}\Gamma\psi\cdot\bar{\psi}\Gamma\psi|0\rangle \tag{21}$$

It was shown in [14] that terms of the first type cancel in the sum rule and only terms of the second type survive. For the latter one may use the factorization hypothesis which reduces all the terms of this type to the square of quark condensate.

A remark is in order here. As was mentioned in the Introduction, the presented approach is invalid at small and large x. No-loop 4-quark condensate contributions are proportional to $\delta(1-x)$ and cannot be accounted. In the same way, the diagrams, which can be considered as a radiative corrections to those, proportional to $\delta(1-x)$, must be also omitted.

All dimension-6 power corrections to the sum rule were calculated in ref.'s 14,15 and the final result is given by:

$$\begin{aligned} xu\ (x) = \frac{3}{2\pi^2}\frac{M^2 e^{m_\pi^2/M^2}}{f^2}x^2(1-x)\Bigg[\Bigg(1+\left(\frac{a_s(M^2)\cdot ln(Q_0^2/M^2)}{3\pi}\right) \\ \times\left(\frac{1+4xln(1-x)}{x}-\frac{2(1-2x)lnx}{1-x}\right)\Bigg)\cdot(1-e^{-s_0/M^2}) \\ -\frac{4\pi\alpha_s(M^2)\cdot 4\pi\alpha_s a^2}{(2\pi)^4\cdot 3^7\cdot 2^6\cdot M^6}\cdot\frac{\omega(x)}{x^3(1-x)^3}\Bigg], \end{aligned} \tag{22}$$

where $\omega(x)$ is 4-order polynomial in x, (for its explicit forms see [14]),

$$a = -(2\pi)^2\langle 0|\bar{\psi}\psi|0\rangle \tag{23}$$

$u\ (x)$ may be used as an initial condition at $Q^2 = Q_0^2$ for solution of QCD evolution equations (DGLAP equations).

In numerical calculations we choose: the effective LO QCD parameter $\Lambda^{LO}_{QCD} = 250MeV$, $Q_0^2 = 2GeV^2$, $\alpha_s a^2(1GeV^2) = 0.34GeV^6$. The value of the latter is taken from the recent best fit of hadronic τ-decay data [19] and corresponds to $\alpha_s\langle\overline{\psi}\psi\rangle^2 = (2.2\pm 0.7)\cdot 10^{-4}$ GeV^6. The continuum threshold was varied in the interval $0.8 < s_0 < 1.2GeV^2$ and it was found, that the results only slightly depend on it. The analysis of the sum rule (22) shows, that it is fulfilled in the region $0.15 < x < 0.7$; the power corrections are less than 30% and the continuum contribution is small ($< 25\%$). The stability in the Borel mass parameter M^2 dependence in the region $0.4GeV^2 < M^2 < 0.6GeV^2$ is good. The result of our calculation of valence distribution in pion $xu\ (x, Q_0^2)$ is shown in Fig.2.

Suppos , that at small $x \lesssim 0.15$ $u\ (x) \sim 1/\sqrt{x}$ according to R gg b haviour and at larg $x \gtrsim 0.7$ $u\ (x) \sim (1-x)^2$ according to quark counting rul s. Th n, matching th s functions with (22), on may find th num rical valu s of th first and s cond mom nts of u-quark distribution

$$\mathcal{M}_1 = \int_0^1 u\ (x)dx \approx 1.1 \tag{24}$$

$$\mathcal{M}_2 = \int_0^1 xu\ (x)dx \approx 0.24 \tag{25}$$

$\mathcal{M}_1$ has th m aning of th numb r of u-quarks in π^+ and should b $\mathcal{M} = 1$. Th d viation of (24) from 1 charact riz s th accuracy of our calculation. $\mathcal{M}_2$ has th m aning of th part of pion mom ntum carri d by val nc u-quark. Th r for , val nc u and $\bar{d}$ quarks ar carrying about 40% of th total mom ntum. In Fig.2 ar plott d also th val nc u-quark distribution found in [4b] by fitting th data on production of $\mu^+\mu^-$ and e^+e^- pairs in pion-nucl on collisions (Dr ll-Yan proc ss).

4. uark distri ution s i -meso

Th choic of hadronic curr nt is vid nt

$$j_\mu = \overline{u}\gamma_\mu d \tag{26}$$

Th matrix l m nt $\langle \rho^+ \mid j_\mu \mid 0 \rangle$ is giv n by

$$\langle \rho^+ \mid j_\mu \mid 0 \rangle = \frac{m^2}{g} e_\mu \tag{27}$$

wh r m is th ρ-m son mass, g is th $\rho - \gamma$ coupling constant, $g^2/4\pi = 1.27$, e is th ρ-m son polarization v ctor. Consid r s parat ly u-quark distributions in longitudinally and transv rsally polariz d ρ-m sons. It was shown in [15], that for d t rmination of u-quark distribution in longitudinal ρ-m son th most suitabl t nsor structur is that, proportional to $P\ P\ P\ P$, whil u-quark distribution in transv rs ρ can b found by consid ring th invariant function at th structur $-P\ P\ \delta$. In cas of longitudinal ρ-m son th t nsor structur , which is s parat d is th sam , as in th cas of pion. Sinc at $m_q = 0$ bar loop contributions for v ctor and axial hadronic curr nts ar qual, th only diff r nc from th pion cas is in th normalization. It can b shown, that u-quark distribution in longitudinal ρ-m son can b found from (22) by substitutions $m \to m_\rho$, $f_\pi \to m_\rho/g$ and th r for

$$xu_L(x) = \frac{f^2}{m^2} g^2 e^{(m_\rho^2 - m_\pi^2)/M^2} xu\ (x), \tag{28}$$

wh r $xu\ (x)$ is giv n by (22). (Th num rical valu s of M^2 and s_0 diff r in th cas s of pion and longitudinal ρ-m son). Aft r s paration of th invariant function at th structur $P\ P\ \delta$ in box diagram w find u-quark distribution in transv rsally polariz d ρ-m son in bar loop approximation:

$$u_T(x)_0 = 3\left[\frac{1}{2} - x(1-x)\right] \equiv \frac{3}{2}\varphi_0(x) \tag{29}$$

$u^{T}(x)_0$ has a minimum at $x = \frac{1}{2}$, unlik $u\ (x)_0$ and $u^{L}(x)_0$. Anoth r ss ntial diff r nc in comparison with pion and longitudinal ρ aris s du to th fact that gluon cond nsat and $\langle G^3 \rangle$ op rator contributions ar nonvanishing in th sum rul for $u^{T}(x)$.

Val nc u-quark distribution in transv rsally polariz d ρ-m son is giv n by

$$xu^{T}(x) = \frac{3}{8\pi^2} g^2 e^{m_\rho^2/M^2} \frac{M^4}{m^4} x \left\{ \varphi_0(x) E_1\left(\frac{s_0}{M^2}\right) \left[1 + \frac{1}{3\pi} ln\left(\frac{Q_0^2}{M^2}\right) \alpha_s(M^2) \left(\frac{(4x-1)}{\varphi_0(x)} + \right.\right.\right.$$

$$\left.\left.\left. + 4ln(1-x) - \frac{2(1-2x+4x^2) lnx}{\varphi_0(x)} \right) \right] - \frac{\pi^2}{6} \frac{\langle 0|(\alpha_s/\pi) G^2|0\rangle}{M^4 x^2} \right.$$

$$+ \frac{1}{2^8 \cdot 3^5 M^6 x^3 (1-x)^3} \langle 0| g^3 f^{abc} G^a\ G^b\ G^c\ |0\rangle \xi(x)$$

$$\left. + \frac{\alpha_s(M^2)(\alpha_s a^2)}{2^5 \cdot 3^8 \pi^2 M^6 x^3 (1-x)^3} \chi(x) \right\} \qquad (30)$$

wh r $\xi(x)$ and $\chi(x)$ ar polynomials in x [15]. In num rical calculations th valu s of param t rs w r chos n: $M^2 = 1$ G V^2 (this valu is in th middl of stability int rval in M^2), $s_0 = 1.5$ G V^2, $Q_0^2 = 4$ G V^2, $\langle 0 \mid (\alpha_s/\pi) G^2 \mid 0 \rangle = 0.006$ G V^2 [20], $\langle G^3 \rangle$ vacuum xp ctation valu was stimat d according to instanton mod l [21]. Th ff ctiv instanton radius, was tak n to b qual $\rho_c = 0.5$ fm. Th oth r param t rs ar th sam as in calculation of $xu\ (x)$. On Fig.3 ar plott d $u^{L}(x), u^{T}(x)$ and, for comparison, $u\ (x)$. As is s n from th Figur , th shap of $u^{T}(x)$ is quit diff r nt from $u\ (x)$ and $u^{L}(x)$ and indicat s, that p rhaps, $u^{T}(x)$ has two humps. $u^{L}(x)$ also has an int r sting f atur : th s cond mom nt of u-quark distribution is qual to 0.4. It m ans, that th part of ρ^{L} mom nta, carri d by val nc – u and $\bar{d}$ quarks compris s 80% and for gluons and s a quarks is l ft about 20% only – a v ry unusual situation !

Th main physical conclusion is: th quark distributions in pion and ρ-m son hav not to much in common. Th sp cific prop rti s of pion, as a Goldston boson manif st th ms lv s in diff r nt quark distributions in comparison with ρ.

5. uark distri ution s i proto

Consid r th 4-curr nt corr lator which corr sponds to th virtual photon scatt ring on th quark curr nt with quantum numb r of proton:

$$T\ (p_1, p_2, q, q') = -i \int d^4x d^4y d^4z \cdot e^{i(p_1 x + qy - p_2 z)} \cdot \langle 0| T\{ J(x),\ j^{u;d}(y), j^{u;d}(0),\ \bar{J}(z)\} |0\rangle, \quad (31)$$

wh r $J(x)$ is th thr -quark curr nt $J(x) = \varepsilon^{abc}(u^a C\gamma\ u^b)\gamma_5 \gamma\ d^c$ [22], $j^u = \bar{u}\gamma\ u$, $j^d = \bar{d}\gamma\ d$. As was shown in [11] and g n raliz d to non qual p_1, p_2 in [16] th sum rul s should b writt n for invariant amplitud at th structur $\hat{p}p\ p$ (it what follows it will b d not d by ImT/ν). Th bar loop diagram is shown on Fig.4. Th r sult of its calculation aft r th doubl or l transformation ar th sam as in th cas for qual $p_1 = p_2$ [11]:

$$ImT^0_{u(d)} = \varphi_0^{u(d)}(x) \frac{M^2}{32\pi^3}\ E_2(s_0/M^2) \qquad (32)$$

wh r

$$\varphi_0^u(x) = x(1-x)^2(1+8x), \quad \varphi_0^d(x) = x(1-x)^2(1-2x), \qquad (33)$$

s_0 is th continuum thr shold

$$E_2(z) = 1 - (1 + z + z^2/2)e^{-z} \tag{34}$$

Th sum rul s for u and d-quark distributions $q(x)^{u(d)}$ in bar loop approximation ar :

$$xq(x)_0^{u(d)} = \frac{2M^6 e^{m^2/M^2}}{\bar{\lambda}_N^2}\, \varphi_0^{u(d)}(x) \cdot E_2(\frac{s_0}{M^2}) \tag{35}$$

H r λ_N is th coupling constant of proton with th curr nt $\langle 0|\eta|p,\sigma\rangle = \lambda_N v^r(p)$, $\overline{\lambda}_N^2 = 32\pi^4\lambda_N^2$. Th first and s cond mom nts of quark distributions, with account of proton mass sum rul in th sam approximation, coincid with quark mod l r sults, as it should b . QCD volution was account d as l ading ord r (LO) corr ction to bar loop formula (s [16]). In OPE th contributions of $d = 4$ op rator (gluon cond nsat) and $d = 6$ op rator (α_s tim s quark cond nsat squar) was calculat d. I pr s nt h r r sults as th ratio to bar loop contributions:

$$\frac{u(x)_{\langle G^2\rangle}}{u_0(x)} = \frac{\langle(\alpha_s/\pi)G^2\rangle}{M^4} \cdot \frac{\pi^2}{12}\, \frac{(11 + 4x - 31x^2)}{x(1-x)^2(1+8x)} \cdot (1 - e^{-s_0/M^2})/E_2(\frac{s_0}{M^2}) \tag{36}$$

$$\frac{d(x)_{\langle G^2\rangle}}{d_0(x)} = -\frac{\langle(\alpha_s/\pi)G^2\rangle}{M^4}\frac{\pi^2}{6}\, \frac{(1-2x^2)}{x^2(1-x)^2(1+2x)}(1 - e^{-s_0/M^2})/E_2(\frac{s_0}{M^2}) \tag{37}$$

$$\frac{u(x)_{s\langle\bar{s}s\rangle^2}}{u_0(x)} = \frac{\alpha_s a^2(215 - 867x + 172x^2 + 288(1-x)ln2)}{M^6 \cdot 81\pi \cdot 8x(1-x)^3(1+8x)}\frac{1}{E_2(s_0/M^2)} \tag{38}$$

$$\frac{d(x)_{s\langle\bar{s}s\rangle^2}}{d_0(x)} = -\frac{\alpha_s a^2(19 - 43x + 36x^2)}{M^6 81\pi x(1-x)^3(1+8x)}\, \frac{1}{E_2(s_0/M^2)} \tag{39}$$

On Fig. 5 ar plott d th obtain d u and d-quark distributions at $Q_0^2 = 5$ G V^2 in comparison with thos , found in [4a] from th data on d p in lastic l pton-nucl on scatt ring. Th sam valu s of QCD param t rs (cond nsat s) w r tak n as in cas s of quark distributions in π and ρ. Th Borl param t r M^2 was chos n in th middl of stability int rval, $M^2 = 1.1$ G V^2.

Th th or tical analysis of th obtain d val nc quark distributions at $Q^2 = 5Ge\bar{V}^2$ show d, that u-quark distribution is r liabl at $0.15 < x < 0.65$, its accuracy is about 25% in th middl of this int rval and d cr as s to 50% at th nds of int rval; d-quark distribution is r liabl at $0.25 < x < 0.55$ with an accuracy of about 30% in th middl and is giv n by a factor of 2 at th nds of int rval. In th limit of th s accuraci s th th or tically calculat d val nc quark distributions ar in agr m nt with thos found from d p in lastic scatt ring and oth r hard proc ss s data.

Th r s arch d scrib d in this publication was mad possibl in part by Award No.RP2-2247 of th US Civilian and D v lopm nt Foundation for th Ind p nd nt Stat s of Form r Sovi t Union (CRDF), by INTAS Grant 2000, Proj ct 587 and by th Russian Found of Basic R s arch, Grant No. 00-02-17808.

References

[1] V.N.Gribov and L.N.Lipatov, Sov. Nucl.Phys. **15** (1972) 438.

[2] Yu.L.Dokshitz r, Sov.Phys. ETP, **46** (1977) 641.

[3] G.Altar lli and G.Parisi, Nucl.Phys. **B126** (1977) 298.

[4] M.Glück, E.R ya and A.Vogt, . Phys. **C53** (1992): a) p.127; b) p.651.

[5] H.L.Lai t al. (CTEQ Collab.) Eur.Phys. . **C12** (2000) 375.

[6] A.D.Martin, R.G.Rob rts, W. Stirling and R S.Thorn , Eur.Phys. .C.**4** (1998) 463.

[7] M.Glück, E.R ya and A.Vogt, Eur.Phys. .**C5** (1998) 461.

[8] A.M.Coop r Sarkar, R.C.E.Dav nish and A.D Ro ck, Int. .Mod.Phys. **A13** (1998) 3385.

[9] Wu-Ki Tung , At th Fronti r of Particl Physics, in: Handbook of QCD, oris Ioff F stschrift, d. by M Shifman, World Sci. 2001, v.2, p.887.

[10] .L.Ioff , Pis'ma h.Eksp.Fiz. **42** (1985) 266, [ETP L tt **42** (1985) 327].

[11] V.M. lya v and .L.Ioff , Nucl.Phys. 310 (1988) 310.

[12] A S.Gorsky, .L.Ioff , A.Yu.Khodjamirian and A.G.Ogan sian, . Phys. **C44** (1989) 523.

[13] .L.Ioff and A.Yu.Khodjamirian, Phys.R v. **D51** (1995) 3373.

[14] .L.Ioff and A.Ogan sian, Eur.Phys. . **C13** (2000) 485.

[15] .L.Ioff and A.G.Ogan sian, Phys.R v. **D63** (2001), 096006.

[16] .L.Ioff and A.Ogan sian, h p-ph/0208192.

[17] V.N.Gribov, .L.Ioff and I.Ya.Pom ranchuk, Yad.Fiz. **2** (1965) 768. [Sov. .Nucl.Phys. **2** (1966) 549].

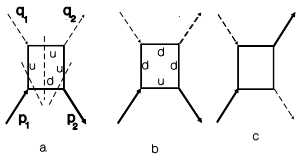
[18] .L.Ioff , Phys.L tt. **30 B** (1969) 123.

[19] .L.Ioff and K.N. yablyuk, Nucl.Phys. **A687** (2001) 437.

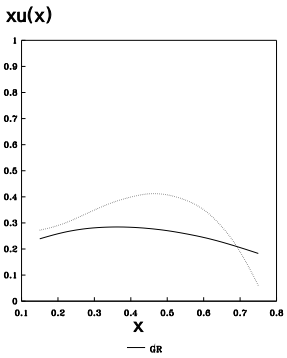
[20] .V.G shk nb in, .L.Ioff and K.N. yablyuk, Phys.R v. **D64** (2001) 093009.

[21] V.A.Novikov, M.A Shifman, A.I.Vainsht in and V.I. akharov, Phys.L tt. **B86** (1979) 347.

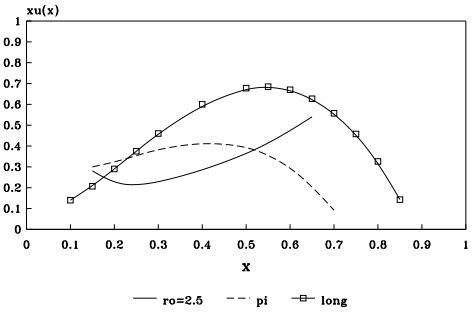
[22] .L.Ioff , Nucl.Phys. **188** (1981) 317.



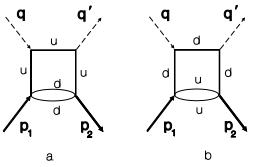
Figur 1: Diagrams, corr sponding to th unit op rator contribution. Dash d lin s with arrows corr spond to th photon, thick solid - to pion or rho curr nt.



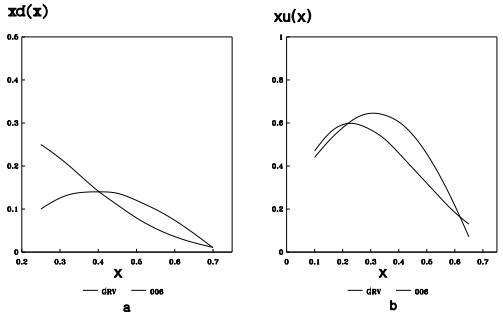
Figur 2: Quark distribution function in pion (dash d lin) and th fit from [4b] - solid lin .



Figur 3: Quark distributions in longitudinal ρ, (solid lin with squar s), transv rsal ρ (solid lin) and pion (dash d lin)



Figur 4: Par loop diagram in cas of proton.



Figur 5: Val nc u- and d-quark distributions in proton (dash d lin s) in comparison with th fit [4a] of d p in lastic scatt ring data (solid lin s)