Soliton solutions of Chiral Born-Infeld Theory and baryons.

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Abstract.

Finite-energy topological spherically symmetrical solutions of Chiral Born-Infeld Theory are studied. Properties of these solution are obtained, and a possible physical interpretation is also given. We compute static properties of baryons (mass,main radius, magnetic main radius, axial coupling constant) whose solutions can be interpreted as the baryons of QCD.

The concept of the baryon as a chiral soliton has a long history. The idea of a unified theory for baryons and mesons that is formulated in terms of chiral field was only proposed for the first time by Skyrme [1]. This theory has a set of topological static solutions and one can classify these solutions by the value of the topological (baryon) charge. In this model such solutions are associated with baryon states with different baryon charges, and soliton with $B = 1$ (skyrmeon) is treated as a nucleon. Such kind of solutions and quantum fluctuations about them is well studied now and results of such analysis are in agreement with the experiment data with accuracy of 30-40 percents for baryonic masses and another static properties of baryons.

But there are many other ways for stabilization of chiral soliton, because from the methodological point of view, arising of the Skyrme part in the lagrangian of effective meson field theory is an "ad hoc" procedure, and there are no real physical bases for this procedure. Of course, one can treat this part in the effective lagrangian as a leading high derivative expansion of effective chiral meson action, but this approach leads to numerous questions so far. Two most important ones concern the physical nature of the scale parameter in this theory and the influence of another terms of expansion on existence and stability of chiral solitons.

The consistent effective low-energy meson's theory construction is a very complicated task that is closely connected to the quark's confinement problem and to the problem of spontaneous breaking of chiral symmetry. It is desirable that such theory should be Lorentz and chiral invariant and should have a set of finite energy stable solutions. In the case of electro-magnetic fields such kind of an effective low-energy theory is the well-known Born-Infeld model [2]. In our work [3] we study the direct analogue of the Born-Infeld action for chiral fields. By construction our model has no singular solutions and looks very attractive as a possible effective action for mesons fields. This model has a set of stable topological solitons. These solutions can be treated as baryons states in our model.

In the well-known paper [2], Born and Infeld proposed a non-linear covariant action for electro-magnetic fields with very attractive features. Firstly, in the framework of BI

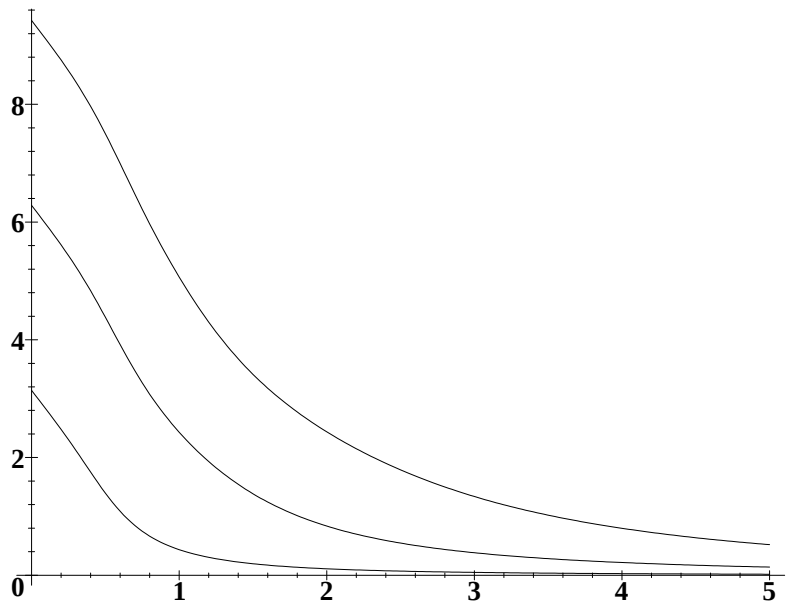

Figure 1: Solitons with $B = 1, 2$ and 3. Horizontal axis: r (in fm).

theory the problem of singular self-energy of electron can be solved. In this theory the electron is a stable finite energy solution of the BI field equation with electric charge. Second, the BI action has a scale parameter β. Using expansion by this parameter one reduces the BI action to the usual Maxwell form in the low-energy limit.

We want to perform a very similar procedure with the chiral prototype lagrangian

$$\mathcal{L}_{pr} = -\frac{f_\pi^2}{4}\mathrm{Tr}L_\mu L^\mu, \tag{1}$$

where $L_\mu = U^+\partial_\mu U$ is a Cartan left-invariant form and $U = \exp(i\frac{\vec{\phi}_\pi\vec{\tau}}{f_\pi})$ where the vector field $\vec{\phi}_\pi$ is associated with π - mesons and $f_\pi = 93$ MeV is the pion decay constant. Like in the case of BI action, our chiral model must have a set of finite energy solutions with integer values of charge (topological or baryon), and in the low-energy limit such a theory must reproduce the prototype lagrangian (1). The model must be Lorentz and chiral invariant. Finally, the form of such theory follows from the analogy with the action of a relativistic particle, the BI action for EM and YM field.

Arguing as above, let us consider a theory with lagrangian

$$\mathcal{L}_{ChBI} = -f_\pi^2\mathrm{Tr}\beta^2\Big(1 - \sqrt{1 - \frac{1}{2\beta^2}L_\mu L^\mu}\Big) \sim -\frac{f_\pi^2}{4}\mathrm{Tr}L_\mu L^\mu, \tag{2}$$

where β is a mass dimensional scale parameter of our model. It is easily shown that the expansion of the lagrangian (2) gives us the prototype theory as the leading order theory by parameter β.

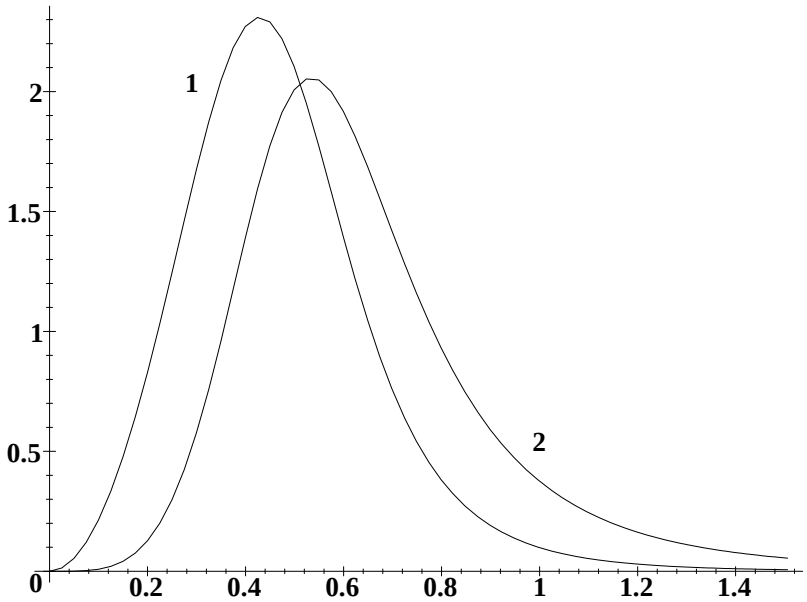

Figure 2: Baryonic density (1) and magnetic moment density (2). Horizontal axis: r (in fm).

Now we consider the spherically symmetrical field configuration $U = \exp(iF(r)(\vec{n}\vec{\tau}))$, $\vec{n} = \vec{r}/|r|$. Using the variation principle, we get the equation of motion

$$(r^2 - \frac{1}{\beta^2}\sin^2 F)F'' + (2rF' - \sin 2F) -$$

$$-\frac{1}{\beta^2}(rF'^3 - F'^2 \sin 2F + 3\frac{1}{r}\sin^2 F - \frac{1}{r^2}\sin 2F \sin^2 F) = 0. \qquad (3)$$

The numerical investigation of the solutions of equation (3) is presented in Fig.1 (solitons with $B = 1, 2$ and 3). The scale parameter $\beta = 807\,\mathrm{MeV}$ is defined from the hypothesis that the soliton with $B = 1$ is a nucleon. Indeed, using now a scale transformation, one gets $\beta = 8\pi f_\pi^2 E(B = 1,\, \beta = 1)/m_p = 807\,\mathrm{MeV}$, where $E(B = 1,\, \beta = 1) = 3.487$ is the energy of this soliton solution for $B = 1$ and $\beta = 1$.

Using this value of the β, one can find another physical properties of the hadron. The baryonic density (see Fig.2) and the main radius of the hadron can be calculate by

$$\rho_B = -\frac{2}{pi}\sin^2 F\, F', \quad < r^2 >= \int_0^\infty r^2 \rho_B(r) dr.$$

For Chiral Born-Infeld model we have $< r^2 >^{\mathrm{ChBI}} \simeq 0.51$fm (Skyrme model: $< r^2 >^{\mathrm{Sk}} \simeq 0.59$fm); experiment data: $< r^2 >^{\mathrm{exp}} \simeq 0.72$fm [4]). The magnetic moment density (see Fig.2) and magnetic main radius of the hadron are

$$\rho_M = \frac{r^2 \sin^2 F\, F'}{\int_0^\infty r^2 \sin^2 F\, F' dr}, \quad < r^2 >_M = \int_0^\infty r^2 \rho_M(r) dr.$$

For Chiral Born-Infeld model we have $< r^2 >_M^{\mathrm{ChBI}} \simeq 0.74$fm (Skyrme model: $< r^2 >^{\mathrm{Sk}} \simeq 0.92$fm); experiment data: $< r^2 >^{\mathrm{exp}} \simeq 0.81$fm [4]).

Using asymptotic of our solutions at infinity, one can find a prediction for the value of axial coupling constant $g_A^{\mathrm{ChBI}} = 0.79$. Such prediction lies much close to the experiment data $g_A^{\mathrm{exp}} = 1.23$ then the prediction of Skyrme model $g_A^{\mathrm{Sk}} = 0.61$ [4].

We do not give a comprehensive investigation of the Chiral Born-Infeld theory. The questions about non-spherical solutions, quantum fluctuations about such solutions and corresponding properties of baryons or about the nucleon-nucleon interaction are not clear now. But maybe the most important question in such investigation is about physical substantiation of such theory. All of these questions should be the themes for a future investigation.

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