Tachyon Condensation in String Theory and Field Theory

N.D. Lambert[1] and I. Sachs[2]

[1]Dept. of Physics and Astronomy
Rutgers University
Piscataway, NJ 08855
USA
nlambert@physics.rutgers.edu

[2]School of Mathematics
University of Dublin, Trinity College
College Green, Dublin 2
Ireland
ivo@maths.tcd.ie

August 22, 2003

Abstract

We review the effective action approach for the tachyon living on a non-BPS brane in superstring theory. In particular we describe a first derivative effective action for the tachyon which admits space-like as well as time-like marginal marginal deformations as classical solutions. The predictions for the physical properties of such kinks are discussed an compared to the predictions from string theory.

1 Tachyons in Superstring Theory

One of the motivations for abandoning bosonic string theory in favour of superstrings is the absence of tachyonic instabilities that plague bosonic strings. The low lying bosonic degrees of freedom in the spectrum of open superstrings are gauge bosons and massless scalars. Furthermore, since Polchinski's discovery of supersymmetric D-branes [1] we know that open superstrings describe just the excitation of these branes leading, in particular, to a simple geometric interpretation of the massless opens string fields. At low energy, the action governing the dynamics of D-branes is well approximated by the so-called Dirac-Born-Infeld action

$$\begin{aligned} S_{BI} &= \tau_p \int \mathrm{d}^{p+1}\sigma \sqrt{\det\left(g_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}\right)} \\ &= \simeq \tau_p \int \mathrm{d}^{p+1}\sigma \sqrt{|g|} + \frac{1}{4g_{eff}^2} \int F_{\mu\nu}F^{\mu\nu} + \cdots \ , \end{aligned} \tag{1}$$

where $F = \mathrm{d}A$ is the field strength, $g_{\mu\nu}$ is the induced metric on the brane. The equations of motion derived from (1) are compatible with the β-function equations so that, in particular, the marginal deformations of the open string Polyakov action are classical solutions of (1). If we include the fermionic degrees of freedom then the low energy effective theory is that of supersymmetric Yang-Mills theory dimensionally reduced to $p + 1$ dimensions. The mass density τ_p is proportional to $1/g_{string}$ implying that D-branes are not perturbative states in string theory.

However, recently it was suggested that D-branes themselves are in fact, solitonic excitations of the tachyonic sector of string theory [2]. To see how this happens we need to go outside the set of supersymmetric backgrounds introduced above. Such non-supersymmetric backgrounds can be realised in terms of Sen's non-BPS branes [3]. In contrast to supersymmetric D-branes, the fluctuations of non-BPS branes are described by two types of open strings. The first type is as above while the second type of open string has the opposite GSO-projection, thus retaining, instead of the massless photon, a real tachyon with mass$^2 = -1/2\alpha'$ in its spectrum. It should be noted that, contrary to bosonic string theory, this tachyon does not mean that the theory is ill defined because in superstring theory a supersymmetric and stable ground state is known to exist. On the other hand the tachyonic sector of superstring theory contains interesting physics some of which will be reviewed in this talk. Let us begin with some general comments about the

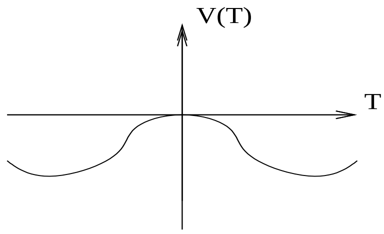

Figure 1: Tachyon Potential on a non-BPS brane.

vacuum structure on non-BPS branes. Just like D-branes, the low energy dynamics of non-BPS D-branes should be described by some (non supersymmetric) field theory. Of course the tachyon, having a mass^2 of the order of the string scale, is not a low energy degree of freedom and correspondingly any effective field theory describing the tachyonic sector will involve higher derivative terms. For the moment let us concentrate on the potential term only. According to Sen's proposal the tachyon potential $V(T)$ should have a minimum at $T = \pm T_0$ (see Fig. 1) such that the negative vacuum energy density exactly cancels the brane tension, i.e.

$$V(T_0) + \tilde{\tau}_p = 0 \ . \tag{2}$$

Here $\tilde{\tau}_p = \sqrt{2}\tau_p$ is the mass density of a non-BPS D-brane. The vacuum $T = \pm T_0$ should then be identified with the closed string vacuum where the non-BPS brane has decayed or, in other words, the open strings have condensed. Furthermore Sen suggests that kink solutions interpolating between $-T_0$ and T_0 should be interpreted as D-branes of one lower dimension.

A central role is played by the tachyon profile

$$T(x) = \chi \sin\left(\frac{x - x_0}{\sqrt{2\alpha'}}\right) \ , \tag{3}$$

which is an exactly marginal deformation of the Polyakov action at tree level in string theory (e.g. see [4]). Correspondingly χ is a modulus interpolating between $T \equiv 0$ and a periodic kink/anti-kink configuration for $\chi = 1$ (it turns out that the physics is in fact periodic in χ) [3].

Marginal deformations can also be used analyse time dependent tachyon backgrounds, or S-branes in string theory [5, 6]. An analysis of the stress tensor obtained from the boundary state for a decaying D-brane [7] shows that the decay of an unstable D-brane results in a gas with finite energy density but vanishing pressure. This suggests that these tachyons could have possible cosmological applications as dark matter candidates. An interesting question, which we will address below, is to what extent these features can be obtained from an effective field theory for the open string tachyon.

2 Tachyon Effective Actions

For general tachyonic profiles which are not almost marginal deformations the quantitative treatment of dynamics of tachyon condensation is a challenging off-shell problem in string theory. In particular the validity of a derivative expansion is questionable.

Before considering the generic situation we can first study the dynamics of the modulus mode χ that appears in (3) in a setting where the tachyonic instability is removed. More precisely we wrap the non-BPS p-brane on an orbifold $T^4/\mathcal{I}(-1)^{F_L}$, where $\mathcal{I}: x^i \to -x^i$ for $i = p-3, \cdots, p$ and F_L is the left-moving fermion number. This has the effect of removing the tachyonic groundstate of the non-BPS brane while retaining the marginal deformation (3) and the corresponding massless mode χ introduced above. In addition we now find one such mode for each of the four directions of the orbifold, χ^i, and these represent the lowest four non-vanishing momentum modes of the tachyon along the orbifold. For appropriately choosen orbifold radii these four modes are exactly massless and the effective action can be unambiguously derived in an analogous manner to the derivation of the Born-Infeld effective action for open strings. The result again has a Born-Infeld form [8]

$$S \propto \tilde{\tau}_p \int d^{p-4}\sigma \mathrm{Tr}\sqrt{-\det(\delta_{mn} + \mathcal{F}_{mn})} \ , \tag{4}$$

where γ^i are four-dimensional Euclidean γ-matrices and

$$\mathcal{F}_{mn} = \begin{pmatrix} 0 & \partial_\mu \chi^i \otimes \gamma^i \\ -\partial_\nu \chi^j \otimes \gamma^j & \{\chi^i, \chi^j\} \otimes \gamma^{ij} \end{pmatrix} \ , \quad i \neq j \ . \tag{5}$$

Here there is no sum over the i, j indices, $m, n = 0, 1, ..., p$ and $\mu, \nu = 0, 1, ..., p-4$. For N parallel non-BPS D-branes so that the fields χ^i take values in a $U(N)$ Lie Algebra. These modes can also be combined with the remaining massless degrees of freedom and form a super-multiplet obtained by dimensional reduction of 10-dimensional $N = 1$ Yang-Mills theory, just as in the case of BPS D-branes. However, the interactions involving χ^i break

SUSY explicitly. The details of the effective action for the massless degrees of freedom can be found in [9]. Thus non-BPS branes provide us with a geometric realisation of non-supersymmetric Yang-Mills-Higgs theory. Note that the interpretation of the scalar degrees of freedom in this theory is rather different from its supersymmetric counter part. While in the latter the scalars correspond to transversal excitation of the D-brane, in this case they describe the marginal deformation of the non-BPS brane into lower dimensional BPS branes. In particular the classically flat directions of the potential correspond to anti-commuting values of the scalars χ^i. Of course since supersymmetry is broken we expect that quantum correction will lift the flat directions described by χ. Indeed, at the loop level an effective potential is created for χ leading to condensation of this mode, or equivalently, to the decay of the non-BPS brane to a brane/anti-brane pair [10].

Let us now return to the problem of truly tachyonic modes on unstable non-BPS D-branes. Concretely we seek an action of the form $S[T, \partial T, \partial^2 T, \cdots]$ which reproduces the desired features of the open string tachyon on non-BPS branes. The neglect of higher derivative terms is inherently ambiguous and therefore there have been several proposals for the tachyon effective action in the literature. The most simplistic ansatz for the tachyon action is to simply adopt the Born-Infeld action for the massless scalars on BPS branes [11]

$$S = \tilde{\tau}_p \int \mathrm{d}^{p+1}\sigma V(T) \sqrt{1 + \pi \partial_m T \partial^m T} \; . \tag{6}$$

Although there is no a priori justification for this ansatz, the predictions obtained from it are in qualitative agreement with some features expected from string theory. On the other hand there is a well defined procedure to extract an effective action from boundary string field theory where the action is defined as the partition function

$$S[T, \partial T, \partial^2 T, \cdots] \equiv Z[T[(x)]] \; . \tag{7}$$

Here $Z[T]$ is the disk partition function with the tachyon profile $T(x)$ inserted at the boundary of the world sheet. In practice, however, explicit results for $Z[T]$ are known only for constant and linear tachyon profiles. In this case the partition function is compatible with the action [12]

$$\begin{aligned} S[T, \partial T] &= \frac{1}{2}\tilde{\tau}_p \int \mathrm{d}^{p+1}\sigma e^{-T^2/2\alpha'} 4^{(\partial T)^2} (\partial T)^2 \frac{\Gamma((\partial T)^2)^2}{\Gamma(2(\partial T)^2)} \\ &= \tilde{\tau}_p \int \mathrm{d}^{p+1}\sigma e^{-\frac{T^2}{2\alpha'}} \left[1 + 4\pi\alpha' \log(2)(\partial T)^2 + O((\partial T)^4)\right] \; . \end{aligned} \tag{8}$$

It should be noted that the tachyon potential obtained in this way is *exact*. The minima of the potential is at $T_0 = \pm\infty$ in these variables. The derivative terms on the other hand are ambiguous due to the problems arising upon partial integration over linear profiles. In particular the tachyon mass term is not reproduced correctly as a result of this ambiguity. Note also that both of the actions presented so far do not allow the marginal profiles as classical solutions. See [13] for a discussion of how the lowest order terms of these actions are related to open string S-matrix elements.

Let us now consider a general ansatz for a first derivative action [14]. The most general form for the effective action of a real T in $p+1$ dimensions that depends at most on first order derivatives and is even in T is given by

$$S = \int d^{p+1}x\mathcal{L} \equiv \int d^{p+1}x \sum_{\alpha,\beta=0}^{\infty} c_{\alpha\beta} T^{2\alpha}(\partial_m T \partial^m T)^{\beta} \ . \tag{9}$$

We now impose compatibility with the β-function equation from string theory. In particular we impose that the marginal deformation (3) solves the field equations derived from (9). It turns out that this requirement completely determines the general action (9) in terms of an arbitrary potential $V(T) = f(\frac{T^2}{2\alpha'})$, i.e. [14]

$$\mathcal{L} = \sum_{\gamma=0}^{\infty} \frac{1}{\gamma!}\frac{1}{2\gamma-1}\frac{d^{\gamma} f(t)}{dt^{\gamma}}(\partial_\mu T\partial^\mu T)^{\gamma} \ , \tag{10}$$

where $t \equiv T^2/2\alpha'$. The function $f(t)$, however, is not determined by this approach. To continue we fix this freedom by choosing the BSFT potential. This then leads to

$$\mathcal{L} = -\tilde{\tau}_p e^{-\frac{T^2}{2\alpha'}} \left[e^{-\partial_\mu T\partial^\mu T} + \sqrt{\pi \partial_\mu T\partial^\mu T}\mathrm{erf}\left(\sqrt{\partial_\mu T\partial^\mu T}\right)\right] \ . \tag{11}$$

We expect that this form for the effective action is valid near the kink solutions.

Let us next verify if our tachyon action is compatible with the expected properties from string theory. It is clear from its construction that the tachyon mass and the marginal deformations are reproduced correctly. In addition we can consider the relevant linear tachyon profile, $T = ux$. Under the renormalisation group flow this profile flows to $u = \infty$. The resulting IR fixed point worldsheet theory should then describe a BPS D$(p-1)$-brane [12]. For finite u this profile cannot be a solution of (11). However, if we scale $u \to \infty$, any function of the form $T(x) = u\tilde{T}(x)$ will be a solution to (11), representing the renormalization group flow on the worldsheet. Provided that $\tilde{T}(x) = 0$ only on discrete set of points, the energy of such a configuration is then given by

$$\begin{aligned} E &= \sqrt{2}\tau_p \int_{-\infty}^{+\infty} e^{-\frac{\kappa u^2}{2\alpha'}\tilde{T}^2} \left(e^{-\kappa u^2(\tilde{T}')^2} + \sqrt{\pi\kappa}u|\tilde{T}'|\mathrm{erf}(\sqrt{\kappa}u|\tilde{T}'|)\right) dx \\ &= \sqrt{2\pi\kappa}u\tau_p \int dx\,|\tilde{T}'|e^{-\frac{\kappa u^2}{2\alpha'}\tilde{T}^2} \\ &= 2\pi N\sqrt{\alpha'}\tau_p \ , \end{aligned} \tag{12}$$

where in the second line we took the limit $u \to \infty$. Here N is the number of times $T(x)$ covers the real line, which in the $u \to \infty$ limit is the number of times $\tilde{T}$ changes sign. This is the correct value to interpret the kink as N $D(p-1)$-branes.[1]

Let us now consider the low energy dynamics on multi-kink $T = u\prod_{i=1}^{N}(x-a_i)$ by letting the zero modes a_i depend on the other coordinates x^μ, $\mu = 0,...,p-1$ of the non-BPS Dp-brane. Substituting in to the action (11) we find, taking the limit $u \to \infty$,

$$\mathcal{L}_{Kink} = \sqrt{2\kappa\pi}\tau_p u \int dx\,|\tilde{T}'|\,e^{-\frac{\kappa u^2}{2\alpha'}\tilde{T}^2}\sqrt{1 + \frac{\partial_\mu\tilde{T}\partial^\mu\tilde{T}}{(\tilde{T}')^2}} \ . \tag{13}$$

We assume that all the a_i are distinct and consider the large u limit, the integral (13) reduces to

$$\mathcal{L}_{Kink} = 2\pi\sqrt{\alpha'}\tau_p\sum_{i=1}^{N}\sqrt{1 + \partial_\mu a_i\partial^\mu a_i} \ . \tag{14}$$

Thus we reproduce the Born-Infeld action (1) for the massless scalar fields on N distinct BPS $D(p-1)$-branes.

Finally we want to discuss perturbative excitations about the true vacuum $T = T_0$. For this we introduce the new variable

$$d\varphi = \sqrt{V(T)}dT \ . \tag{15}$$

The effective action now takes the form

$$\mathcal{L} = -V(\varphi)K(\frac{\partial_\mu\varphi\partial^\mu\varphi}{V}) \ , \tag{16}$$

[1] Note that a closer analysis shows that the resulting configuration is that of parallel branes and anti-branes. The difference, however, will not affect our dicussion here.

for some $K(y) = 1 + \kappa_1 y + \ldots$. Thus for $(\partial\varphi)^2 << V$ (*i.e.* $(\partial T)^2 << 1$),

$$\mathcal{L} \simeq -\kappa_1 \partial_\mu \varphi \partial^\mu \varphi - V(\varphi) \ , \tag{17}$$

with

$$V(\varphi) \propto \varphi^2 \log\left(\frac{\varphi}{\sqrt{\alpha'}}\right) \ . \tag{18}$$

Thus we see that the second derivative of the potential diverges at $T = T_0$ implying that perturbative excitation are indeed absent in the true vacuum. This is necessary in order to identify the this vacuum with the closed string vacuum.

It should be noted that these results are also reproduced by a rather large class of effective actions [15] including the actions (6) (assuming the same potential) and (8). In addition a numerical comparison between the BI-type action (6), the BSFT-action (8) and (11) shows that they are almost identical on space-like, linear profiles [14]. Thus, although these actions all differ and represent different trucations of the true effective action, they share remarkably similar properties for static tachyons.

3 Tachyon Dynamics

Let us now consider the case of dynamical tachyon solutions. A suitable class of time dependent tachyon profiles is obtained by a Wick rotation of (3)

$$T(x^0) = A\sinh(\frac{x^0}{\sqrt{2\alpha'}}) + B\cosh(\frac{x^0}{\sqrt{2\alpha'}}) \ , \tag{19}$$

and it is easy to see that these are exact solutions of the equation of motion of (11). Let us now determine the energy and presure of this solution. We have

$$E = -T_{00} = \tilde{\tau}_p e^{-\frac{T^2}{2\alpha'} + \dot{T}^2} \ . \tag{20}$$

Conservation of energy then implies that (19) is the only regular solution of the equation of motion. In particular, as the tachyon rolls to the minimum $\dot{T}$ diverges in agreement with the conformal field theory approach [5, 7].

We also find

$$T_{ij} = \delta_{ij} \tilde{\tau}_p e^{-\frac{T^2}{2\alpha'}} \left(e^{\dot{T}^2} + i\sqrt{\pi \dot{T}^2} \mathrm{erf}\left(i\sqrt{\dot{T}^2} \right) \right) \ . \tag{21}$$

Now, for large $\dot{T}$,

$$T_{ij} \simeq \frac{T_{00}}{2\dot{T}^2}\delta_{ij} \quad \text{for} \quad \dot{T} \to \infty \ . \tag{22}$$

Thus, the action (11) predicts that at large times the tachyon condensation produces a gas with non-vanishing energy and vanishing pressure. In particular for large x^0, where $T \simeq \chi e^{x^0/\sqrt{2\alpha'}}/2$,

$$p \simeq \frac{2E}{\chi^2} e^{-\sqrt{\frac{2}{\alpha'}}x^0} \ . \tag{23}$$

This property is interesting in view of possible cosmological applications as dark matter candidates. Note also that this exponential fall-off agrees exactly with the string theory result from the boundary state [7]. On the other hand, in the boundary state approach, the pressure is always negative, whereas here we find that the pressure approaches zero from above.

We can contrast these predictions with the BI-type and BSFT effective actions. Note that in these cases the profile will no longer have the form (19). In particular $\dot{T}$ approaches a constant [7, 16, 17]. For the BI-type action with the potential (8) one finds that the pressure depends exponentally on the *square* of x^0 [16]. However this can be cured by choosing a potential $V = e^{-\sqrt{1+T^2/2\alpha'}}$ (see also [18]), but then one no longer finds that the effective action reproduces the correct properties of static tachyon profiles. The same $(x^0)^2$ dependence of the pressure was also observed in [16, 17] for the BSFT effective action. Thus the effective actions discussed above differ in their behaviour of time dependent tachyons. While none of these actions produce results which are in exact agreement with the conformal field theory calculations, in our opinion the effective action (11) most faithfully reproduces the correct physical predictions.

Acknowledgements

I.S. would like to thank the organisers of QUARKS-2002 for a very interesting and stimulating conference.

References

[1] J. Polchinski, Phys. Rev. Lett. **75** (1995) 4724 [arXiv:hep-th/9510017]

[2] A. Sen, JHEP **9809**, 023 (1998) [arXiv:hep-th/9808141].

[3] A.Sen, JHEP **9808** (1998) 010 [arXiv:hep-th/9805019]; JHEP **9812** (1998) 021 [arXiv:hep-th/9812031]; [arXiv:hep-th/9904207].

[4] A. Recknagel and V. Schomerus, Nucl. Phys. B **545** (1999) 233 [arXiv:hep-th/9811237]; A. Sen, Int. J. Mod. Phys. A **14** (1999) 4061 [arXiv:hep-th/9902105]; [arXiv:hep-th/9811237].

[5] A. Sen, JHEP **0204** (2002) 048 [arXiv:hep-th/0203211].

[6] M. Gutperle and A. Strominger, JHEP **0204** (2002) 018 [arXiv:hep-th/0202210].

[7] A. Sen, [arXiv:hep-th/0203265].

[8] N. D. Lambert and I. Sachs, JHEP **0008** (2000) 024 [arXiv:hep-th/0006122].

[9] N. D. Lambert and I. Sachs, JHEP **0003** (2000) 028 [arXiv:hep-th/0002061].

[10] N. D. Lambert and I. Sachs, JHEP **0102** (2001) 018 [arXiv:hep-th/0010045].

[11] M. R. Garousi, Nucl. Phys. B **584** (2000) 284 [arXiv:hep-th/0003122].

[12] D. Kutasov, M. Marino and G. W. Moore, [arXiv:hep-th/0010108].

[13] M. R. Garousi, [arXiv:hep-th/0209068].

[14] N. D. Lambert and I. Sachs, JHEP **0106** (2001) 060 [arXiv:hep-th/0104218].

[15] N. D. Lambert and I. Sachs, [arXiv:hep-th/0208217].

[16] S. Sugimoto and S. Terashima, JHEP **0207** (2002) 025 [arXiv:hep-th/0205085].

[17] J. A. Minahan, JHEP **0207** (2002) 030 [arXiv:hep-th/0205098].

[18] A. Sen, [arXiv:hep-th/0204143].