Solutions of Vacuum Superstring Field Theory

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Abstract

In this report we review[1] a structure of cubic Vacuum Superstring Field Theory and known solutions to its equation of motion.

1 Introduction

During the last two years the bosonic vacuum string field theory (VSFT) proposed to describe physics around the bosonic tachyon vacuum [4] has been investigated in many papers [5]-[11]. VSFT action has the same form as the original Witten SFT action [12], but with a new differential operator $\mathcal{Q}$ (for a review of SFT see [13, 14, 15] and references therein). The absence of physical open string excitations around the tachyon vacuum [16, 17] supports a suggestion [4] that after some field redefinition $\mathcal{Q}$ can be written as a pure ghost operator. Under this assumption solutions to VSFT equation of motion admit a factorized form with the projector-like matter part.

A generalization of VSFT to superstrings has been discussed in [4] and more recently in [1]-[3],[11] and [18] in the context of cubic SSFT [19, 20] and non-polynomial SSFT [21], respectively. Open fermionic string in the NSR formalizm has a tachyon in the GSO$-$ sector that leads to a classical instability of the perturbative vacuum in the theory without supersymmetry. It has been proposed [16] to interpret the tachyon condensation in the GSO$-$ sector of the NS string as a decay of unstable non-BPS D9-brane.

In this note we consider a construction of cubic Vacuum Superstring field Theory and solution to its equation of motion. Actually, this means a construction of a new BRST charge while the structure of the action will be the same. This question is considered in Section 2. In Section 3 we consider solution to the matter part of the fermionic sector of the NS string, the NS sliver. In Section 4 NS ghost sliver is considered.

2 Cubic Vacuum String Field Theory on a non-BPS D-brane

To describe the open string states living on a single non-BPS D-brane one has to consider GSO$\pm$ states [16]. GSO$-$ states are Grassmann even, while GSO$+$ states are Grassmann odd. The unique (up to rescaling of the fields) gauge

[1]This report is based on the papers [1]-[3].

invariant cubic action unifying GSO+ and GSO− sectors is [22]

$$
\begin{aligned}
S[A_+,A_-]=\frac{1}{g_o^2}\Bigg[&\frac{1}{2}\langle\langle Y_{-2}|A_+,Q_BA_+\rangle\rangle+\frac{1}{3}\langle\langle Y_{-2}|A_+,A_+,A_+\rangle\rangle\\
&+\frac{1}{2}\langle\langle Y_{-2}|A_-,Q_BA_-\rangle\rangle-\langle\langle Y_{-2}|A_+,A_-,A_-\rangle\rangle\Bigg]. \qquad (1)
\end{aligned}
$$

Here the factors before the odd brackets are fixed by the constraint of gauge invariance, that is specified below, and reality of the string fields $A_\pm$. Variation of this action with respect to A_+, A_- yields the following equations of motion (see [22] for details)

$$
Q_BA_\pm+A_+\star A_\pm-A_-\star A_\mp=0 \qquad (2)
$$

The action (1) is invariant under the gauge transformations

$$
\delta A_\pm=Q_B\Lambda_\pm+[A_\pm,\Lambda_+]+\{A_\mp,\Lambda_-\}
$$

where $[\,,]$ ($\{\,,\}$) denotes $\star$-(anti)commutator and $\Lambda_\pm$ are gauge parameters.

The action (1) can be rewritten in the matrix form as

$$
S[\hat{A}]=\frac{1}{2g_o^2}\,\mathrm{Tr}\left[\frac{1}{2}\langle\langle Y_{-2}|\hat{A},\hat{Q}_B\hat{A}\rangle\rangle+\frac{1}{3}\langle\langle Y_{-2}|\hat{A},\hat{A},\hat{A}\rangle\rangle\right], \qquad (3)
$$

$\hat{Q}_B=Q_B\otimes a$, $\hat{Y}_{-2}=Y_{-2}\otimes a$, $\hat{A}=A_+\otimes a+A_-\otimes b$ and a and b are 2×2 matrices such that $a^2=1$, $b^2=-1$ and $\{a,b\}=0$.

The action (3) is invariant under GSO symmetry transformation given by $\hat{A}\mapsto((-1)^F\otimes1)\hat{A}$, and twist symmetry transformation Ω which action on the string field is given via conformal transformation $M(z)=e^{-\pi i}z$. One can check that the BRST charge $\hat{Q}_B$ commutes with $(-1)^F\otimes1$ and Ω.

Let $\hat{A}_0$ be a solution to the equations (2). A shift of a string field $\hat{A}=\hat{A}_0+\hat{\mathcal{A}}$ yields the following form of the action (3)

$$
S[\hat{A}_0,\hat{\mathcal{A}}]=S[\hat{A}_0]+\frac{1}{2g_o^2}\,\mathrm{Tr}\left[\frac{1}{2}\langle\langle Y_{-2}|\hat{\mathcal{A}},\hat{Q}\hat{\mathcal{A}}\rangle\rangle+\frac{1}{3}\langle\langle Y_{-2}|\hat{\mathcal{A}},\hat{\mathcal{A}},\hat{\mathcal{A}}\rangle\rangle\right], \qquad (4)
$$

where $\hat{Q}$ is “a new BRST charge” of the form

$$
\hat{Q}=\hat{Q}_B+\{\hat{A}_0,\cdot\}. \qquad (5)
$$

Further we will refer to $\hat{Q}$ as a kinetic operator. One can check that the equation $\hat{Q}^2=0$ yields the equation of motion for the field $\hat{A}_0$ and therefore $\hat{Q}$ is nilpotent.

The kinetic operator can be written in the form

$$
\hat{Q}=Q_{\mathsf{odd}}\otimes a+Q_{\mathsf{even}}\otimes b. \qquad (6)
$$

The nilpotency of the $\hat{Q}$ yields the following identities for the operators Q_{odd} and Q_{even}

$$
Q_{\mathsf{odd}}^2-Q_{\mathsf{even}}^2=0 \quad \text{and} \quad [Q_{\mathsf{odd}},Q_{\mathsf{even}}]=0. \qquad (7)
$$

Equations of motion following from the VSFT action (4) have the same form as for the action (3) but with the shifted BRST operator $\hat{Q}$. In components these equations are

$$Q_{\mathsf{odd}}\mathcal{A}_{\pm} - Q_{\mathsf{even}}\mathcal{A}_{\mp} + \mathcal{A}_{+}\star\mathcal{A}_{\pm} - \mathcal{A}_{-}\star\mathcal{A}_{\mp} = 0. \tag{8}$$

According to Sen conjectures [16] the solution $\hat{A}_0$ represents the vacuum without open string excitations[2], and therefore the cohomology of the kinetic operator $\hat{Q}$ must be zero.

In proposing a simple form of the vacuum SSFT action, we have in mind field redefinition, which preserves the form of the cubic action, but simplifies the expression for the kinetic operator $\hat{Q}$. By an appropriate field redefinition

$$\hat{\mathcal{U}} = \mathcal{U}_{\mathsf{even}}\otimes 1 + \mathcal{U}_{\mathsf{odd}}\otimes ab \tag{9a}$$

we will assume a $\star$-algebra homomorphism $\hat{\mathcal{U}}(\hat{\mathcal{A}}\star\hat{\mathcal{B}}) = (\hat{\mathcal{U}}\hat{\mathcal{A}})\star(\hat{\mathcal{U}}\hat{\mathcal{B}})$, which satisfies two additional conditions:

$$\mathrm{Tr}\int'\hat{\mathcal{U}}\hat{\mathcal{A}} = \mathrm{Tr}\int'\hat{\mathcal{A}} \text{ and an existance of } \hat{\mathcal{U}}^{-1}: \hat{\mathcal{U}}\hat{\mathcal{U}}^{-1} = 1. \tag{9b}$$

The $\hat{}$ in the expressions for the field redefinition $\hat{\mathcal{U}}$ is very important since this transformation acts in both GSO+ and GSO$-$ sectors. Using (9) one can check that after the field redefinition $\hat{\mathcal{A}}\mapsto\hat{\mathcal{U}}\hat{\mathcal{A}}$ the kinetic operator transforms into $\hat{\mathcal{Q}} = \hat{\mathcal{U}}^{-1}\hat{Q}\hat{\mathcal{U}}$. Note that the transformation $\hat{\mathcal{U}}$ is highly non-trivial and mixes GSO+ and GSO$-$ sectors.

Consider the standard BRST charge in the superconformal field theory

$$Q_B = \frac{1}{2\pi i}\oint d\zeta\Big[c(T_B + T_\phi + T_{\eta\xi} + \frac{1}{2}T_{bc}) - \eta e^{\phi}T_F + \frac{1}{4}b\partial\eta\eta e^{2\phi}\Big]. \tag{10}$$

One can check that after the homogenous field redefinition [23]

$$\mathcal{U} = e^{-R}, \quad \text{where} \quad R = \frac{1}{2\pi i}\oint d\zeta\Big[cT_F e^{-\phi}e^{\chi} + \frac{1}{4}\partial(e^{-2\phi})e^{2\chi}c\partial c\Big] \tag{11}$$

the BRST charge (10) takes the form

$$\mathcal{Q} = \mathcal{U}^{-1}Q_B\mathcal{U} = \frac{1}{2\pi i}\oint d\zeta\, b\gamma^2(\zeta). \tag{12}$$

Following the idea of the paper [4], which is based on Sen conjectures, gauge invariance and algebraic properties of the BRST charge, we require $\hat{\mathcal{Q}}$ to satisfy the following properties:

1. $\hat{\mathcal{Q}} = \mathcal{Q}_{\mathsf{odd}}\otimes a + \mathcal{Q}_{\mathsf{even}}\otimes b$;

[2] This conjecture has been checked for the non-BPS brane decay only at the first non-trivial level [15].

2. Both $\mathcal{Q}_{\mathsf{odd}}$ and $\mathcal{Q}_{\mathsf{even}}$ have superghost number equal to one, but $\mathcal{Q}_{\mathsf{odd}}$ is Grassmann odd, while $\mathcal{Q}_{\mathsf{even}}$ is Grassmann even;
3. $\hat{\mathcal{Q}}$ is a nilpotent operator, that in components means the identities
$$\mathcal{Q}_{\mathsf{odd}}^2 - \mathcal{Q}_{\mathsf{even}}^2 = 0 \quad \text{and} \quad [\mathcal{Q}_{\mathsf{odd}}, \mathcal{Q}_{\mathsf{even}}] = 0;$$
4. $\hat{\mathcal{Q}}(\hat{A}\star\hat{B}) = (\hat{\mathcal{Q}}\hat{A})\star\hat{B} + (-1)^{|\hat{A}|}\hat{A}\star(\hat{\mathcal{Q}}\hat{B})$, In particular, this identity means that operators $\mathcal{Q}_{\mathsf{odd}}$ and $\mathcal{Q}_{\mathsf{even}}$ also satisfy the Leibnitz rule;
5. $\mathrm{Tr}\int' \hat{\mathcal{Q}}(\hat{\mathcal{A}}\star\hat{\mathcal{B}}) = 0$;
6. The operator $\hat{\mathcal{Q}}$ must be universal, what means that it has to be written without reference to the brane boundary CFT;
7. The operator $\hat{\mathcal{Q}}$ must have vanishing cohomology;
8. $[\hat{Y}_{-2},\, \hat{\mathcal{Q}}] = 0$ or $\{\hat{Y}_{-2},\, \hat{\mathcal{Q}}\} = 0$. We need this axiom to relate the axiom 5 with the fact that $\hat{\mathcal{Q}}$ annihilates the identity $|\mathcal{I}\rangle$. Therefore we can have several variations of this axiom and in general we only need something like the following
$$\mathcal{Q}_{\mathsf{odd}}Y_{-2} \pm Y_{-2}\mathcal{Q}_{\mathsf{odd}} = 0 \quad \text{and} \quad \mathcal{Q}_{\mathsf{even}}Y_{-2} \pm Y_{-2}\mathcal{Q}_{\mathsf{even}} = 0;$$
Plus/minus in these formulae can be chosen independently.
9. $\hat{\mathcal{Q}}$ is a hermitian operator, which means that both Q_{odd} and Q_{even} are hermitian ones.

Since $A_{0,+} \neq 0$ and $A_{0,-} \neq 0$ we believe that after the field redefinition both charges $\mathcal{Q}_{\mathsf{odd}}$ and $\mathcal{Q}_{\mathsf{even}}$ are non zero.

The following ghost kinetic operator satisfies all above axioms [1, 11]

$$\mathcal{Q}_{\mathsf{odd}} = \frac{\mu^2}{4i}\big[c(i) - c(-i)\big] + \frac{1}{2\pi i}\oint b(z)\gamma^2(z)dz, \tag{13a}$$

$$\mathcal{Q}_{\mathsf{even}}^{+} = \frac{\mu}{2i}\big[\gamma(i) - \gamma(-i)\big],\ \mathcal{Q}_{\mathsf{even}}^{-} = \frac{\mu}{2}\big[\gamma(i) + \gamma(-i)\big], \tag{13b}$$

where $\mathcal{Q}_{\mathsf{even}}^{\pm}$ means the restriction of the operator $\mathcal{Q}_{\mathsf{even}}$ to GSO$\pm$ sectors and μ is a complex number.

In some sense (13) is the only form for the kinetic operator which satisfies the twist invariance and the above conditions. One can explain it as follows. Following [9] consider an original (before field redefinition) BRST charge Q defined as

$$Q = \sum_r \frac{1}{2\pi i}\oint d\zeta\, a_r(\zeta)\mathcal{O}_r(\zeta) \tag{14}$$

where a_r are smooth forms of ζ and $\mathcal{O}_r(\zeta)$ are some local conformal operators of ghost number 1. It was shown in [9] that after a singular field redefinition the dominant contribution to the transformed charge $\mathcal{Q}$ will come from the lowest dimensional conformal operators. This has led to the choice of $c(i)$ and $c(-i)$ in the bosonic case, and this also leads to our choice of $\mathcal{Q}_{\mathsf{even}}$, since γ is the lowest dimensional even primary operator of ghost number 1.

3 NS Matter Sliver

While after the field redefinition the kinetic operator of VSFT has a pure ghost form it is natural to search for solutions to VSFT equation of motion in the factorized form $\Phi = \Xi_{matter} \otimes \Phi_{ghost}$, where Ξ_{matter} satisfies a projector-like equation:

$$\Xi_{matter} = \Xi_{matter} \star \ \Xi_{matter}. \tag{15}$$

An equation similar to (15) has appeared in a construction of solitonic solutions in noncommutative field theories in the large non-commutativity limit [24].

A way to solve projection equation (15) for the bosonic matter has been proposed by Rastelli and Zwiebach [5]. They have constructed a solution to (15) as the $n \to \infty$ limit of the wedge states $|n\rangle$. The wedge states are defined on CFT language and they satisfy the algebra

$$|n\rangle \star |m\rangle = |n + m - 1\rangle. \tag{16}$$

From algebra (16) it immediately follows that $|\infty\rangle$, the so-called sliver state, satisfies (15).

Now we are going to construct the fermionic sliver state using CFT methods. We refer reader to [2] in order to find the algebraic construction of the fermionic sliver state. We have to note that numeric calculations show a conspicous agreement between algebraic and CFT methods [2].

A generalization of the bosonic wedge states [5, 6] to the fermionic wedge states is straightforward. Wedge states $|n\rangle$ are defined by

$$\langle n|\phi^{\psi}\rangle = \langle f_n \circ \phi^{\psi}(0)\rangle, \tag{17}$$

where $|\phi^{\psi}\rangle$ is an arbitrary state which belongs to the fermionic subspace, $f_n \circ \phi^{\psi}(\xi)$ denotes the conformal transform of $\phi^{\psi}(\xi)$ and $f_n(\xi)$ is the same as in the bosonic case, i.e.

$$f_n(\xi) = \frac{n}{2} \tan\left(\frac{2}{n} \tan^{-1} \xi\right). \tag{18}$$

The wedge state with $n = 1$ corresponds to the identity of the star algebra and with $n = 2$ corresponds to the vacuum.

Taking the limit $n \to \infty$ in (18) one derives the conformal map for the sliver state $|\infty\rangle$

$$w(\xi) = \tan^{-1}(\xi). \tag{19}$$

For a state $|\Lambda\rangle \propto \exp(1/2\psi_r^{\dagger}\Lambda_{rs}\psi_s^{\dagger})|0\rangle$, corresponding to a conformal map $\lambda(\xi)$ one gets

$$\Lambda_{rs} = \oint \frac{d\xi}{2\pi\imath} \oint \frac{d\xi'}{2\pi\imath} \xi^{-r-\frac{1}{2}} \xi'^{-s-\frac{1}{2}} \left(\frac{\partial\lambda(\xi)}{\partial\xi}\right)^{\frac{1}{2}} \frac{1}{\lambda(\xi) - \lambda(\xi')} \left(\frac{\partial\lambda(\xi')}{\partial\xi'}\right)^{\frac{1}{2}}. \tag{20}$$

Here $\oint$ denotes the contour integration around the origin. Substituting the sliver conformal map (19) one gets that the conformal sliver $|\tilde{\Xi}^{\psi}\rangle \equiv |\infty\rangle$ is defined as

$$|\tilde{\Xi}^{\psi}\rangle = \tilde{\mathcal{N}}^{10} \exp(\frac{1}{2}\psi_r^{\dagger}\tilde{S}_{rs}\psi_s^{\dagger})|0\rangle, \tag{21}$$

$$\tilde{S}_{rs} = \oint \frac{d\xi}{2\pi\imath}\frac{d\xi'}{2\pi\imath}\xi^{-r-\frac{1}{2}}\xi'^{-s-\frac{1}{2}}\frac{2\imath}{\sqrt{1+\xi^2}\sqrt{1+\xi'^2}}\ln\left(\frac{(1+i\xi)(1-i\xi')}{(1-i\xi)(1+i\xi')}\right). \quad (22)$$

The matrix $\tilde{S}_{rs}$ can be calculated explicitly. Only coefficients with $r+s=$ even differ from zero.

4 NS ghost sliver

Ghost part of VSFT equations of motion has been studied in [8, 9]. It was observed that a sliver constructed in the twisted conformal theory with new $SL(2,\mathbb{R})$ invariant vacuum solves the ghost part of VSFT equation of motion. This equation is a usual SFT equation of motion with a canonical choice of ghost kinetic term that is a local insertion at the string midpoint.

We present here the twisted superghost conformal theory and derive corresponding equations in analogy with the one constructed by Gaiotto, Rastelli, Sen and Zwiebach [9]. We refer the reader to [3] where algebraic construction of the NS ghost sliver can be found.

A twisted CFT is introduced by subtracting from the stress energy tensor $T(w)$ of the (β,γ) system the derivative of $U(1)$ ghost number current j as follows

$$T'(w) = T(w) - \partial j(w), \quad \bar{T}'(\bar{w}) = \bar{T}(\bar{w}) - \partial\bar{j}(\bar{w}), \quad j = -\beta\gamma. \quad (23)$$

More explicitly for the holomorphic stress energy tensor one obtains

$$T(w) = -\frac{3}{2}\beta\partial\gamma(w) - \frac{1}{2}\partial\beta\gamma(w), \quad \text{with } c = 11, \quad (24)$$

$$T'(w) = -\frac{1}{2}\beta'\partial\gamma'(w) + \frac{1}{2}\partial\beta'\gamma'(w), \quad \text{with } c = -1, \quad (25)$$

where (β',γ') denotes the superghosts of the twisted CFT and c is the central charge. The weights of these new β' and γ' become equal to 1/2 and the superghost current $j' = -\beta'\gamma'$ has no anomaly. Fermionic ghosts in the original theory are bosonised as

$$\gamma(w) = \eta e^{\phi}(w), \quad \beta(w) = e^{-\phi}\partial\xi(w), \quad (26)$$

so that the ghost number current is expressed in the form $j = -\partial\phi$. The Euclidean world-sheet actions S and S' for the fields ϕ and ϕ' correspondingly are related as

$$S[\phi] = S'[\phi] - \frac{1}{2\pi}\int_\Sigma d^2\zeta\,\sqrt{g}R^{(2)}(\phi+\bar{\phi}), \quad (27)$$

where ζ denotes the world-sheet coordinates, g denotes the Euclidean world-sheet metric and $R^{(2)}$ is the scalar curvature.

We assume that scalar curvature is proportional to δ-function, which has a support on the infinity in some coordinates on Σ. Therefore we can identify the fields ϕ and ϕ' of two CFTs. The states in the two theories can be identified by the following map between the oscillators and the vacuum states

$$\beta_n \leftrightarrow \beta'_n, \quad \gamma_n \leftrightarrow \gamma'_n, \quad |-1\rangle \leftrightarrow |0'\rangle, \quad \langle -1| \leftrightarrow \langle 0'|, \quad \langle 0'|0'\rangle = 1, \tag{28}$$

where $|0\rangle$ and $|0'\rangle$ are the $SL(2,\mathbb{R})$ invariant vacua of two theories and $|-1\rangle = e^{-\phi(0)}|0\rangle$.

In the CFT$'$ the fields β', γ' are bosonized as in the original theory

$$\gamma'(w) = \eta e^{\phi}(w), \quad \beta'(w) = e^{-\phi}\partial\xi(w). \tag{29}$$

Notice that we do not introduce new notations for the (ξ, η) system because it has not changed.

The advantage of the CFT method in comparison with the operator method, that we have used in Section 2, is that we do not have to postulate the sliver equation from the very beginning. The aim of this section is to define a sliver state as a surface state over $SL(2,\mathbb{R})$ invariant vacuum in CFT and CFT$'$, correspondingly, by the conformal map used in the matter case.

First we define the surface state for the original (β, γ) system. The fermionic ghost surface state corresponding to the conformal map $\lambda(\xi)$ is defined as

$$\langle \Lambda| = \mathcal{N}_{\beta\gamma}\langle 0| \exp(-\sum_{\substack{r\geq 3/2\\ s\geq -1/2}} \gamma_r \Lambda_{rs}\beta_s), \tag{30}$$

where $\mathcal{N}_{\beta\gamma}$ is a normalization factor and the matrix Λ_{rs} is defined so that the following identity holds

$$\langle 0| e^{-\sum\limits_{\substack{r\geq 3/2\\ s\geq -1/2}} \gamma_r\Lambda_{rs}\beta_s} \gamma(w)\beta(z)e^{-Q\phi(0)}|0\rangle = \langle \lambda \circ \gamma(w) \lambda \circ \beta(z) \lambda \circ e^{-Q\phi(0)}\rangle. \tag{31}$$

One can evaluate Λ_{rs} explicitly. To this end one has to calculate the left hand side and right hand side of (31). Substitution of $\gamma(w) = \sum_r \gamma_{-r} w^{r+1/2}$ and $\beta(z) = \sum_s \beta_{-s} z^{s-3/2}$ into the left hand side of (31) yields

$$h(z,w) \equiv \langle 0| e^{-\gamma_r\Lambda_{rs}\beta_s}\gamma(w)\beta(z)e^{-Q\phi(0)}|0\rangle = -\sum_{r,s} w^{r+1/2} z^{s-3/2}\Lambda_{rs}, \tag{32}$$

therefore

$$\Lambda_{rs} = -\oint \frac{dz}{2\pi i}\frac{1}{z^{r-1/2}} \oint \frac{dw}{2\pi i}\frac{1}{w^{s+3/2}} h(z,w). \tag{33}$$

Further one evaluates the correlation function in the right hand side of (31)

$$\begin{aligned}\langle \lambda \circ \gamma(w) \lambda \circ \beta(z) \lambda \circ e^{-Q\phi(0)}\rangle \\ = \left(\frac{\partial\lambda(w)}{\partial w}\right)^{-1/2} \left(\frac{\partial\lambda(z)}{\partial z}\right)^{3/2} \frac{1}{\lambda(w)-\lambda(z)} \left(\frac{\lambda(w)-\lambda(0)}{\lambda(z)-\lambda(0)}\right)^{-Q}.\end{aligned} \tag{34}$$

One gets the following answer for Λ_{rs}

$$\oint \frac{dz}{2\pi i}\frac{z^{\frac{1}{2}}}{z^r}\frac{dw}{2\pi i}\frac{w^{-\frac{3}{2}}}{w^s}\left(\frac{\partial\lambda(w)}{\partial w}\right)^{-\frac{1}{2}}\left(\frac{\partial\lambda(z)}{\partial z}\right)^{\frac{3}{2}}\frac{1}{\lambda(z)-\lambda(w)}\left(\frac{\lambda(z)-\lambda(0)}{\lambda(w)-\lambda(0)}\right)^2. \quad (35)$$

The fermionic ghost surface state in CFT' corresponding to the conformal map $\lambda(\xi)$ is defined as

$$\langle\Lambda'| = \mathcal{N'}_{\beta\gamma}\langle 0'|\exp(-\sum_{\substack{r\geq 1/2\\ s\geq 1/2}}\gamma_r\Lambda'_{rs}\beta_s), \quad (36)$$

where $\mathcal{N'}_{\beta\gamma}$ is a normalization factor and the matrix Λ'_{rs} is defined so that the following identity holds

$$\langle 0'|\exp(-\sum_{\substack{r\geq 1/2\\ s\geq 1/2}}\gamma_r\Lambda'_{rs}\beta_s)\gamma'(w)\beta'(z)|0'\rangle = \langle\lambda\circ\gamma'(w)\lambda\circ\beta'(z)\rangle'. \quad (37)$$

Substitution of $\gamma'(w)=\sum_r\gamma_{-r}w^{r-1/2}$ and $\beta'(z)=\sum_s\beta_{-s}z^{s-1/2}$ into the left hand side of (37) yields

$$h'(z,w)\equiv\langle 0'|e^{-\gamma_r\Lambda'_{rs}\beta_s}\gamma'(w)\beta'(z)|0'\rangle = -\sum_{r,s}w^{r-1/2}z^{s-1/2}\Lambda'_{rs}, \quad (38)$$

therefore

$$\Lambda'_{rs} = -\oint\frac{dz}{2\pi i}\frac{1}{z^{r+1/2}}\oint\frac{dw}{2\pi i}\frac{1}{w^{s+1/2}}h'(z,w). \quad (39)$$

Evaluating the correlation function in the right hand side of (37) one finds

$$\langle\lambda\ \circ\ \gamma'(w)\lambda\ \circ\ \beta'(z)\rangle' = \left(\frac{\partial\lambda(w)}{\partial w}\right)^{1/2}\left(\frac{\partial\lambda(z)}{\partial z}\right)^{1/2}\frac{1}{\lambda(w)-\lambda(z)}. \quad (40)$$

One gets the following answer

$$\Lambda'_{rs} = \oint\frac{dz}{2\pi i}\frac{1}{z^{r+1/2}}\frac{dw}{2\pi i}\frac{1}{w^{s+1/2}}\left(\frac{\partial\lambda(z)}{\partial z}\right)^{1/2}\left(\frac{\partial\lambda(w)}{\partial w}\right)^{1/2}\frac{1}{\lambda(z)-\lambda(w)}. \quad (41)$$

It should be mentioned here that the matrix (41) coincides with the matrix of the matter sliver.

Acknowledgments

The author would like to thank I.Ya. Arefeva, D.M. Belov, A.A. Giryavets and P.B. Medvedev for many valuable discussions and comments. This work was supported in part by RFBR grant 02-01-00695, RFBR grant for leading scientific schools and by INTAS grant 99-0590.

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