

Topological and non-topological solutions for the chiral bag model with constituent quarks

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September 27, 2002

1 Introduction

At present the models of quark bags turn out to be one of the most perspective approaches to the study of the low-energy structure of baryons. The most promising results have been obtained within so called hybrid chiral bag models (HCBM, [1-4]). In HCBM free and massless quarks and gluons are confined in a chirally invariant way in a spatial volume, surrounded by the colorless purely mesonic phase. Mesons are described by some nonlinear theory like the Skyrme model ([5-8]). However, in such 2-phase HCBM there is no place for massive constituent quarks, whose concept is one of the cornerstones in the hadronic spectroscopy ([9-12]). Thus the most attractive situation should be the one, where the initially free and almost massless current quarks at first transmute by interaction into massive constituent quarks with the same quantum numbers, and only afterwards the purely mesonic colorless phase emerges.

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2 Intermediate phase

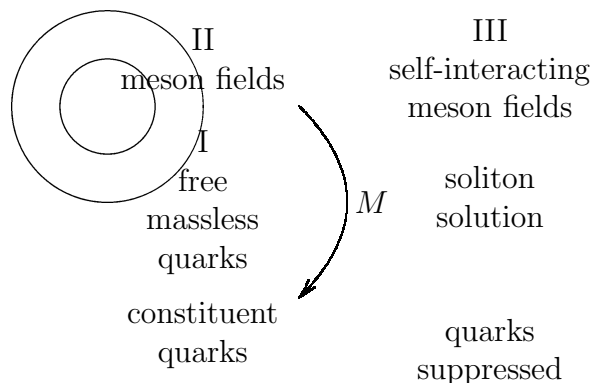


Fig. 1. Bag with intermediate phase

The first step towards such a version of the bag is made in a 3-phase chiral model ([13]), where an additional intermediate phase of interacting quarks and mesons is introduced. Asymptotically free massless quarks live in the first (inner) phase. Second (intermediate) phase contains constituent quarks acquiring effective mass due to the chirally invariant interaction with the meson fields. Hadronization takes place in the third (outer) phase where the quark degrees of freedom are completely suppressed, while a nonlinear dynamics of meson fields leads to the appearance of the c-number boson condensate in a form of a classical soliton solution. This soliton solution keeps up the topological nature of the model as well as the relevant quantum numbers.

3 3-phase HCBM in (1+1)D

We consider a toy model of such kind in (1+1)D with one-flavor fermion field as quarks, and real scalar field as mesons. For this model the self-consistent solutions with different values of topological charge (namely 1, 2, and 0) has been found ([13-15]). For these solutions renormalized total energy of the bag can be studied as a function of its geometry and topological charge. It has been shown that for non-zero topological charge there exists a set of configurations being the local minima of the total energy of the bag and containing all the three phases, while in the nontopological case the minimum

of the bag's energy corresponds to the asymptotic freedom phase of vanishing size.

The model is described by the shown Lagrangian:

$$\begin{aligned} \mathcal{L} = & \bar{\psi}i\hat{\partial}\psi + \frac{(\partial_\mu\varphi)^2}{2} - \frac{m_0^2\varphi^2}{2}\theta_{\text{I}} - \frac{M}{2} \left[\bar{\psi}, e^{i\gamma_5\varphi}\psi \right]_- \theta_{\text{II}} \\ & - \frac{M_0}{2} \left[\bar{\psi}, e^{i\gamma_5\varphi}\psi \right]_- \theta_{\text{III}} - V(\varphi)\theta_{\text{III}}. \end{aligned} \quad (1)$$

Theta-functions with indices I, II and III select the corresponding region. First and second terms are standard kinetic terms for fermion and scalar fields. It must be noted that unlike other models kinetic terms and therefore fields exist in a whole space though they may be suppressed in some regions in terms of physical observables. Third term assigns mass m_0 to the scalar field in the inner region. Forth and fifth terms describe chirally invariant interaction between fermions and mesons with the constant M and M_0 in the regions II and III correspondently.

Last term is a non-linear self-interaction potential of the scalar field in the outer region. It is even and leads to soliton-like solutions. Thus we suppose it have at least two stationary points (like well-known potential φ^4) at values of φ equal to $\pm\pi$. Then the scalar field could be either odd (the topological charge is nonzero) or even (null in the simplest case). Various configurations are shown on the figure 2.

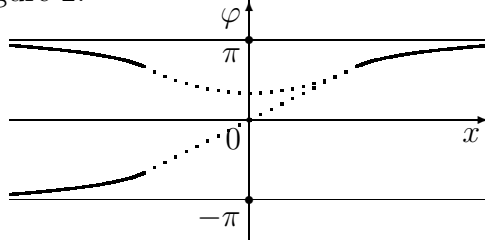


Fig. 2. Scalar field asymptotics

We consider the topological case at first and later we'll shortly discuss non-topological one. The soliton is always approaching quadratic stationary values exponentially:

$$\varphi(x) \sim \pm\pi \left(1 - e^{-m(|x| - x_0)} \right). \quad (2)$$

m here is a second-order derivation of potential V in the stationary points.

$$m^2 = \left. \frac{\partial^2 V}{\partial \varphi^2} \right|_{\varphi = \pm\pi}. \quad (3)$$

To form the bag we suppose interaction constant M_0 and scalar field mass m_0 to be very large, that dynamically suppresses fermions in the outer region and meson field in the inner region. According to the general approach accepted in HCBM, the boson field is treated in the mean-field approximation, i.e. it is assumed to be a c -number field being a stationary classical background for fermions.

4 Self-consistent solution

The essential feature of this model is that the equations of motion in the intermediate region possess simple and physically meaningful solution. In order to obtain it let us assume the linear behavior for the scalar field φ in the intermediate region:

$$\varphi' = 2\lambda = \text{const.} \quad (4)$$

Then the Skyrme rotation

$$\psi = e^{-i\gamma_5\varphi/2}\chi \quad (5)$$

transforms the fermion equations in the intermediate region into the equations for free fermions χ with mass M , and eigenvalues $\nu = \omega - \lambda$:

$$\begin{aligned} x \in \text{I} : & \quad \begin{cases} i\hat{\partial}\chi = 0, \\ \varphi = 0; \end{cases} \\ x \in \text{II} : & \quad \begin{cases} i\left(\hat{\partial} - \gamma_0\lambda - M\right)\chi = 0, \\ \varphi'' = M \cdot \langle J_5 \rangle; \end{cases} \\ x \in \text{III} : & \quad \begin{cases} \chi = 0, \\ \varphi'' = \partial V / \partial \varphi. \end{cases} \end{aligned} \quad (6)$$

So the fermions being massless in the inner region, acquire the mass M in the region II due to the coupling to the field φ , whence the intermediate phase describing massive quasifree constituent quarks emerges (see fig. 3).

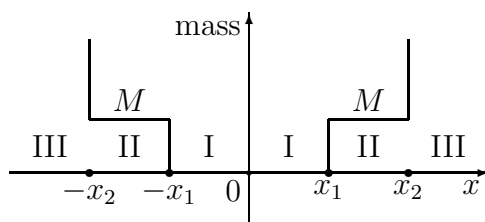


Fig. 3. Fermion mass profile

5 Condition on bag's geometry

Right-hand side of the equation of motion for the field φ in the intermediate region is proportional to the vacuum expectation value for the C -odd chiral current J_5 :

$$\varphi'' = M \cdot \langle J_5 \rangle. \quad (7)$$

Assuming every level is either occupied or empty this chiral current can be found from the following expression:

$$\langle J_5 \rangle = \left(\frac{1}{2} \sum_{n < 0} - \frac{1}{2} \sum_{n > 0} \right) \bar{\chi}_n \gamma_5 \chi_n. \quad (8)$$

where the first summation includes occupied levels and the second one includes unoccupied ones.

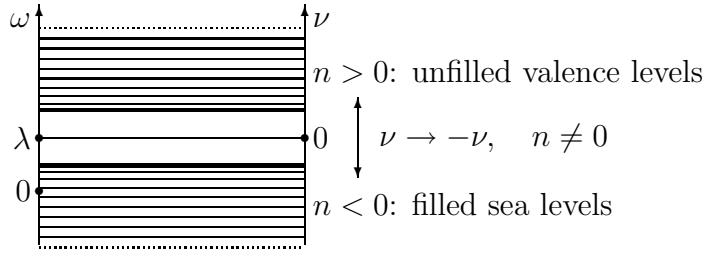


Fig. 4. Energy levels

The equation for the fermion field χ in the *intermediate* region contains the important symmetry changing the sign of eigenvalues ν and leaving the chiral current additive $\bar{\chi} \gamma_5 \chi$ intact. Thus if there is a pair with opposite ν for each energy level (see fig. 4), additives of the pair will cancel each other in the expression (8) for J_5 , and the chiral current will vanish; linear function will satisfy the equation of motion (7), and our solution will be self-consistent.

However, fermion equation in the *inner* region generally does not obeys this symmetry. Hence we receive an additional condition on the bag geometry for spectrum to be symmetrical:

$$4\lambda x_1 = \pi s, \quad s = 2, 4, 6 \dots, \quad (9)$$

where $2x_1$ is the length of the inner region. Spectra corresponding to the odd values of s although symmetrical include a single level with zero value of ν for which there's no pair, so we consider only the even values of s here. Note that for large enough λ some levels with positive ω must be occupied for the cancelation of the chiral current.

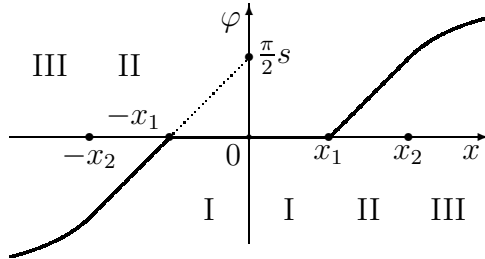


Fig. 5. Scalar field profile

6 Solution keypoints

Thus we obtained a series of self-consistent solutions. There are the following keypoints that make these solutions meaningful. The first is the finiteness of the intermediate region size, because for an infinite region the linear solution would be unacceptable. In our case, however, the size of the intermediate region is always finite by construction, while the boson field acquires the solitonic behavior in the outer region due to the self-interaction. Here the following circumstance manifests again: in (1+1)D the chiral coupling itself cannot cause the solitonic behavior of the scalar field by virtue of the effects of fermion-vacuum polarization only, i.e. without any additional self-interaction of the bosons.

The second point is the discreteness and the symmetry of the fermionic spectrum, what leads in turn to a reasonable method for the calculation of the chiral current average over the filled Dirac's sea, as well as of other C -odd observables like the total fermion number. After all, in this case we consider the boson field to be continuous everywhere and so it is topologically equivalent to that odd soliton, which would take place due to the self-interaction in the absence of fermions. So a topological number of the boson field doesn't depend on the existence and sizes of the spatial regions containing fermions ($I \cup II$). On the other hand, the baryon number of the hybrid bag is, by definition, the sum of the topological charge of the boson soliton and the fermion number of the bag interior. In our case the latter is null (for the ground state), hence the baryon number of the bag is determined by the topological charge of the boson field only and doesn't depend on the sizes of the regions containing fermions, what meets the general requirements for hybrid models. Additional details concerning the status of this solution can be found in publications [13-15].

The third point is the required absence of level with zero value of ν because this non-degenerate level prevents the cancellation of the chiral current. In our framework only configurations with even values of s do not contain this level and therefore self-consistent. However, other bags exist with only odd values of s allowed instead of even.

7 Bag energy as function of it's size

Because the additional condition (9) must hold, and we expect the scalar field to pass smoothly into it's asymptotics (2). we have only one geometrical degree of freedom left: size of the bag compared to the fermion mass M . We denote it as ρ :

$$\rho = 2x_2M, \quad (10)$$

and also introduce dimensionless ratio μ of the two mass parameters of the model:

$$\mu = m/2M, \quad (11)$$

The value of μ can be chosen as ratio of masses of the physical mesons and constituent quarks; we take the value of $1/4$. Now we can examine a dependence of a total energy of the bag E_{bag} on it's size ρ . For the model to be meaningful the energy must have a distinct minimum.

The total energy of the bag is the sum of the bosonic field energy E_φ and the fermionic contribution E_ψ :

$$E_{\text{bag}} = E_\varphi + E_\psi. \quad (12)$$

For the total energy of the boson field with the help of the virial theorem one finds the following expression:

$$E_\varphi = m\pi^2 \frac{2s+1}{\mu\rho+1}. \quad (13)$$

This value is decreasing as the size of the bag grows thus the soliton itself is non-stable in the absence of the fermion fields. The C -odd expression for the total energy of the fermion field reads as shown:

$$E_\psi = \left(\frac{1}{2} \sum_{n<0} - \frac{1}{2} \sum_{n>0} \right) \omega_n. \quad (14)$$

Energy eigenvalues ω_n can be obtained numerically. We must also apply a renormalization procedure to this sum (14), though I will not go into details in this report.

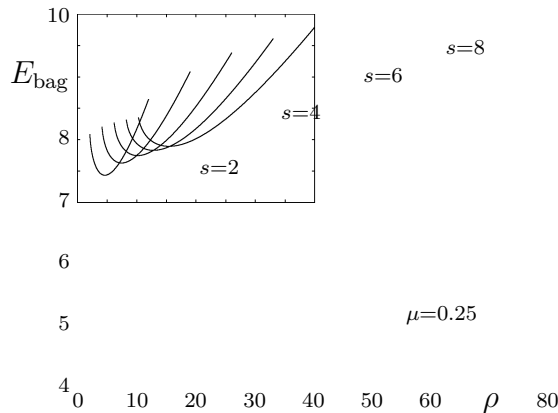


Fig. 6. Total bag energy in units of M

Numerical results for different values of s are shown on the figure 6. It can be seen that for each value of s there is a minimum in total bag energy. The size and the energy of the solution determined from the minimum of energy grows continuously for increasing s , what suggests the interpretation of configurations with $s > 2$ as excited states of the bag.

8 Non-topological case

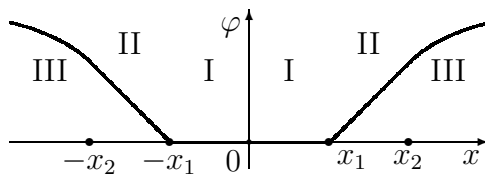


Fig. 7. Scalar field profile

Now a brief description of the non-topological case. The scalar field profile is shown on the figure 7. In our framework such configurations correspond to mesons. This case is different from the previous one in the following points: (1) there're no additional conditions on bag's geometry and (2) the chiral current never vanishes. Because of the later point calculations become much more numerical, though linear function turns out to be a very good

approximation to exact solution in the interior region. Because of the lack of additional conditions we've got two degrees of freedom now: size of the interior region and size of the intermediate region. We denote correspondent dimensionless ratios as α and β .

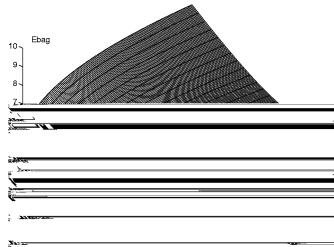


Fig. 8. Total bag energy

Total bag energy with the same value of μ as a function of α and β is shown on the figure 8. It can be seen that minimum of energy corresponds to a finite size of intermediate region and vanishing size of inner region. So for the bags with zero topological charge the considered 3-phase model predicts, that the main role should be played by the intermediate phase of constituent quarks, what is quite consistent with semi-phenomenological quark models of mesons.

9 Conclusion

This work was aimed at the construction of a 3-phase version of a hybrid chiral bag with both current and constituent quarks. Our results show, that such a model can be formulated in a quite consistent fashion and in the topological case leads to the series of configurations with reasonable behaviour of the total bag's energy as a function of its size, which takes the form of an infinitely deep potential well with a distinct minimum, whereas in the non-topological case the minimal energy of the bag corresponds to the configuration, where the phase of asymptotic freedom disappears.

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