Instanton Corrections
to the Quark Form Factor

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Abstract

The nonperturbative effects in the quark form factor are considered in the Wilson loop formalism, within the framework of the instanton liquid model. For the integration path corresponding to this form factor, the explicit expression for the vacuum expectation value of the Wilson operator is found to the leading order. It is shown that the instantons produce the power-like corrections to the perturbative result, which are comparable in magnitude with the perturbative part at the scale of order of the inverse average instanton size. It is demonstrated that the instanton contributions to the quark form factor are exponentiated to high orders in the small instanton density parameter.

1 Introduction

In the present report, we start the investigation of the instanton induced effects in the high-energy QCD processes by means of the Wilson integral formalism [1, 2]. The basic object of study in such an approach is the gauge invariant vacuum average of the Wilson loop operator

$$W(C) = \frac{1}{N_c} \mathrm{Tr} \langle 0 | \mathcal{P} \exp \left(ig \int_C dx_\mu \hat{A}_\mu(x) \right) |0\rangle \ , \tag{1}$$

where the integration goes along the closed contour C and the gauge field

$$\hat{A}_\mu(x) = T^a A^a_\mu(x) \ , \quad T^a = \frac{\lambda^a}{2} \quad , \tag{2}$$

belongs to the Lie algebra of the gauge group $SU(N_c)$, while the Wilson loop operator $\mathcal{P}e^{ig\int dx A(x)}$ lies (for quark lines) in its fundamental representation. We propose an approach which allows one to evaluate the instanton contributions to the Wilson integrals made of several (in)finite lines containing specific cusp, and/or cross singularities. For this purpose, we start with one of the simplest configurations, *i. e.,* the angle with infinite sides that corresponds to the integration path for the Wilson operator describing the soft part of the quark Sudakov form factor [3].

2 Perturbative contribution

For brevity, we omit here the detailed discussion on the renormalization of cusp singularities, just say that it can be proven that there exists the consistent renormalization procedure for such quantities [4]. For details, see [5].

The leading nontrivial cusp dependent term in the expansion of $W(C)$ (1) in powers of g^2 for the angle with two infinite straight line rays (Fig. 1a) contains the contributions from both perturbative (Fig. 1b) and nonperturbative (Fig. 1c) fields, which can be expressed in the following form:

$$W^{\text{LO}}(\gamma) = -\frac{g^2 C_F}{2} \int_{C_\gamma} dx_\mu \int_{C_\gamma} dy_\nu \ D_{\mu\nu}(x-y) \ , \tag{3}$$

where $C_F = \frac{N_c^2-1}{2N_c}$. It is convenient to present the gluon propagator $D_{\mu\nu}(z)$ in the form

$$D_{\mu\nu}(z) = \delta_{\mu\nu}\partial_z^2 d_1(z^2) - \partial_\mu\partial_\nu d_2(z^2) \ . \tag{4}$$

Here and in what follows, we use the dimensional regularization with $n = 4-2\varepsilon$, $\varepsilon < 0$ in order to control the IR-divergent terms in the integrals. The remaining UV singularity (due to the infinitely small z^2 in the vicinity of the cusp) will be regularized by the corresponding UV cutoff.

The trajectories of the incoming and outgoing quarks (Fig. 1a) may be parameterized as $x = v_1 s \ (0 < s < \infty) \quad , \quad y = v_2\tau \ (-\infty < \tau < 0)$. The angle between the vectors v_1 and v_2 is given in the Minkowski space by

$$\cosh \chi = (v_1 v_2) = \frac{(p_1 p_2)}{m^2} = 1 + \frac{Q^2}{2m^2} \ , \quad -Q^2 = (p_2 - p_1)^2 \ , \quad v_{1,2}^2 = 1, \quad (5)$$

where the quark momenta are supposed to be on-shell: $p_1^2 = p_2^2 = m^2$. The continuation to the Euclidean space is defined as [7, 8] $\chi \to i\gamma$. Then, we have to consider the quantity [4, 7]:

$$W^{\rm LO}(\gamma) = \widetilde{W}^{\rm LO}(\gamma) - \widetilde{W}^{\rm LO}(0) \ , \quad (6)$$

where [5]

$$\widetilde{W}^{\rm LO}(\gamma) = -g^2 C_F \left[(n-2) d_1(0) \gamma \cot\gamma + d_2(0)\right] \ . \quad (7)$$

It follows from Eq. (6) that the integrals (3) in which both points x and y belong to the same side of the angle do not contribute to the quantity $W^{\rm LO}(\gamma)$. Hence we have within the one loop accuracy

$$W(\gamma) = 1 - 4\pi\alpha_S C_F (n-2) h(\gamma) d_1(0) \ , \quad (8)$$

where

$$h(\gamma) = \gamma\cot\gamma - 1 \quad (9)$$

is the universal cusp factor. We should emphasize here that the expression (8) holds for perturbative as well as for nonperturbative part depending on the value $d_1(0)$. For the perturbative field, Eq. (8) reflects the explicit gauge invariance in the set of covariant gauges, since the gauge fixing parameter ξ enters only in the function $d_2(z^2)$.

Let us consider first the perturbative part $W_P^{\rm LO}(\gamma)$. By using the free propagator in the Euclidean space the IR-regularized value of $d_1(0)$ can be written in the form:

$$d_1(0; \varepsilon, \lambda) = \frac{(\lambda^2 \pi)^\varepsilon}{16\pi^2} \int_0^\infty d\alpha \alpha^{-(1+\varepsilon)} \ , \quad (10)$$

where λ^2 is the IR regularization parameter. This integral diverges at the upper (UV) limit for $\varepsilon < 0$, and hence we must regularize it. To this end, we may introduce the UV cutoff μ^2, and finally we get

$$d_1(0; \varepsilon, \mu/\lambda) = -\frac{1}{\varepsilon} \frac{1}{16\pi^2} \left(\frac{\lambda^2 \pi}{\mu^2}\right)^\varepsilon \ . \quad (11)$$

Thus one obtains the perturbative part with one loop accuracy:

$$W_P(\gamma,\mu) = 1 - \frac{\alpha_S}{2\pi} C_F h(\gamma) \ln \frac{\mu^2}{\lambda^2} \ . \tag{12}$$

The one-loop cusp anomalous dimension which satisfies the RG equation reads: Hence, we reproduce the Wilson operator value for the infinite contour with the Euclidean cusp parameter γ.

3 Instanton contribution

Let us estimate the nonperturbative contribution to $W(C)$ in the instanton model. The instanton field is given by

$$\hat{A}_\mu(x;\rho) = A_\mu^a(x;\rho)\frac{\sigma^a}{2} = \frac{1}{g}\mathbf{R}^{ab}\sigma^a \eta^{\pm b}_{\ \mu\nu}(x-z_0)_\nu \varphi(x-z_0;\rho), \tag{13}$$

where $\mathbf{R}^{ab}$ is the color orientation matrix $(a,b = 1,2,3)$, σ^a's are the Pauli matrices, and $(\pm)$ corresponds to the instanton, or anti-instanton. The averaging of the Wilson operator over the nonperturbative vacuum is reduced to the integration over the coordinate of the instanton center z_0, the color orientation and the instanton size ρ. The measure for the averaging over the instanton ensemble reads $dI = d\mathbf{R}\ d^4z_0\ dn(\rho)$, where $d\mathbf{R}$ refers to the averaging over color orientation, and $dn(\rho)$ depends on the choice of the instanton size distribution. Taking into account (13), we write the Wilson integral (1) in *the single instanton approximation* in the form:

$$w_I(C) = \frac{1}{N_c}\langle 0|\mathrm{Tr}\exp\left(i\sigma^a\phi^a\right)|0\rangle \ , \tag{14}$$

where the phase is

$$\phi^a = \mathbf{R}^{ab}\eta^{\pm b}_{\ \mu\nu}\int_{C_\gamma} dx_\mu\ (x-z_0)_\nu \varphi(x-z_0;\rho) \ . \tag{15}$$

Thus we obtain the all-order single instanton contribution to the cusp-dependent part of Wilson loop (1):

$$w_I(\gamma) = \int d^4z_0 \int dn(\rho)\ \left[\cos\ \phi(\gamma,z_0,\rho) - \cos\ \phi(0,z_0,\rho)\right] \ , \tag{16}$$

where the expression for squared phase $\phi^2 = \phi^a\phi^a$ can be found in [5].

Although the expression (16) gives the complete formula for the all-order single instanton contribution, in what follows we restrict ourselves to the investigation of the weak field limit. In this limit, the leading instanton induced term reads:

$$w_I^{(1)}(\gamma) = -\frac{g^2\lambda^{n-4}}{2} \int dn(\rho) \int_{C_\gamma} dx_\mu \int_{C_\gamma} dy_\nu \int \frac{d^n k}{(2\pi)^n} \tilde{A}^a_\mu(k;\rho)\tilde{A}^a_\nu(-k;\rho)\mathrm{e}^{-ik(x-y)} . \tag{17}$$

By using the Fourier transform of the instanton field

$$\tilde{A}^a_\mu(k;\rho) = -\frac{2i}{g}\eta^{\pm a}_{\mu\sigma} k_\sigma \tilde{\varphi}'(k^2;\rho) , \tag{18}$$

Eq. (17) can be written in the form of Eq. (4) with the instantonic analogue of the function $d_1(z^2)$: $d_1(z^2) \to d_1^I(z^2)$, where

$$d_1^I(z^2) = -\frac{1}{g^2 C_F} \int dn(\rho) D_I(z^2;\rho,\lambda) = -\frac{\lambda^{4-n}}{g^2 C_F} \int dn(\rho) \frac{d^n k}{(2\pi)^n} \mathrm{e}^{-ikz} \left(2\tilde{\varphi}'(k^2;\rho)\right)^2 \tag{19}$$

Above, $\tilde{\varphi}(k^2;\rho)$ is the Fourier transform of the instanton profile function $\varphi(z^2;\rho)$ and $\tilde{\varphi}'(k^2;\rho)$ is it's derivative with respect to k^2. Now using the result (8) of the previous Section, we get the instanton contribution in the form:

$$w_I(\gamma;\varepsilon,\lambda) = (n-2)h(\gamma) \int dn(\rho)\ D_I(0;\varepsilon,\lambda,\rho) . \tag{20}$$

Consider now the renormalization of the nonperturbative part for the instanton field in the singular gauge, where the profile function is:

$$\varphi(u;\rho) = \frac{\rho^2}{z^2(z^2+\rho^2)} . \tag{21}$$

For the complete expression for $D_I(0;\varepsilon,\lambda,\rho)$, see [5]. Applying the renormalization procedure as described in the previous Section, we find in the leading order the instanton contribution to the Wilson loop:

$$w_I^{(1)}(\gamma,\lambda) = 1 + \pi^2 h(\gamma) \int dn(\rho)\ \rho^4 \ln(\rho\lambda) . \tag{22}$$

In order to estimate the magnitude of the instanton induced effect we consider the distribution function which has been suggested in [15] (and discussed in [13] in the framework of constrained instanton model) in order to describe the lattice data [10]:

$$dn(\rho) = \frac{d\rho}{\rho^5}\, C_{N_c} \left(\frac{2\pi}{\alpha_S(\rho)}\right)^{2N_c} \exp\left(-\frac{2\pi}{\alpha_S(\rho)}\right) \exp\left(-2\pi\sigma\rho^2\right) \ , \qquad (23)$$

where the numerical constant C_{N_c} is determined by the number of colours $C_{N_c} \approx 0.0015$ and the string tension is accepted to be $\sigma \approx (0.44\ GeV)^2$ [12, 15]. Then, using the one loop expression for the running coupling constant we find the instanton contribution (22) in the form (in the distribution (23), the slow varying logarithmic factor due to the power of the coupling α_S is assumed to be constant, and taken at the point of the mean instanton size $\bar{\rho}$):

$$w_I^{(1)}(\gamma, \lambda) = 1 + \pi^2 h(\gamma) \frac{C_{N_c}\Gamma(\beta_0/2)}{4} \left(\frac{2\pi}{\alpha_S(\bar{\rho})}\right)^{2N_c} \left(\frac{\Lambda_{QCD}}{\sqrt{2\pi\sigma}}\right)^{\beta_0} \ln\frac{\lambda^2}{2\pi\sigma} \ , \qquad (24)$$

where β_0 is the first coefficient of perturbative β-function. The expression (24) shows explicitly that the instantons yield the power-like corrections to the perturbative result, what is expected from general consideration, *e. g.*, from the renormalon analysis (see, *e.g.*, [16]).

It is instructive to express the result (24) in terms of the mean instanton size $\bar{\rho}$ and the instanton density $\bar{n}$ calculated directly from the distribution (23). To compare the instanton induced and perturbative parts, we assume that the factorization scale μ (which divides the soft and hard regions of momenta in the factorized quark form factor) is of order of the inverse instanton size $\mu \approx \bar{\rho}^{-1} \approx 0.6\ GeV$. Then we write the total leading order contribution to the Wilson loop expectation value in the form:

$$W(\gamma, \rho_0\lambda) = 1 + \frac{\alpha_S(\bar{\rho}^{-1})}{2\pi} C_F h(\gamma) \ln\left(\bar{\rho}^2\lambda^2\right) \left(1 + K\ \frac{S_0 \pi^2 \bar{n}\bar{\rho}^4}{C_F}\right) \ , \qquad (25)$$

where $S_0 = \frac{8\pi^2}{g^2(\bar{\rho}^{-1})} \approx 10$ is the "classical enhancement" factor with the renormalized coupling constant $g(\mu)$ at the energy scale $\mu \approx \bar{\rho}^{-1}$, and $K \approx 0.74$. The ratio of the instanton correction to the perturbative leading term is about 0.5, what is estimated using the conventional value for the packing fraction [17] $\pi^2\bar{n}\bar{\rho}^4 \approx 0.1$. One can see using the main formula (24) that the strong power suppression of the instanton part is partially compensated by the large factor $S_0^{2N_c+1}$. This means that at the energy scale of order of $\bar{\rho}^{-1}$

the magnitude of the instanton induced effects is comparable to the leading perturbative part, and must be taken into account as well. It is possible also to estimate numerically the two-loop order perturbative contribution which appears to be of the same order as the instanton part [5]. Thus, the complete consideration of the quark form factor at the low momentum scale must include both the two-loop perturbative part and the leading order instanton one, which appear to be of the same order of magnitude.

Expression (25) defines the first terms of the Wilson loop expansion in gauge fields. On the basis of the exponentiation theorem [14] for the non-abelian path-ordered exponentials it is well known that perturbative corrections to the Sudakov form factor are exponentiated to high orders in the QCD coupling constant. Let us describe briefly how the single instanton contribution is exponentiated in the small instanton density parameter, treating the instanton vacuum as a dilute medium [18]. For details, see [5]. The gauge field is taken to be the sum of individual instanton fields in the singular gauge, (13, 21), with their centers at the points z_j's. Since the parametrization of the loop integral along rays of the angle plays the role of the proper time, a time-ordered series of instantons arises and has an effect on the Wilson loop. Then, the expression is simplified when averaging over the gauge orientations of instantons: the entire loop integral collapses to a product of traces,

$$W_I^{(n)}(\gamma) \to \lim_{n\to\infty} \prod_{j=1}^{n} w_I^{(j)}(\gamma). \tag{26}$$

Since the individual instantons are considered to be decoupled in the dilute medium, the total multiple instanton contribution to the vacuum average of the Wilson operator simply exponentiates the all-order single instanton term $w_I(\gamma)$ in (16), and one has

$$W_I(\gamma) \;=\; \lim_{n\to\infty} \left\{1 + \frac{1}{n} w_I(\gamma)\right\}^n = \exp[w_I(\gamma)]. \tag{27}$$

Thus, we have proved that in the dilute regime, the full instanton contribution to the quark form factor is given by the exponent of the all-order single instanton result. The exponentiation arises due to taking into account the many-instanton configurations effect.

4 Conclusion

To summarize, within the instanton vacuum model we have developed an approach which allows one to calculate the nonperturbative contributions to the Wilson integrals over the infinite contour with a cusp that represent, *e. g.,* the classical trajectories of partons participating in hard collisions. We have calculated the instanton contribution to the soft part of the quark form factor, described in terms of the vacuum expectation value of the Wilson loop for the contour of a special form (16). We have proved that in the dilute regime, the full instanton contribution to the quark form factor is given by the exponentiated all-order single instanton result, see (27). In the weak-field limit, the instanton contribution to the soft part of the color singlet quark form factor is found explicitly in terms of the instanton profile function in the singular gauge. It is shown that the instanton induced effects are of a power type (24), but nevertheless they are comparable in magnitude to the perturbative ones at the scale of order of the inverse average size of the instanton in the instanton vacuum, see (25).

5 Acknowledgements

We are indebted to the Organizers of QUARKS'2002 for hospitality and financial support. We are grateful for the fruitful discussions to D. Antonov, N.I. Kochelev, S.V. Mikhailov and S. Tafat. This work is partially supported by RFBR (nos. 02-02-16194, 01-02-16431), INTAS (no. 00-00-366), and Heisenberg-Landau program HL-2002-13. The work of I. Ch. is supported also by the RFBR grant no. 00-15-96577.

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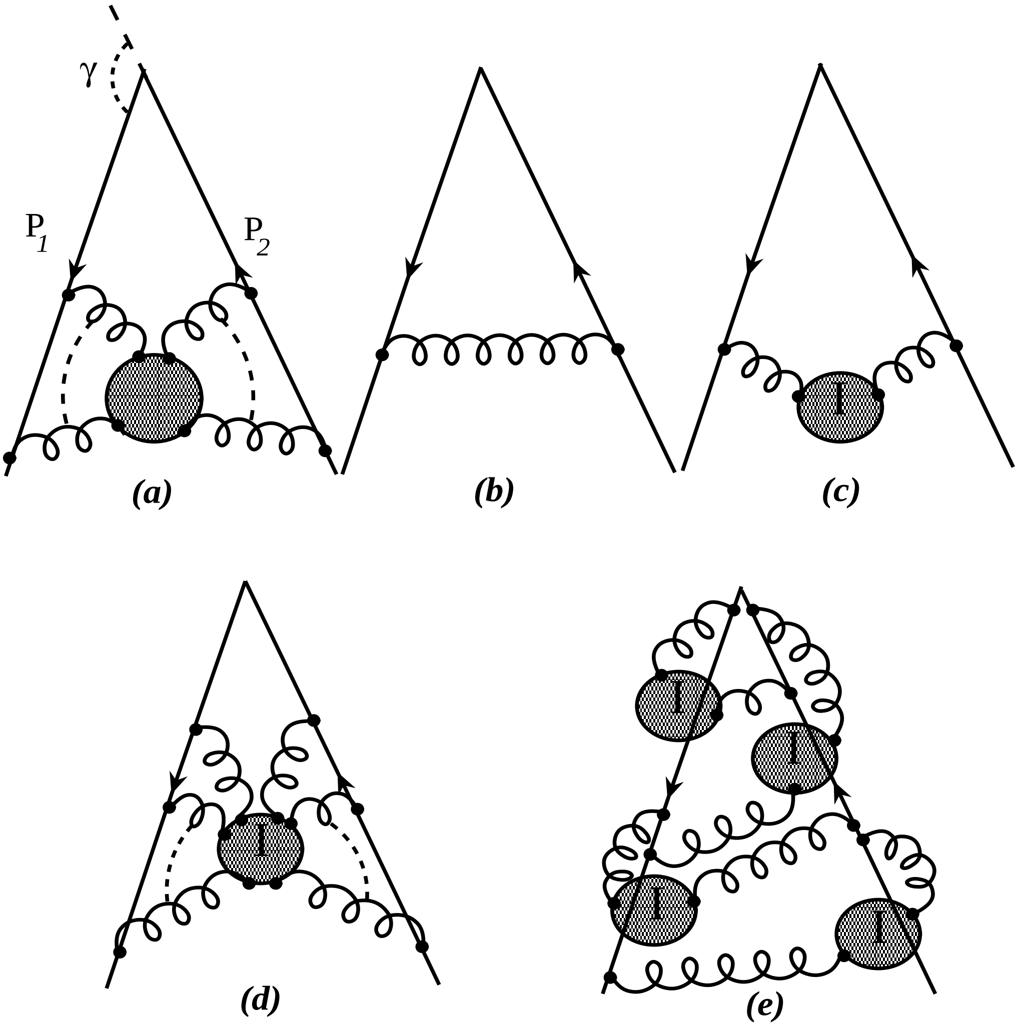

Fig. 1: The notations for the quark momenta and the total cusp-dependent part of the Wilson loop integral for the quark form factor (a); the leading order contributions of the perturbative (b) and nonperturbative (single-instanton) (c) fields; (d) the all-order single instanton result; (e) the exponentiation of the single instanton result.