Effective expansion parameter near the phase transition temperature in the O(N)-model

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Abstract

The temperature induced phase transition in the N-component scalar field theory with spontaneous symmetry breaking is investigated in the perturbative approach. The second Legendre transform is used together with the consideration of the gap equations in the extrema of the free energy. Resummations are performed on the super daisy level. The phase transition turns out to be of a weak first order. For large N, a new effective expansion parameter, $(1/N)^{1/3}$, is found giving the opportunity to account for higher graphs perturbatively. In the limit N goes to infinity the phase transition becomes second order. A comparison with other approaches is done.

1 Introduction

Investigations of the phase transition in the N-component ϕ^4-theory at finite temperature have been carried out for many years by various methods. The physical importance of the theory arises from the fact that for $N = 4$ it describes the scalar sector of the standard electroweak theory and for some larger N corresponds to its different extensions. For a negative bare mass squared the theory exhibits a spontaneous breaking of the $O(N)$ symmetry to $O(N-1)$. At sufficiently high temperature the original symmetry is restored [1, 2]. The type of the

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phase transition determined in perturbative (see, for example, [3, 4, 5] and references therein) and non perturbative (lattice simulations, average action) methods [6, 7, 8, 9, 10], large-N expansion is different. In the former case a first order transition was determined whereas in the latter one it has been found to be of second order. As a general believe, one finds the cause of the discrepancy in the breakdown of the perturbative expansion near the critical temperature. In papers [11, 4, 12] dealing with perturbative methods the so-called "daisy" diagrams have been summed up. In papers [13, 14] by an auxillary-mass method it was found that the phase transition in either the $O(1)$- or the $O(N)$-model is of second order. We would like to mention that the calculated in this way effective potential has an imaginary part in its minima. This means an inconsistency of calculation procedure adopted therein.

In our previous paper [15] a detailed analysis of the phase transition in $O(1)$-model in the "super daisy" resummations and beyond has been done. To elaborate that a new method combining the second Legendre transform with considering the gap equation in the extrema of the free energy functional was developed. This allows for elimination of the condensate as a variable from the gap equation and considerably simplifies computations. Whithin this approach we found the first order phase transion and the expected disappearance of an effective expansion parameter near the phase transition temperature.

In the present paper we apply this method to the $O(N)$-model. We shall show in what follows that investigation of the gap equations in the super daisy approximation in the minimum of the free energy results in a first order phase transition for any finite N. It turns into a second order one in the limit of N goes to infinity. The observed phase transition is very close to a second order one and differs from it in the next-to-leading order in $1/N$ for finite but large N. For large N the effective expansion parameter $(\frac{1}{N})^{1/3}$ is derived that gives possibility for perturbative calculations near the phase transition transition temperature.

2 General consideration

We consider a scalar N-component field theory in $(3 + 1)$ dimensions in the Eucleadean version. The action reads

$$S[\phi] \quad = \quad \int \mathrm{d}x \left(\frac{1}{2}\phi(x)\mathbf{K}\phi(x) - \frac{\lambda}{4N}(\phi(x))^4 \right), \tag{1}$$

where $\mathbf{K} = \Box - m^2$ is the kernel of the free action. The sign is chosen so that the vacuum Green functions are given by the functional representation

$$Z \equiv e^W = \int D\phi \; e^S \, , \tag{2}$$

where W is the functional of the connected Greens functions. It is connected by

$$W = -FT \tag{3}$$

with the free energy F. We consider the theory at finite temperature T in the imaginary time formalism so that the loop integrations are given by

$$Tr_p = T \sum_{l=-\infty}^{\infty} \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3} \tag{4}$$

with the momentum $p = (2\pi Tl, \vec{p})$ $(l \in Z)$. The symbol ϕ is here a condensed notation for N scalar fields, $\phi = \{\phi_1, \phi_2, \ldots\}$. In the action (1) the following summation over the internal indicees is assumed: $\phi^2 = \sum_{a=1}^N \phi_a^2$ and $\phi^4 = \left(\sum_{a=1}^N \phi_a^2\right)^2$. Renormalization we perform at zero temperature.

We introduce the spontaneous symmetry breaking by turning the sign of the mass term in the free propagator, $m^2 \to -m^2$ in Δ, and $\Delta = -K^{-1}$ is the free propagator. It reads now $\Delta = p^2 - m^2$ in the momentum representation. Then, on the tree level, the minimum of the energy can be reached by shifting the fields, $\phi_1 \to \eta + v$, where v is the condensate. After that we change the notations according to $\{\phi_1, \phi_2, \ldots\} \to \{\eta + v, \phi_1, \phi_2, \ldots\}$, where η is the Higgs field and $\{\phi_1, \phi_2, \ldots\}$ $(a = 1, 2, \ldots, N-1)$ are the Goldstone fields which are symmetric under the residual $O(N-1)$ symmetry.

After that the action reads

$$\begin{aligned} S[\eta, \Phi] &= \int dx \left\{ \frac{m^2}{2} v^2 - \frac{\lambda}{4N} v^4 + \eta(m^2 - \frac{\lambda}{N} v^2) v \right. \\ &\quad + \frac{1}{2}\eta(\Box - \mu_\eta^2)\eta + \frac{1}{2}\phi(\Box - \mu_\phi^2)\phi \\ &\quad \left. - \frac{\lambda}{4N} \left(\eta^4 + 4\eta^3 v + 2\left(\eta^2 + 2\eta v\right)\phi^2 + \phi^4 \right) \right\} \end{aligned} \tag{5}$$

with $\mu_\eta = -m^2 + 3\frac{\lambda}{N} v^2$ and $\mu_\phi = -m^2 + \frac{\lambda}{N} v^2$. Again, the sum over the internal indicees is assumed. In momentum representation, the corresponding free propagators are

$$\Delta_\eta = p^2 + \mu_\eta^2 \qquad \text{and} \qquad \Delta_\phi = p^2 + \mu_\phi^2. \tag{6}$$

The propagator for the Goldstones is diagonal, $(\Delta_\phi)_{ab} = \delta_{ab}\, \Delta_\phi$.

With these notations, the second Legendre transform results in the representation for the connected vacuum Green functions

$$\begin{aligned} W &= S[0] + \frac{1}{2} Tr \log \beta_\eta + \frac{N-1}{2} Tr \log \beta_\phi \\ &\quad - \frac{1}{2} Tr \Delta_\eta^{-1} \beta_\eta - \frac{N-1}{2} Tr \Delta_\phi^{-1} \beta_\phi + W_2[\beta_\eta, \beta_\phi], \end{aligned} \tag{7}$$

where $W_2[\beta_\eta, \beta_\phi]$ is the sum of all two particle irreducible (2, I) graphs taken out of the connected Green functions with the full propagators β_η and β_ϕ on the lines correspopnding to the fields η and ϕ. The propagators β_η and β_ϕ are subject to the

Schwinger-Dyson (SD) equations. In the considered case of a broken symmetry, two equations remain:

$$\begin{aligned}
\beta_\eta^{-1}(p) &= \Delta_\eta^{-1} - \Sigma_\eta[\beta_\eta, \beta_\phi](p), \\
\beta_\phi^{-1}(p) &= \Delta_\phi^{-1} - \Sigma_\phi[\beta_\eta, \beta_\phi](p),
\end{aligned} \tag{8}$$

where the self energy functionals are given by

$$\begin{aligned}
\Sigma_\eta[\beta_\eta, \beta_\phi](p) &= 2\frac{\delta W_2[\beta_\eta, \beta_\phi]}{\delta\beta_\eta(p)}, \\
\Sigma_\phi[\beta_\eta, \beta_\phi](p) &= \frac{2}{N-1}\frac{\delta W_2[\beta_\eta, \beta_\phi]}{\delta\phi_\eta(p)}.
\end{aligned} \tag{9}$$

Here, the simple relation $(\Sigma_\phi)_{ab} = \delta_{ab}\Sigma_\phi = 2\frac{\delta W_2}{\delta(\beta_\phi)_{ab}} = \delta_{ab}\frac{2}{N-1}\frac{\delta W_2}{\delta\beta_\phi}$ was taken into account.

At sufficiently high temperature the symmetry is restored. This means that the value of the condensate is zero, $v = 0$, and the masses of the Higgs and the Goldstone particles are equal, $M_\eta = M_\phi$. Then the SD-equations (8) reduce to one equation. At low temperature the symmetry is broken and we have a non zero value of v, the masses are different and we have two equations, Eqs. (8), to consider. The free energy, then, is a function of the condensate v and has a minimum at $v > 0$ which can be seen already on the tree level. It may have more extrema. In that case due to the continuity of the free energy as a function of v an additional extremum for $v > 0$ must be a maximum indicating a first order phase transition.

Following [15], we consider the free energy in its extrema. We take the derivative of W with respect to the condensate v^2,

$$\frac{d}{dv^2}W[\beta_\eta, \beta_\phi] = \frac{m^2}{2} - \frac{\lambda}{2N}v^2 - \frac{3\lambda}{2N}\mathrm{Tr}\beta_\eta - \frac{\lambda(N-1)}{2N}\mathrm{Tr}\beta_\phi. \tag{10}$$

In general, in this expression there are contributions proportional to $\partial\beta_\eta/\partial v^2$ and $\partial\beta_\phi/\partial v^2$. But they vanish by means of the SD-equations (8). Here, we also have taken into consideration that in the super daisy approximation of interest $W_2[\beta_\eta, \beta_\phi]$ does not depend on v^2 (see below Eq.(13)).

Demanding now the derivative given by Eq. (10) to vanish,

$$\frac{dW[\beta_\eta, \beta\phi]}{dv^2} = 0, \tag{11}$$

we obtain the following equation

$$\frac{\lambda}{N}v^2 = m^2 - 3\frac{\lambda}{N}\mathrm{Tr}\beta_\eta - \frac{\lambda(N-1)}{N}\mathrm{Tr}\beta_\phi, \tag{12}$$

which has to be considered together with the SD-equations (8).

Now we turn to the approximation of "super daisy" resummations. It is given by keeping in the functional W_2 graphs with one vertex only,

$$W_2^{\rm SD} = -\frac{3\lambda}{4N}(\Sigma_\eta^{(0)})^2 - \frac{\lambda}{2}\frac{N-1}{N}\Sigma_\eta^{(0)}\,\Sigma_\phi^{(0)} - \frac{\lambda}{2}\frac{N^2-1}{N}(\Sigma_\phi^{(0)})^2. \tag{13}$$

Here we introduced the notations $\Sigma_\eta^{(0)} = Tr\beta_\eta$ and $\Sigma_\phi^{(0)} = Tr\beta_\phi$ for the graphs consisting of one line closed over one vertex.

Now we turn to the SD equations. In the given approximation, the self energy graphs in the rhs of Eqs. (8) are independent of the momentum p. Therefore, the simple ansatz

$$\beta_\eta^{-1} = p^2 + M_\eta^2, \qquad \beta_\phi^{-1} = p^2 + M_\phi^2 \tag{14}$$

for the propagators becomes exact and the SD equations reduce to gap equations for the corresponding masses.

In the restored phase we have $v = 0$ and the masses are equal, $M_\eta = M_\phi \equiv M_r$. There is only one equation

$$M_r^2 = -m^2 + \lambda\frac{N+2}{N}\Sigma^{(0)}, \tag{15}$$

where in $\Sigma^{(0)}$ the propagator $\beta = 1/(p^2 + M_r^2)$ has to be inserted.

In the broken phase in the extremum of of the free energy the condensate is given by Eq. (12) and we rewrite Eqs. (8) as

$$M_\eta^2 = -m^2 + \frac{3\lambda}{N}v^2 + \frac{3\lambda}{N}\Sigma_\eta^{(0)} + \lambda\frac{N-1}{N}\Sigma_\phi^{(0)}, \tag{16}$$

$$M_\phi^2 = -m^2 + \frac{\lambda}{N}v^2 + \frac{\lambda}{N}\Sigma_\eta^{(0)} + \lambda\frac{N+1}{N}\Sigma_\phi^{(0)}. \tag{17}$$

The Eq. (12) resulting from the condition of the extremum takes the form

$$\frac{\lambda v^2}{N} = m^2 - \frac{3\lambda}{N}\Sigma_\eta^{(0)} - \lambda\frac{N-1}{N}\Sigma_\phi^{(0)}. \tag{18}$$

Now we use Eq. (18) to rewrite the first gap equation, (16), in the form

$$M_\eta^2 = 2\left(m^2 - \frac{3\lambda}{N}\Sigma_\eta^{(0)} - \lambda\frac{N-1}{N}\Sigma_\phi^{(0)}\right), \tag{19}$$

In addition, by means of Eq. (16), we rewrite Eq. (18) as

$$\frac{\lambda}{N}v^2 = \frac{M_\eta^2}{2}. \tag{20}$$

The second equation can be rewritten in a similar way. We obtain in this case

$$M_\phi^2 = \frac{2\lambda}{N}\left(\Sigma_\phi^{(0)} - \Sigma_\eta^{(0)}\right). \tag{21}$$

In this way all necessary general formulas are collected and we turn to the solution of these equations.

3 Solution of gap equations

The solution of gap equations (15), (19) and (21) can be investigated numerically for any λ and T. We are interested in small bare coupling λ. Near the phase transition the temperature is large, $T \sim 1/\sqrt{\lambda}$. Hence we take the known higt temperature expansion for the function $\Sigma^{(0)}$,

$$\Sigma^{(0)} = \frac{T^2}{12} - \frac{MT}{4\pi} + \ldots, \tag{22}$$

where the corresponding masses M_r, M_η or M_ϕ have to be inserted. In Eq. (22), the dots denote contributions suppressed by higher powers of M/T. In this approximation the gap equations read in the restored phase

$$M_r^2 = -m^2 + \frac{\lambda(N+2)T^2}{12N} - 2M_r \frac{\lambda(N+2)T}{8\pi N} \tag{23}$$

and in the broken phase

$$\begin{aligned} M_\eta^2 &= 2m^2 - \frac{\lambda(N+2)T^2}{6N} + 2M_\eta \frac{3\lambda T}{4\pi N} + M_\phi \frac{\lambda(N-1)T}{2\pi N}, \\ M_\phi^2 &= \frac{\lambda T}{2\pi N}\left(M_\eta - M_\phi\right). \end{aligned} \tag{24}$$

In the restored phase, Eq. (23) can be solved simply with the result

$$M_r = -\frac{\lambda(N+2)T}{8\pi N} + \sqrt{\left(\frac{\lambda(N+2)T}{8\pi N}\right)^2 - m^2 + \frac{\lambda(N+2)T^2}{12N}}. \tag{25}$$

We observe that it has a single solution for positive mass as long as $T > T_-$, where

$$T_- = \sqrt{\frac{12N}{\lambda(N+2)}}\, m \tag{26}$$

is the lower spinodal temperature.

In the broken phase, the two gap equations (24) form a system of algebraic equations. To solve it we proceed as follows. The second equation can be solved with respect to M_ϕ,

$$M_\phi(M_\eta) = -\frac{\lambda T}{4\pi N} + \sqrt{\left(\frac{\lambda T}{4\pi N}\right)^2 + \frac{\lambda T}{2\pi N} M_\eta}. \tag{27}$$

This is a unique solution $M_\phi(M_\eta)$ for all T. Note the special value $M_\phi(0) = 0$ independent of T.

Now we insert $M_\phi(M_\eta)$ from (27) into the first equation in (24),

$$M_\eta^2 = 2m^2 - \frac{\lambda(N+2)T^2}{6N} + 2M_\eta \frac{3\lambda T}{4\pi N} + \frac{\lambda(N-1)T}{2\pi N} M_\phi(M_\eta). \tag{28}$$

N	0	1	2	3	4	5
t_N	0.0569932	0.029511	0.0203949	0.0158314	0.0130797	0.0112318

Table 1: Some numerical values of the coefficients t_N in Eq. (29)

This equation has an explicit solution which is, however, too large to be displayed here. It can be easily plotted and is shown in Figure 1. We have chosen $\lambda = 1$ in order to make the details near the phase transition better visible. But we can obtain the basic properties of this solution in the follwing way. Assume we plot the rhs. versus lhs. of Eq. (28). We observe that for small T there is one solution. At $T = T_-$ a second solution appears and at some temperature T_+ both solutions merge and disappear. So T_+ has to be interpreted as the upper spinodal temperature. In this way we observe a first order phase transition. Once we have an algebraic solution of the gap equations, we can give an algebraic expression for T_+. Again, it is too complicated to be displayed. However, for small λ, T_+ can be expressed as

$$\frac{T_+}{T_-} = 1 + t_N \lambda + O(\lambda^2), \tag{29}$$

where the numbers t_N are algebraic expressions in N. Some numerical values are given in Table 1. For $N = 1$ we reobtain with $t_1 = \frac{9}{16\pi^2}$ the known result from the one component case in [15].

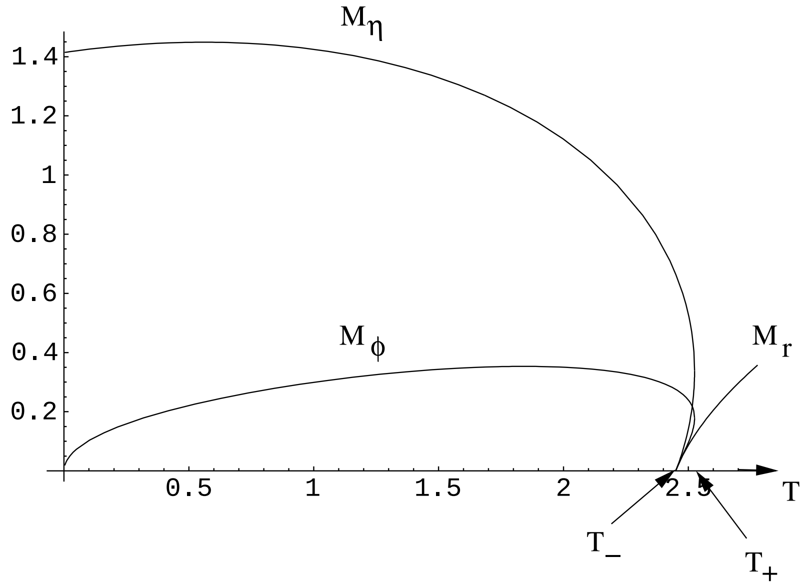

Figure 1: The masses M_η, M_ϕ and M_r as function of the temperature T for $\lambda = 1$, $N = 2$. The upper parts of the curves for M_η and M_ϕ show the corresponding masses in the minimum of the free energy, the lower parts show the masses in the maximum.

Now, having solved the gap equation, we consider the functional W. Let

W_b be its value in the minimum (broken phase) with the masses inserted from the solutions discussed above and the condensate v from Eq. (18). Let $W_r = W(0, M_r, M_r)$ be the corresponding value in the restored phase. Again, the analytic expressions are too large to be displayed. So, we restrict us to some numerical values. For example, for $\lambda = 1$ and $N = 2$ we have $T_- = 2.44949$ and $T_+ = 2.52514$. In $T = T_-$ we have $W_b = -8.15081$ and $W_r = -8.14568$ (note that the free energy differs in sign from W, see Eq. (3)). In $T = T_+$ the corresponding values are $W_b = -8.15081$ and $W_r = -9.1671$. The temperature T_c of the phase transition follows from $\Delta W \equiv W_b - W_r = 0$ to be $T_c = 2.467$.

4 Large N limit

In this section we investigate the limit of large N within the super daisy approximation. Here explicite results can be obtained. We start from the gap equations, (28), where the solution (27) of the second equation had been inserted. It is useful to introduce the following notations

$$\mu = \frac{8\pi N}{\lambda T} M_\eta, \tag{30}$$

$$h_0 = \left(\frac{8\pi N}{\lambda T}\right)^2 \left(2m^2 - \frac{6(N+2)T^2}{6N} - \frac{\lambda^2(N-1)T^2}{8\pi^2 N^2}\right). \tag{31}$$

Now Eq. (28) can be rewritten in the form

$$f(\mu) = 0 \tag{32}$$

with

$$f(\mu) = -\mu^2 + 12\mu + h_0 + 8(N-1)\sqrt{\mu+1}. \tag{33}$$

We consider this equation at $T = T_+$. At this temperature the two solutions existing for $T_- < T < T_+$ describing the minimum and the maximum of the free energy, merge. So, two solutions of equation (32) coincide.

It is easy to obtain the expansion of the solution for large N. We find h_0 from the first equation (32):

$$h_0 = -3(2N)^{\frac{4}{3}} - 14(2N)^{\frac{2}{3}} + \dots. \tag{34}$$

For T_+ following from Eq. (31) we get

$$T_+ = \sqrt{\frac{12}{\lambda}} m \left(1 + \frac{9\lambda}{16\pi^2} \frac{1}{(2N)^{\frac{2}{3}}} + \dots\right). \tag{35}$$

Together with Eq. (26) we can write down it as follows

$$\frac{T_+}{T_-} = 1 + \frac{9\lambda}{16\pi^2} \frac{1}{(2N)^{\frac{2}{3}}} + \dots. \tag{36}$$

This gives at once the limit of the coefficients t_N in Eq. (29) for large N.

Now we rewrite Eq. (30) as

$$M_\eta = \frac{\lambda T}{8\pi N}\mu$$

and find for the Higgs boson mass

$$M_\eta = \frac{\sqrt{3\lambda}}{2\pi}\frac{m}{(2N)^{\frac{1}{3}}} + \dots \tag{37}$$

and from Eq. (27) the Goldstone boson mass

$$M_\phi = \frac{\sqrt{3\lambda}}{2\pi}\frac{2m}{(2N)^{\frac{2}{3}}} + \dots\ . \tag{38}$$

In this way we obtained the asymptotic expansions of all quantities in the limit of large N at $T = T_+$. From Eq. (36) it follows that the gap between the two spinodal temperatures closes at $N \to \infty$. So, the phase transitions becomes second order in that limit in agreement with leading order in the standard treatment of the $1/N$-expansion.

Now we discuss the effective expansion parameter of a perturbative series near the phase transition. We remind that by means of solving of the gap equation we made an infinite resummation of the perturbative expansion, where, of course, a remainder with a still infinite number of graphs is left unsummed. It is generated by all graphs in W_2 not contained in $W_2^{\rm SD}$. Let us estimate a generic graph of this kind. Let V, C resp. L be the number of vertices, loops resp. lines. These numbers are connected by $C = L - V + 1$ and, because the graphs of W_2 do not have external lines, by $2V = L$ if we restrict ourselves to graphs with quartic vertices, only. Now, for small λ, and, according to Eq. (26) high T, we take in leading order the zeroth Matsubara frequency. Then we rescale the 3-dimensional momenta by means of $\vec{p} \to M\vec{p}$. The factors in front of a graph are $\left(\frac{\lambda}{N}\right)^V$ from the vertex factors, T^C from (4) and M^{3C-2L} from $\mathrm{d}\vec{p}$ and from the lines,

$$\left(\frac{\lambda}{N}\right)^V T^C M^{3C-2L} \sim \left(\frac{\lambda}{N}\right)^V \sqrt{\lambda}^C \left(\frac{\sqrt{\lambda}}{N^{\frac{2}{3}}}\right)^{3C-2L} = \lambda^1\ \left(\frac{1}{N}\right)^{\frac{1}{3}V+2}, \tag{39}$$

where we inserted T_+ from Eq. (35) and M_ϕ from Eq. (38).

From Eq. (39) we have two conclusions.

1. λ does not provide a good expansion parameter near the phase transition. This is the same as we observed in [15] in the one component case and is generally quite well known.

2. $\left(\frac{1}{N}\right)^{\frac{1}{3}}$ provides a good expansion parameter as it goes to the power of the number of vertices. Therefore, the remaining uncalculated part of the perturbative expansion can be considered as small. Note that this power $\frac{1}{3}$ is a non-perturbative effect that is in no way connected with the standard $1/N$-expansion.

We made the estimation at T_+. As it can be shown, in the range of $T_- < T < T_+$ the masses are heavier than at T_+ so that T_+ is the 'worst case'.

Conclusions

In the super daisy approximation for small coupling λ we obtained clearly a first order phase transition. It should be noticed that the first order character is quite weak. The change in the free energy between T_c and T_- is numerically small. Also, the difference between the two spinodal temperatures, Eq. (29), is of order λ. So, to leading order in λ the transition is of second order and the first order character is a next-to-leading effect in λ.

The next step of approximation was done when we considered the SDA for large N. All interesting quantities turn out to be expandable in powers of $\left(\frac{1}{N}\right)^{\frac{1}{3}}$. Again, as for $\lambda \to 0$ so here for $N \to \infty$ the transition becomes second order so that the first order character is also next-to-leading in $N \to \infty$. The most interesting result in this section is on the effective expansion parameter appearing in this approximation for the infinite set of diagrams left uncalculated. Near the phase transition, it is shown to be at least $\left(\frac{1}{N}\right)^{\frac{1}{3}}$ in the sense that each graphs goes with a factor $\lambda^1 \left(\frac{1}{N}\right)^{\frac{1}{3}C}$ where C is the number of loops. So with respect to λ there is no good expansion parameter but with respect to $\left(\frac{1}{N}\right)^{\frac{1}{3}}$ there is one. The existence of this effective expansion parameter is a quite strong indication that the phase transition is indeed first order starting from the next-to-leading order for large N. Really, the graphs left beyond the super daisy approximation can, in principle, be accounted for in perturbation theory in $(1/N)^{1/3}$ that could not change qualitatively the character of the phase transition.

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