Quantum gravity as a consistent local field theory

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Abstract

We show that the Einstein-Hilbert action for the gravitational field can be obtained as a linear low-energy approximation for the dynamical massless fields in the theory with the lagrangian quadratic in the gauge field strength-tensor of spin connection under the spontaneous breaking of symmetry with a vacuum state described by an ansatz for nontrivial background strength-tension.

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1 Introduction

There is the only physical argument for the development of string, superstring and M-theories with extended objects in spaces with extra dimensions as well as some other popular topics of mathematical physics [1]. The motivation is the problem on a consistent local quantum field theory of gravitational interactions, since, under the quantization, the classic Einstein–Hilbert action for the gravitational field leads to the nonrenormalizable theory. The nonrenormalizability implies that the quantum loop corrections cannot be calculated in terms of finite number of physical parameters determining the coupling constant of interaction and the normalization of fields. Instead of that, some infinite set of constants appears in the series of operators with growing powers of energy. In this way, a small contribution of such corrections as observed empirically, can be explained by the fact that the standard gravitational interaction is the particular contribution giving the valid description in the restricted region of energy, only. This contribution is originated from a consistent full theory, so that the matching of full theory parameters with the couplings of higher operators results in the suppression of higher operators. The physical reason for the nonrenormalizability is caused by the dimension of gravitational constant determining the interaction of dynamical metric field with the energy-momentum tensor. The presence of fundamental dimensional parameter can be naturally introduced[1] in the theories with the extended objects like the strings, membranes and so on, so that the consistent quantization of such the objects[2] can be done only in the spaces of higher dimensions. In this way, the principles of renormalizability and gauge invariance lose their fundamental role in the construction of action and become auxiliary in the reduction of the theory to the observed four-dimensional world. Thus, the quantum gravity makes the question on the principal consistency and completeness of local field theory.

In this paper we develop the idea by Utiyama on the gauge invariance of gravitational interactions [2] and show how the action quadratic in the strength tensor is reduced to the standard Einstein–Hilbert action with the massless gravitons in the low-energy limit under the spontaneous breaking of symmetry by introducing an ansatz for the vacuum state. As was demonstrated by Stelle [3], the introduction of terms quadratic in the Riemannian tensor of curvature provides the renormalizability of quantum theory, that is rather natural for the general nonabelian gauge theory. Therefore, we get the real possibility to solve the problem on the formulation of quantum gravity in the framework of local field theory.

2 Spontaneous breaking of symmetry

The starting point of consideration is the action of spin-connection gauge-field $\mathcal{A}$ interacting with the spinor ,

$$\mathbf{S} = \int \det[\mathbf{h}]\, \mathbf{d}^4\mathbf{x} \quad \frac{1}{4\mathbf{g}_{\rm Pl}^2}\, \mathcal{F}_{\mu\nu,mn}\mathcal{F}^{\mu\nu,mn} +$$

[1]In the renormalizable gauge theories with local, point-like, fields the dimensional quantities appear due to the spontaneous breaking of symmetry or in the dimensional transmutation in the renormalization group, but in the bare lagrangians.

[2]We mean the quantization with no anomalies.

$$+\ \frac{\mathrm{i}}{2}[\bar{\ }(\mathbf{x})\ \bar{\ }^{\mu}\nabla_{\mu}\ (\mathbf{x}) + \ (\mathbf{x})\ \ ^{\mu}\overline{\nabla}_{\mu}\ \bar{\ }(\mathbf{x})]\ \ , \qquad (1)$$

where we accept the usual notations in the textbook by Wess and Bagger [4], $\mathbf{h}^{\mu}_{m}$ is a tetrad, $\mathbf{g}_{\rm P1}$ is a coupling constant of the gauge interaction. The lagrangian in (1) is invariant under the local gauge transformations by the group SL$(2,\mathbb{C})$ acting on the Weyl spinors

$$\begin{aligned} {}' &= \mathbf{M}\ , & \bar{\ }' &= \bar{\ }\mathbf{M}^{\dagger}, \\ {}'^{\mu} &= \mathbf{M}\ \ ^{\mu}\mathbf{M}^{\dagger}, & \bar{\ }'^{\mu} &= [\mathbf{M}^{\dagger}]^{-1}\ \bar{\ }^{\mu}\mathbf{M}^{-1}, \end{aligned} \qquad (2)$$

whereas the infinitesimal transformations with the parameters ω_{mn}

$$\begin{aligned} \mathbf{M} &= 1 + \ ^{nm}\omega_{nm}, \\ [\mathbf{M}^{\dagger}]^{-1} &= 1 + \bar{\ }^{nm}\omega_{nm}, \end{aligned} \qquad \omega_{nm} \to 0, \qquad (3)$$

are given by the generators

$$\begin{aligned} [\ ^{nm}]_{\alpha}^{\ \beta} &= \frac{1}{4}\Big[\ ^{n}_{\alpha\dot\alpha}\ \bar{\ }^{m\dot\alpha\beta} - \ ^{m}_{\alpha\dot\alpha}\ \bar{\ }^{n\dot\alpha\beta}\Big], \\ [\bar{\ }^{nm}]^{\dot\alpha}_{\ \dot\beta} &= \frac{1}{4}\Big[\bar{\ }^{n\dot\alpha\beta}\ \ ^{m}_{\beta\dot\beta} - \bar{\ }^{m\dot\alpha\beta}\ \ ^{n}_{\beta\dot\beta}\Big]. \end{aligned} \qquad (4)$$

which satisfy the ordinary commutation relations for the spin operators. The covariant derivatives on the left-handed and right-handed chiral spinors

$$\begin{aligned} \nabla_{\mu\alpha}^{\ \ \ \beta} &= \ _{\alpha}^{\beta}\,\partial_{\mu} + \mathcal{A}_{\mu,nm}\ \ ^{nm\ \beta}_{\ \alpha}, \\ \overline{\nabla}_{\mu\ \dot\beta}^{\ \dot\alpha} &= \ ^{\dot\alpha}_{\dot\beta}\,\partial_{\mu} + \mathcal{A}_{\mu,nm}\ \bar{\ }^{nm\dot\alpha}_{\ \ \ \ \dot\beta}, \end{aligned} \qquad (5)$$

determine the strength tensor of gauge field $\mathcal{A}$

$$\begin{aligned} [\nabla_{\mu}, \nabla_{\nu}] &= \mathcal{F}_{\mu\nu,mn}\ \ ^{mn}, \\ [\overline{\nabla}_{\mu}, \overline{\nabla}_{\nu}] &= \mathcal{F}_{\mu\nu,mn}\ \bar{\ }^{mn}, \end{aligned} \qquad (6)$$

so that

$$\mathcal{F}_{\mu\nu,mn} = \partial_{\mu}\mathcal{A}_{\nu,mn} - \partial_{\nu}\mathcal{A}_{\mu,mn} + 2(\mathcal{A}_{\mu,mk}\mathcal{A}_{\nu,ln} - \mathcal{A}_{\nu,mk}\mathcal{A}_{\mu,ln})\,\mathbf{g}^{kl}. \qquad (7)$$

The Noether current, which follows from the gauge invariance under the change of equivalent representations of spinor algebra, has the form

$$\mathbf{j}^{\mu,nm} = \frac{\mathrm{i}}{2}[\bar{\ }(\mathbf{x})\ \bar{\ }^{\mu}\ \ ^{nm}\ (\mathbf{x}) + \ (\mathbf{x})\ \ ^{\mu}\ \bar{\ }^{nm}\ \bar{\ }(\mathbf{x})]. \qquad (8)$$

In the Pauli gauge for the -matrices, $^{\mu} = (1; \boldsymbol{\sigma})$, we get

$$
j^{\mu,nm} = \frac{1}{2}\; {}^{\mu n m \nu}\, j^{\lambda}\, g_{\nu\lambda}; \quad j^{\lambda} = {}^{-\;-\lambda} \;\; : \tag{9}
$$

Further, in the usual way we can derive the current conservation and the Slavnov–Taylor identities (see discussions in the extended version of [5]).

Introduce some nonzero vacuum fields leading to the spontaneous breaking of symmetry, so that

$$
\mathcal{F}_{\mu\nu,mn} \;=\; \mathcal{R}^{\mathrm{vac}}_{\mu\nu,mn} + \tfrac{1}{2}\mathcal{R}_{\mu\nu,mn}[\Gamma] - \tfrac{1}{2}\;{}_{\mu\nu}{}^{\alpha\beta}\,\tfrac{1}{2}\mathcal{R}_{\alpha\beta,mn}[\Gamma^{D}]; \tag{10}
$$

where we suppose the following ansatz:

$$
\mathcal{R}^{\mathrm{vac}}_{\mu\nu,mn} \;=\; -(g_{\mu m} g_{\nu n} - g_{\mu n} g_{\nu m})\, g^2_{\mathrm{Pl}} v^2 - \;{}_{\mu\nu mn} g^2_{\mathrm{Pl}} v^2; \tag{11}
$$

while Γ and Γ^D are dynamical fields redefined in accordance with $\Gamma_{\mu,nm} = 2\mathcal{A}_{\mu,nm}$, and $\mathcal{F}_{\mu\nu,mn}[\mathcal{A}] = \frac{1}{2}\mathcal{R}_{\mu\nu,nm}[\Gamma]$, so that $\mathcal{R}_{\mu\nu,nm}[\Gamma]$ represents the Riemannian tensor of curvature as we will see below. Then the contribution into the lagrangian linear in the vacuum fields[3], takes the form

$$
\mathcal{L}_G = -\frac{1}{2}\, v^2\, \mathcal{R}[\Gamma] - \frac{1}{2}\, v^2\, \mathcal{R}[\Gamma^D]; \tag{12}
$$

where we have introduced the Ricci tensor $\mathcal{R}_{\mu\nu}[\Gamma] = \mathcal{R}^{\gamma}{}_{\mu,\gamma\nu}[\Gamma]$ and the scalar curvature $\mathcal{R}[\Gamma] = \mathcal{R}_{\mu\nu}\, g^{\mu\nu}$. Following Palatini [6], we find that the variation of action over the connection gives zero covariant derivative of metric, i.e. it leads to the metric connection expressed in terms of Cristoffel symbols for both Γ and Γ^D, and they are coherent $\Gamma = \Gamma^D$. The variation over the metric gives the Einstein equations.

The lagrangian of (12) coincides with the Einstein–Hilbert lagrangian of general relativity, if we put $v^2 = \frac{1}{16\pi G};$ where G is the gravitational constant, and then

$$
\mathcal{L}_G = -\frac{1}{16\;\; G}\, \mathcal{R}[\Gamma]: \tag{13}
$$

Let us stress the most important features of suggested mechanism for the spontaneous breaking of gauge symmetry:

1. The vacuum expectation value of (11) is covariant, and it preserves the symmetry with respect to general transformations of coordinates.

[3]We remove the terms like $v^2\epsilon^{\mu\nu mn}\mathcal{R}_{\mu\nu mn}$, since they are equal to zero with the symmetric connection in the world indices.

2. The expression of (11) represents the sum of the Riemannian tensor for the space-time with the constant curvature (the first term) and that of dual one.

3. The field of (11) can satisfy the gauge-field equations at zero external sources, if the covariant derivative of metric is equal to zero.

4. The cosmological constant caused by the term quadratic in the vacuum strength-tensor, is equal to zero.

3 Conclusion

In this paper we have considered an ansatz for the vacuum state leading to the spontaneous breaking of gauge symmetry in the theory with the spin connection. To the linear approximation in the background vacuum fields this mechanism leads to the Einstein–Hilbert action for the gravitons. Therefore, we find the opportunity to solve the problem on the formulation of quantum gravity in the framework of local field theory.

If there are no anomalies, the gauge invariance insures the renormalizability of the theory. However, we have to point out the necessity of investigations concerning for massive modes appearing under the spontaneous breaking of symmetry, the canonical quantization and the role of duality, which is discussed in the extended version [5]. In addition, the nonabelian character of gauge group leads to the asymptotic freedom of running coupling constant in the renormalization group. Thus, the principal idea on the quantum gravity supposed in this paper demands a further consideration.

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