Hadronic string and chiral symmetry breaking

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Abstract

We assume that QCD can be effectively described with string-like variables. The hadronic string is built over the chirally non-invariant QCD vacuum by means of the boundary interaction with background chiral fields associated with pions. By making this interaction compatible with the conformal symmetry of the string and with the unitarity constraint on chiral fields we reconstruct the equations of motion for the latter ones and furthermore recover the Lagrangian of non-linear sigma model of pion interactions. The estimated chiral structural constants of Gasser and Leutwyler fit well the phenomenological values.

1. Introduction

The history of attempts to describe the hadrons in the framework of a string theory beyond or within QCD encompasses already more than 30 years (see,[1]-[8] as well as the reviews [9]-[11]). The commonly cited arguments to justify the stringy description of QCD are the dominance of planar gluon diagrams in the large N limit[12] being interpreted as the world-sheet of a string, the expansion in terms of surfaces built out of plaquettes in strong-coupling lattice QCD[13], and the incarnation of Regge phenomenology[10] within QCD[14].

There is a motivated agreement that in a certain kinematic regime the Nambu-Goto or the Polyakov string action may be satisfactory. Here we focus on low-energy properties of string-generated particle states and it is known for a long time that the hadronic amplitudes derived from such type of strings are not quite physically consistent. To illuminate their flaws we recall the original Veneziano amplitude[1], which can be derived from Nambu-Goto string and supposedly describes the scattering amplitude of four pions. One can show that in this amplitude the scalar resonance is a tachyon and the vector state (which we should identify with the rho particle) is massless.

At last such an amplitude does not have the appropriate Adler zero, i.e. the property that at $s = t = 0$ the pion scattering amplitude vanishes.

It is quite conceivable that the main reason for the presence of a tachyon in the spectrum and the wrong chiral properties lies in a wrong choice of the vacuum[15]. A possible way to take into account the non-perturbative properties of the QCD vacuum was suggested in [16] and developed in [17]. Namely, one can assume that in QCD chiral symmetry breaking takes place and the massless (in the chiral limit) pseudoscalar mesons form the background of the QCD vacuum, whereas other massive excitations are assembled into a string. The massless pion fields can be collected in a unitary matrix $U(x)$ belonging to $SU(2)$ group (here we consider non-strange Goldstone mesons only). It describes excitations around the non-perturbative vacuum breaking the chiral symmetry. From the string point of view $U(x)$ is nothing but a bunch of couplings involving the string variable $x_\mu(\tau, \sigma)$. It has to be coupled to the boundary of the string where flavour is attached. Our goal is to find a consistent string propagation in this non-perturbative background.

An essential property of string theory is conformal invariance. Since it must hold when perturbing the string around any vacuum we demand the new coupling to chiral fields, living on the boundary, to preserve it.

Thus our proposal is to introduce the general reparameterization-invariant boundary interaction to chiral fields and derive all the divergences induced by this interaction. We shall need additional dimensional operators in the boundary action to renormalize divergences. From the condition of vanishing β functions for $U(x)$ the equations of motion for chiral fields are obtained in the low-momentum (derivative) expansion. We consistently implement the unitarity constraint on the chiral fields and locality of the chiral Lagrangian and finally calculate the $O(p^4)$ terms of the Gasser and Leutwyler[18] effective Lagrangian. A strikingly good correspondence with their phenomenological values is found.

2. Pion interaction to the QCD string and diagrammar

The hadronic string in the conformal gauge is described by the following conformal field theory action which has four dimensional Euclidean space-time as target space

$$\mathcal{W}_{str} = \frac{1}{4\pi\alpha'} \int d^{2+\epsilon}\sigma \left(\frac{\varphi}{\mu}\right)^{-\epsilon} \partial_i x_\mu \partial_i x_\mu, \tag{1}$$

where for $\epsilon = 0$ one takes $x_\mu = x_\mu(\tau,\sigma)$, $-\infty < \tau < \infty, 0 < \sigma < \infty$, $i = \tau,\sigma$, $\mu = 1,...,4$. The conformal factor $\varphi(\tau,\sigma)$ is introduced to restore the conformal invariance in $2+\epsilon$ dimensions. The Regge trajectory slope (related to the inverse string tension) is known to be universal $\alpha' \simeq 0.9$ GeV^{-2} [19].

We would like to couple in a chiral invariant manner the matrix in flavor space $U(x)$ containing the meson fields to the string degrees of freedom while preserving general covariance in the two dimensional coordinates and conformal invariance under local scale transformations of the two-dimensional metric tensor.

Since the string variable x does not contain any flavor dependence, we introduce two dimensionless Grassmann variables ('quarks') living on the boundary of the string sheet: $\psi_L(\tau), \psi_R(\tau)$. They transform in the fundamental representation of the light flavor group ($SU(2)$ in the present paper). A local hermitean action $S_b = \int d\tau L_f$ is then introduced on the boundary $\sigma = 0$ to describe the interaction with background chiral fields $U(x(\tau)) = \exp(i\pi(x)/f_\pi)$, where the normalization scale is set to $f_\pi \simeq 93MeV$, the weak pion decay constant.

The boundary Lagrangian is chosen to be reparameterization invariant and in its minimal form reads

$$\begin{aligned} L_f &= \frac{1}{2} i \left(\bar{\psi}_L U(1-z)\dot{\psi}_R - \dot{\bar{\psi}}_L U(1+z)\psi_R \right. \\ &\quad \left. + \bar{\psi}_R U^+(1+z^*)\dot{\psi}_L - \dot{\bar{\psi}}_R U^+(1-z^*)\psi_L \right), \end{aligned} \tag{2}$$

herein and further on a dot implies a τ derivative: $\dot{\psi} \equiv d\psi/d\tau$.

A further restriction is obtained by requiring CP invariance,

$$U \leftrightarrow U^+, \quad \psi_L \leftrightarrow \psi_R. \tag{3}$$

The above Lagrangian is CP symmetric for $z = -z^* = ia$. The fulfillment of this symmetry happens to be crucial to preserve conformal symmetry in the presence of the added boundary interaction.

Now we expand the function $U(x)$ in powers of the string coordinate field $x_\mu(\tau) = x_{0\mu} + \tilde{x}_\mu(\tau)$ around a constant x_0,

$$U(x) = U(x_0) + \tilde{x}_\mu(\tau)\partial_\mu U(x_0) + \frac{1}{2}\tilde{x}_\mu(\tau)\tilde{x}_\nu(\tau)\partial_\mu\partial_\nu U(x_0) + \dots . \tag{4}$$

and look for the potentially divergent one particle irreducible diagrams.The two-fermion, N-boson vertex operators are generated by the expansion (4), from the generating functional $Z_b = \langle \exp(iS_b) \rangle$ and eq.(2). Each additional loop comes with a power of α'. One can find a resemblance to the familiar derivative expansion of chiral perturbation theory [18].

The free fermion propagator is

$$\langle \psi_R(\tau)\bar{\psi}_L(\tau')\rangle = \langle \psi_L(\tau)\bar{\psi}_R(\tau')\rangle^\dagger = U^{-1}(x_0)\theta(\tau-\tau'), \tag{5}$$

if we impose CP symmetry for unitary chiral fields $U(x)$.

The free boson propagator projected on the boundary is

$$\langle x_\mu(\tau)x_\nu(\tau')\rangle = \delta_{\mu\nu}\Delta(\tau-\tau') = -2\delta_{\mu\nu}\alpha' \ln(|\tau-\tau'|\mu). \tag{6}$$

The normalization of the string propagator is inferred [17] from the definition of the kernel of the N-point tachyon amplitude for the open string [9]. In dimensional regularization one adopts $\Delta(0) \sim \alpha'/\epsilon$ and $\Delta'(0) = 0$.

To implement the renormalization process we perform a loop (equivalent to a derivative) expansion, proceed to determine the counterterms required to make the theory finite and further on to impose a vanishing beta functional for the coupling $U(x)$ to implement the absence of conformal anomaly.

3. Renormalization at one and two loops

Using the above set of Feynman rules one arrives at the one-loop divergent part of the propagator,

$$-\theta(A-B)U^{-1}\delta U U^{-1}, \quad \delta U \equiv \Delta(0)\left[\frac{1}{2}\partial_\mu^2 U - \frac{3+z^2}{4}\partial_\mu U U^{-1}\partial_\mu U\right]. \tag{7}$$

This divergence is eliminated by introducing an appropriate counterterm $U \to U + \delta U$. Conformal symmetry is restored (the beta-function is zero) if the above contribution vanishes, $\delta U = 0$.

Let us find out for which value of z this variation of U is compatible with its unitarity.

$$\delta(UU^+) = U \cdot \delta U^+ + \delta U \cdot U^+ = 0. \tag{8}$$

A simple calculation shows that this takes place for $z = \pm i$. The related local classical action which has $\delta U = 0$ as equation of motion is

$$W^{(2)} = \frac{f_\pi^2}{4}\int d^4x \mathrm{tr}\left[\partial_\mu U \partial_\mu U^+\right], \tag{9}$$

i.e. the well known non-linear sigma model of pion interactions.

We have thus found the chiral action induced by the QCD string. It has all the required properties of locality, chiral symmetry and proper low momentum behavior (Adler zero) and describes massless pions. However f_π, the overall normalization scale, cannot be predicted from these arguments.

Before proceeding to a full two loop calculation we have to check whether the minimal Lagrangian (2) is sufficient to renormalize also the vertices containing the boson legs. It turns out that it is not.

To obtain the divergences for vertices with external boson lines we introduce an external background boson field $\bar{x}_\mu$ and split $x_\mu = \bar{x}_\mu + \eta_\mu$. The free propagator for the fluctuating field η_μ coincides with the one for x_μ.

The total one-loop divergence in the vertex with two fermions and one boson line can be represented by the following vertex operator in the Lagrangian

$$\frac{i}{2}\left(\bar{\psi}_L \Phi^{(1)} \dot{\psi}_R - \dot{\bar{\psi}}_L \Phi^{(2)} \psi_R\right) + \text{h.c.}, \quad \Phi^{(1,2)} \equiv \bar{x}_\mu(\tau)(1 \mp z)\left[\partial_\mu\left(\delta U\right) \mp \phi_\mu\right]. \tag{10}$$

The terms proportional to derivatives of δU are automatically eliminated by the renormalization of the one-loop propagator. But the part proportional to ϕ_μ remains and to absorb these divergences new counterterms are required. The latter ones can be parameterized with three bare constants g_1 , g_2 and g_3, which are real if the CP symmetry for $z = -z^*$ holds

$$\begin{aligned} \Delta L_{bare} &= \frac{i}{8}(1-z^2)\bar{\psi}_L \Big((g_1 - z g_2)\partial_\nu \dot{U} U^{-1} \partial_\nu U - (g_1 + z g_2)\partial_\nu U U^{-1} \partial_\nu \dot{U} \\ &\quad + 2 z g_3 \partial_\nu U U^{-1} \dot{U} U^{-1} \partial_\nu U\Big)\psi_R + \text{h.c.} \end{aligned} \tag{11}$$

Renormalization is accomplished by subtraction,

$$g_i = g_{i,r} - \Delta(0). \tag{12}$$

The constants $g_{i,r}$ are finite, but in principle scheme dependent. The counterterms are of higher dimensionality than the original Lagrangian (2) and the couplings g_i are of dimension α'. Since (2) was the most general coupling permitted by the symmetries of the model, one concludes that conformal symmetry seems to be broken by these boundary couplings already at tree level. However in spite of the fact that the new couplings are dimensionful, it

turns out [17] that their contribution into the trace of the energy-momentum tensor vanishes once the requirements of unitarity of U and CP invariance are taken into account. Therefore conformal invariance is not broken at the order we are working.

On the other hand the appearance of new vertices changes the fermion propagator. One obtains from such terms the following contribution to the propagator

$$\begin{aligned}
& \theta(A-B)\frac{1}{16}\Delta(0)(1-z^2)U^{-1}\Big\{2(g_{1,r}-z^2g_{2,r})\partial_\rho UU^{-1}\partial_\mu\partial_\rho UU^{-1}\partial_\mu U \\
& -(1+z)(g_{1,r}+zg_{2,r})\partial_\rho UU^{-1}\partial_\mu UU^{-1}\partial_\rho\partial_\mu U \\
& -(1-z)(g_{1,r}-zg_{2,r})\partial_\rho\partial_\mu UU^{-1}\partial_\rho UU^{-1}\partial_\mu U \\
& +4z^2g_{3,r}\partial_\rho UU^{-1}\partial_\mu UU^{-1}\partial_\rho UU^{-1}\partial_\mu U\Big\}U^{-1} \\
\equiv\ & -\theta(A-B)\Delta(0)U^{-1}\delta^{(4)}UU^{-1}, \qquad (13)
\end{aligned}$$

One should add this divergence to the one-loop result, thereby modifying the U field renormalization and equations of motion

$$\bar{\delta}U = \Delta(0)\left[\frac{1}{2}\partial_\mu^2 U - \frac{3+z^2}{4}\partial_\mu UU^{-1}\partial_\mu U + \delta^{(4)}U\right] = 0. \qquad (14)$$

This is one source of $O(p^4)$ terms and we shall see that there is another contribution at two loops.

As to other vertices it can be proved [17] that any diagram with an arbitrary number of external boson lines and two fermion lines, i.e. any vertex of those generated by the perturbative expansion of (2) is rendered finite by the previous counterterms. This completes the renormalization program at one loop.

There are 10 two-loop one-particle irreducible diagrams which are analytically calculated in [17]. The divergences in the propagator consist of the double divergent part, $\sim \Delta^2(0)$ and of the single divergent contributions, $\sim \Delta(0)$. The substantial part of these divergences is fully renormalized by performing the one-loop renormalization and taking into account the renormgroup evolution.

Some single-pole divergences remain however. Namely, there are divergences linear in $\Delta(0)$ which come from irreducible two-loop diagrams with maximal number of vertices. These divergences appear to be

$$
\begin{aligned}
-\Delta(0)U^{-1}\delta^{(4)}_{2-l}U^{-1} &\equiv c\Delta(0)\left[U^{-1}\partial_\rho U U^{-1}\partial_\mu U U^{-1}\partial_\mu U U^{-1}\partial_\rho U U^{-1}-\right.\\
&\left.-U^{-1}\partial_\rho U U^{-1}\partial_\mu U U^{-1}\partial_\rho U U^{-1}\partial_\mu U U^{-1}\right]. \qquad (15)
\end{aligned}
$$

with $c = \alpha'(1-z^2)^2/8 = \alpha'/2$ for $z = \pm i$. This term survives after adding all the counterterms. It must therefore modify the equation of motion (refeom4) at the next order in the α' expansion, $\delta^{(4)}U \to \delta^{(4)}U + \delta^{(4)}_{2-l}$. Its presence allows for non zero solutions for the coupling constants g_i and therefore for nonzero values for the Gasser-Leutwyler $O(p^4)$ coefficients.

4. Local integrability of equations of motion

The equation of motion, $\delta U = 0$, can be obtained from the dimension-two local action (9), involving a unitary matrix $U(x)$, only for $z = \pm i$. If the four-derivative part of equations of motion can be derived from dimension-four operators in a local effective Lagrangian then certain constraints are to be imposed on constants $g_{i,r}$.

Such a Lagrangian has only two terms compatible with the chiral symmetry,

$$
\mathcal{L}^{(4)} = f_\pi^2 \mathrm{tr}\left(K_1\partial_\mu U\partial_\rho U^+\partial_\mu U\partial_\rho U^+ + K_2\partial_\mu U\partial_\mu U^+\partial_\rho U\partial_\rho U^+\right). \qquad (16)
$$

Other terms containing $\partial_\mu^2 U$ are reduced to the set (16) with the help of the dimension-two equations of motion.

Variation of the previous Lagrangian supposedly saturate the dimension-four component of the equations of motion. Therefrom we identify this parameterization of constants with the coupling constants arising from the equations of motion (14) supplemented with (15) and after applying the $O(p^2)$ equations of motion. Then one obtains the following set of coefficients for the various chiral field structures

$$
\begin{aligned}
-2(2K_1+K_2) &= \frac{1}{16}(1-z^2)(1\pm z)(g_{1,r}\pm z g_{2,r});\\
-4K_2 &= \frac{1}{8}(1-z^2)(-g_{1,r}+z^2 g_{2,r}); \quad 2[(1-z^2)K_1+K_2] = -c;\\
-2z^2K_2 &= 0; \quad 4[K_1+K_2] = -\frac{1}{4}(1-z^2)z^2 g_{3,r} + c, \qquad (17)
\end{aligned}
$$

For $z^2 = -1$ only one solution is possible,

$$K_2 = 0, \quad K_1 = -\frac{1}{4}c = -\frac{\alpha'}{8}; \qquad g_{1,r} = -g_{2,r} = -g_{3,r} = 4c. \tag{18}$$

Thus, comparing eq.(16) with the usual parameterization of the Gasser and Leutwyler Lagrangian[18],

$$L_1 = \frac{1}{2}L_2 = -\frac{1}{4}L_3 = -\frac{1}{2}K_1 f_\pi^2 = \frac{f_\pi^2 \alpha'}{16}. \tag{19}$$

For $\alpha' = 0.9$ GeV^{-2} and $f_\pi \simeq 93$ MeV it yields $L_2 \simeq 0.9 \cdot 10^{-3}$ which is quite a satisfactory result[20].

The relation $L_1 = 1/2L_2 = -1/4L_3$ was established earlier in bosonization models [21] and in the chiral quark model[22] by means of a derivative expansion of quark determinant. However at that time its possible connection with a string description of QCD was not recognized. The first attempt to derive the chiral coefficients from the Veneziano-type dual amplitude was undertaken in[23] where a similar relation was found but with different numerical values for the L_i. However the specific choice of dual amplitude in [23] cannot be related to any known hadron string.

Another check comes from the compatibility of the unitarity of U and the equations of motion at the two-loop level. It turns out that if one accepts arbitrary real coefficients in the set of dimension-four operators then the only solution compatible with the unitarity is given by the parameterization with constants K_1 and K_2.

In our talk we have reported on a simplified model of the QCD string. Requiring of its conformal invariance around a chirally non-invariant vacuum leads to the Gasser and Leutwyler lagrangian. However the bosonic string action used here does not prevent large Euclidean world sheets from crumpling [24]. It does not also describe correctly the high-temperature behavior of large N QCD [25]. To correct it, a QCD induced string must be modified [24, 26] including operators breaking manifestly conformal symmetry on the world-sheet for large strings. Nevertheless we are concerned here with the low-energy string properties and therefore do not expect that the strategy and technique to derive the chiral field action needs any significant changes to be adjusted to a modified QCD string action.

We have restricted ourselves here to the $SU(2)$ global flavor group. In this case only parity-even terms in the equations of motion can be revealed from the simple fermion Lagrangian (2) and to obtain the parity-odd WZW lagrangian relevant for the case of three flavors one has to extend the boundary fermion action supplementing one-dimensional fermions with true spinor degrees of freedom.

This talk is supported by MCyT FPA2001-3598 and CIRIT 2001SGR-00065 and the European Networks EURODAPHNE and EUROGRID, as well as by Grant RFFI 01-02-17152, Russian Ministry of Education Grant E00-33-208 and by The Program *Universities of Russia: Fundamental Investigations* (Grant 992612). By this work A.A. and D.E. contribute to the fulfillment of Project INTAS 2000-587.

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