

New developments in nonperturbative QCD

Yu.A.Simonov
State Research Center
Institute of Theoretical and Experimental Physics,
Moscow, 117218 Russia

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Abstract

Basic developments in the analytic study of the QCD vacuum structure and of the QCD spectrum, including glueballs and hybrids are reviewed.

1 Introduction

One of the main problems of QCD is the development of quantitative analytic nonperturbative methods. Recently a very promising new method has been suggested [1, 2] (for a review see [3]), called the Field Correlator Method (FCM) which is fast progressing now.

FCM has a wide scope and allows to study all nonperturbative problems from the QCD vacuum structure [4, 5] to the details of the QCD spectrum [6]-[9], and high-energy scattering [10]. A practical usefulness of the method is connected to the number of gauge-invariant field correlators which should be included as the input of the method. Recently the lattice study [11] has confirmed the phenomenon of the Casimir scaling for the static $Q\bar{Q}$ interaction, [5], which implies the dominant role of the quadratic (Gaussian) field correlators with accuracy of order of 1%. In this way a very simple picture of the QCD vacuum emerges, called the Gaussian Stochastic Approximation (GSA), which implies short range correlations between field strength operators in the vacuum and the simple spectrum described by the only scale

parameter – string tension σ (with the α_s and correlation length T_g defining some 10% corrections to the spectrum).

In this talk we shall review the situation with the QCD vacuum structure and the derivation of the QCD spectrum.

2 The QCD vacuum structure. Stochastic vs coherent.

The basic quantity which defines the vacuum structure in QCD is the field correlator (FC)

$$D^{(n)}(x_1, \dots, x_n) \equiv \langle F_{\mu_1\nu_1}(x_1)\Phi(x_1, x_2)F_{\mu_2\nu_2}(x_2)\Phi(x_2, x_3)\dots F_{\mu_n\nu_n}(x_n)\Phi(x_n, x_1) \rangle, \quad (1)$$

$$\Phi(x, y) = P \exp ig \int_y^x A_\mu(z) dz_\mu.$$

The set of FC (1) for $n = 2, 3, \dots$ gives a detailed characteristic of vacuum structure, including field condensates (for coinciding $x_1 = x_2 = \dots x_n$). On general grounds one can distinguish two opposite situations: 1) stochastic vacuum 2) coherent vacuum. In the first case FC form a hierarchy with the dominant lowest term $D^{(2)}(x_1, x_2) = D^{(2)}(x_1 - x_2)$, while higher FC are fast decreasing with n . In the second case all FC are comparable, and expansion of physical amplitudes as the series of FC is impractical. This is the case for the gas/liquid of classical solutions, e.g. of instantons, magnetic monopoles etc. The physical picture behind the situation of nonconverging FC series is that of the coherent lump(s), when all points in the lump are strongly correlated.

To understand where belongs the QCD vacuum one can start with the Wilson loop in the representation D of the color group $SU(3)$,

$$W_D(C) = \langle tr_D \exp(ig \int_C dz_\mu A_\mu^a T_a^{(D)}) \rangle \quad (2)$$

The Stokes theorem and the cluster expansion identity allow to obtain the basic equation, which is used in most applications of the FCM (for more details see [3])

$$W(C) = \exp \sum_n \frac{(ig)^n}{n!} \int \tilde{D}^{(n)}(x_1, \dots, x_n) d\sigma_{\mu_1\nu_1}(x_1) \dots d\sigma_{\mu_n\nu_n}(x_n). \quad (3)$$

Here integration is performed over the minimal surface S_{min} inside the contour C defined in (2) and \tilde{D}_n is the so-called cumulant or the connected correlator, obtained from the FC Eq.(1) by subtracting all disconnected averages. From (3) one easily obtains that the Wilson loop has the area-law asymptotics, $W(C) \sim \exp(-\sigma S_{min})$, for any finite number of terms $n_{max}; n \leq n_{max}$ in the exponent (3).

The string tension is expressed through $\tilde{D}^{(n)}$.

$$\sigma = \frac{1}{2} \int D^{(2)}(x_1 - x_2) d^2(x_1 - x_2) + 0(\tilde{D}^{(n)}, n \geq 4) = \sigma_2 + \sigma_4 + \dots \quad (4)$$

Eq.(4) has several consequences: 1) confinement appears naturally for $n = 2$, i.e. in the GSA 2) the lack of confinement can be due to vanishing of all FC, or due to the special cancellation between the cumulants, as it happens for the instanton vacuum [4], 3) for static quarks in the representation D of the color group SU(3), the string tension $\sigma_2^{(D)}$ is proportional to the quadratic Casimir factor (the Casimir scaling)

$$\sigma_2^{(D)} = \frac{C_D^{(2)}}{C_D^{(fund)}} \sigma_2^{(fund)}, C_D^{(2)} = \frac{1}{3}(\mu^2 + \mu\nu + \nu^2 + 3\mu + 3\nu). \quad (5)$$

However for larger $n, n \geq 4$ the Casimir scaling is violated:

$$\sigma_n^{(D)} = a_1 C_D^{(2)} + a_2 (C_D^{(2)})^2 + a_3 C_D^{(3)} + \dots \quad (6)$$

It is remarkable that perturbative interaction of static quarks $V^{(D)}(r)$ satisfies the Casimir scaling to the order $O(g^6)$ considered so far [5], so the total potential $V^{(D)}(r) = V_{pert}^{(D)}(r) + \sigma^{(D)}r + const$ is also Casimir scaling, if GSA works well.

This picture was tested recently on the lattice [11] and confirmed the Casimir scaling with the accuracy around 1% in the range $0.1 \leq r \leq 1.1$. fm. The full theoretical understanding of this fundamental fact is still lacking, both for the perturbative part and for the string tension. On the pedestrian level the Casimir scaling and the quadratic (Gaussian) correlator dominance implies that the vacuum is highly stochastic and any quasiclassical objects, like instantons, are strongly suppressed in the real QCD vacuum. The vacuum consists of small white dipoles of the size T_g made of neighboring field strength operators. The smallness of T_g might be an explanation for the Gaussian dominance since higher correlator terms in σ are proportional to

$(FT_g^2)^n(T_g^2)^{-1}$, where F is the estimate of the average nonperturbative vacuum field, $F \sim 500 \text{ (MeV)}^2$.

Lattice calculations of FC have been done repeatedly during last decades, using cooling technic [12] and with less accuracy without cooling [13]. (Recently another approach based on the so-called gluelump states was exploited on the lattice [14] and analytically [15], which has a direct connection to FC).

The basic result of [12] is that FC consists of perturbative part $O(1/x^4)$ at small distances and nonperturbative part $O(\exp(-x/T_g))$ at larger distances with T_g in the range $T_g = 0.2 \text{ fm}$ (quenched vacuum) and $T_g = 0.3 \text{ fm}$ (2 flavours).

Calculations in [13] and [14, 15], as well as sum rule estimates [16] yield a smaller value, $T_g \approx 0.13 \text{ fm}$ to 0.17 fm . This enables us in what follows to take the limit $T_g \rightarrow 0$ keeping $\sigma = \text{const} \approx 0.18 \text{ GeV}^2$.

3 Relativistic dynamics of confinement

The Gaussian dominance and the small T_g limit greatly simplify the relativistic dynamics of hadrons.

As it was shown both for heavy quarks and for light quarks (see [6] for a review), in the limit of small T_g one has local relativistic dynamics which can be described by a local Hamiltonian. Before entering into the details of the method, it is useful to list a number of problems, present in all dynamical calculations of hadrons, which have been easily solved in our Hamiltonian method:

- 1) the problem of the constituent quark and gluon mass
- 2) the problem of Regge slope ($\alpha' = \frac{1}{8\sigma}$ for most calculations while in nature one has $\alpha' = \frac{1}{2\pi\sigma}$)
- 3) the problem of Regge intercept (an arbitrary constant appears in the Hamiltonian in most calculations to shift the masses down to the experimental values).

Finally, previous calculations are based on model assumptions and are not derived, in contrast to our Hamiltonian directly from the QCD Lagrangian. This results in a large number of fitting parameters, whereas in our case we have none.

- The basic scheme of the Hamiltonian derivation consists of three steps:
- i) Definition of initial and final states.

ii) Exact representation of the gauge-invariant Green's function as a path integral over Wilson loops.

iii) Derivation of the Hamiltonian from the Green's function in the Gaussian approximation and the small T_g limit.

We start with gauge-invariant asymptotic states, which are built with the use of parallel transporters $\Phi(x, y)$ for nonlocal states. This is done in the same way as on the lattice (but without fuzzing links etc.) Thus for the meson one has

$$\Psi_M(x, y) = \bar{q}(x)\Gamma\Phi(x, y)q(y), \quad \Gamma = (1, \gamma_\mu, \dots) \otimes 1, D_\nu, D_\mu D_\nu \dots \quad (7)$$

Gluons can be created gauge-covariantly by $F_{\mu\nu}$ and covariant derivatives e.g. $(D)^n F_{\mu\nu}$. At this point it is advantageous to separate perturbative (valence) a_μ and nonperturbative (background) gluonic field B_μ , [17]

$$A_\mu = B_\mu + a_\mu \quad (8)$$

which are integrated independently in the partition function due to the 'tHooft identity [17]

$$Z = \frac{1}{N} \int DA e^{-S(A)} = \frac{1}{N'} \int DB \int Da e^{-S(B+a)}. \quad (9)$$

Assigning to a_μ homogeneous gauge transformations, one can create valence gluonic states with the operators $a_\mu, (D)^n a_\mu$ etc. where $D = \partial_\mu - igB_\mu$. In this way glueball [18], hybrid [6, 19] and gluelump [15] states have been introduced in good agreement with lattice data.

As the next step one introduces FFSR for the Green's function [2], for a review and earlier refs. see [20]. E.g. for the meson it is

$$G_{q\bar{q}}^\Gamma(x, y) = \langle G_q(\cdot, y)\Gamma G_{\bar{q}}(x, y)\Gamma - G_q(x, x)\Gamma G_{\bar{q}}(\cdot, y)\Gamma \rangle_A \quad (10)$$

where FFSR for the (anti) quark Green's function is

$$G_q(x, y) = (m - \hat{D}) \int_0^\infty ds (Dz)_{xy} e^{-K} \Phi_\sigma(x, y), \quad K = \frac{1}{4} \int_0^\infty \left(\frac{dz_\mu}{d\tau} \right)^2 d\tau, \quad (11)$$

$$\Phi_\sigma(x, y) = P_A \exp(ig \int_y^x A_\mu dz_\mu) P_F \exp(g \int_0^s d\tau \sigma_{\mu\nu} F_{\mu\nu}) \quad (12)$$

and $(Dz)_{xy}$ implies the path integral from y to x . Here P_A, P_F are ordering operators. A similar representation exists for the valence gluon Green's

function, with 3 changes; factor $(m - \hat{D})$ in front in (11) is missing; $A_\mu, F_{\mu\nu}$ are in the adjoint representation and $g\sigma_{\mu\nu}F_{\mu\nu}$ in (12) should be replaced by $2gF_{\sigma\rho}$.

Insertion of (11) into (10) yields the main result to be used below; all gluonic field is present in the gauge-invariant form of the closed Wilson loop (modulo factor $(m - \hat{D})$ and σF insertions which are treated separately). Using for the latter the area law, found on the lattice or analytically in the Gaussian approximation, one has symbolically (neglecting spin terms)

$$G_{q\bar{q}}^\Gamma(x, y) \sim \int_0^\infty ds \int_0^\infty d\bar{s} (Dz)_{xy} (D\bar{z})_{xy} e^{-K-\bar{K}} e^{-\sigma S_{min}(C)}. \quad (13)$$

Here $S_{min}(C)$ is the minimal area inside the contour C made of the variable paths $C_z, C_{\bar{z}}$ integrated in (13). As the next step one goes over to the Hamiltonian H to escape from the path integrals, using the standard definition

$$G_{q\bar{q}}^\Gamma(x, y) = \langle x | e^{-HT} | y \rangle. \quad (14)$$

Here T is the evolution parameter associated with the chosen hypersurface, i.e. for the c.m. Hamiltonian it is the c.m. time coordinate. To proceed one should tackle the proper time s (or τ) in (11), (13), and here enters a new fundamental quantity which connects proper and actual (Euclidean) time t ,

$$2\mu(t) = \frac{dt}{d\tau}, \quad t \equiv z_4(\tau) \quad (15)$$

so that the integrals in (11), (13) can be rewritten as (see [20, 21] for details)

$$\int_0^\infty ds (D^4 z)_{xy\dots} = const \int D\mu(t) (D^3 z)_{xy}. \quad (16)$$

Here $\mu(t)$ enters as an einbein variable [22], physically the stationary point μ_0 of $\mu(t)$ will play the role of the constituent mass. Another einbein variable $\nu(\beta)$ enters from the representation of the string action in (13) and has the physical meaning of the string energy density along the string coordinate $\beta, 0 \leq \beta \leq 1$. The equal quark mass $m_1 = m_2 = m$ Hamiltonian was obtained in [21] and has the form

$$H = \frac{p_r^2 + m^2}{\mu(\tau)} + \mu(\tau) + \frac{\hat{L}^2/r^2}{\mu + 2 \int_0^1 (\beta - \frac{1}{2})^2 \nu(\beta) d\beta} +$$

$$+\frac{\sigma^2 r^2}{2} \int_0^1 \frac{d\beta}{\nu(\beta)} + \int_0^1 \frac{\nu(\beta)}{2} d\beta, \quad (17)$$

where $p_r^2 = (\mathbf{pr})^2/r^2$, and L is the angular momentum, $\hat{L} = (\mathbf{r} \times \mathbf{p})$.

The physical meaning of the terms $\mu(t)$ and $\nu(\beta)$ can be understood when one finds their extremal values. E.g. when $\sigma = 0$ and $L = 0$, one finds from (17)

$$H_0 = 2\sqrt{\mathbf{p}^2 + m^2}, \quad \mu_0 = \sqrt{\mathbf{p}^2 + m^2} \quad (18)$$

so that μ_0 is the energy of the quark. Similarly in the limiting case $L \rightarrow \infty$ the extremum over $\nu(\beta)$ yields:

$$\nu_0(\beta) = \frac{\sigma r}{\sqrt{1 - 4y^2(\beta - \frac{1}{2})^2}}, \quad M^2(L) = 2\pi\sigma\sqrt{L(L+1)} \quad (19)$$

so that ν_0 is the energy density along the string with the β playing the role of the coordinate along the string.

A similar form has the two-gluon (glueball) Hamiltonian, which obtains from (17) putting $m \equiv 0$ and $\sigma \rightarrow \sigma_{adj} = \frac{3}{4}\sigma$.

The generalization to more quarks and unequal masses and valence gluons is straight-forward, one can find the corresponding examples in [6].

4 Solution of the problems 1)-3)

1) To find the stationary values of $\mu = \mu_0$ one can use an approximate relation $\langle \frac{\partial H}{\partial \mu} |_{\mu=\mu_0} \rangle \approx \frac{\partial E(\mu)}{\partial \mu} |_{\mu=\mu_0} = 0$ which has a 5% accuracy [6], and the resulting μ_0 which can be considered as a constituent mass, is expressed entirely through σ . Taking $\sigma = 0.18 \text{ GeV}^2$ one obtains for constituent quark mass from the lowest meson state $\mu_0^q (\text{meson}) = 0.35 \text{ GeV}$ and for the constituent gluon mass from lowest glueball state, $\mu_0^g (\text{glueball}) = \frac{3}{2}\mu_0^q = 0.52 \text{ GeV}$. The proof that μ_0 is indeed constituent mass is obtained from the calculation of baryon magnetic moments which are expressed entirely through μ_0 in [23], and agree with experiments within 10%.

2) The Regge-slope problem is actually solved by (19), which yield $\alpha' = \frac{1}{2\pi\sigma}$ at large L due to the third term on the r.h.s. of (17), containing the string moment of inertia in the denominator. This is just the term, which is absent in the usual relativistic quark model (RQM) predicting $\alpha' = \frac{1}{8\sigma}$. For not large L the Regge slope (and the physical meson masses) have been

calculated in [7, 8] in good agreement with experimental meson masses and α'_{exp} .

3) the Regge-intercept problem is connected to the fact that actually the hadron mass is not given by the eigenvalue M_0 of H (17), but rather by the renormalized value $M = M_0 - \Delta M$. In all RQM calculations ΔM is considered as a phenomenological parameter and is large, $\Delta M \sim 0.7$ GeV for mesons, whereas its physical origin was obscure.

In [24] it was found that ΔM appears due to the nonperturbative quark mass renormalization, namely the selfenergy due to the term $\sigma_{\mu\nu}F_{\mu\nu}$ in the second order.

This mechanism gives $\Delta M = \frac{4\sigma}{\pi} \sum_{i=1}^n \frac{1}{2\mu_i} \eta_i$, where μ_i is the constituent mass of the i -th quark, the sum is over all quarks in the hadron and $\eta_i(m_i)$ depends only on the current quark mass m_i and $\eta_i(0) = 1$ for light quarks. This expression yields correct values for ΔM for both mesons and baryons [7]-[9].

It is fascinating that i) μ_i , entering the denominator of ΔM , help to keep intact the Regge-slope in $M = M_0 - \Delta M$. ii) the quark mass renormalization $\delta m_q^2 = -\frac{4\sigma}{\pi}$ is twice as large as in the 1+1 QCD. Thus also the third problem – the absolute values of hadron masses – is simply solved within the method. The overall agreement of calculated masses of light mesons in [7, 8], heavy quarkonia in [25], baryons in [9], glueballs in [18], hybrids in [19] with experimental and/or lattice data is better than 10% and is surprising since the method exploits only two inputs σ and α_s (or Λ_{QCD}) (in addition to current quark masses renormalized at around 1 GeV).

Here we have touched only upon two basic topics in the framework of the fast developing FCM, more details and applications can be found in the review [3] and cited literature.

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